# Investigation of decay process $\psi(2s) \to \pi^+\pi^-$ and branching ratio calculation

N. Ghahramany* and R. Khosravi

*Physics Department , Shiraz University, Shiraz 71454, Iran*

## Abstract

According to existing PDG information, the branching ratio for $\psi(2s) \to \pi^+\pi^-$ is equal to $(8 \pm 5) \times 10^{-5}$ [1]. Recently experimental groups [2, 3, 4] have reported two different values for this decay namely $(0.8 \pm 0.8 \pm 0.2) \times 10^{-5}$, $0.84 \pm 0.55^{+0.16}_{-0.35} \times 10^{-5}$ and $1.8 \times 10^{-5}$. In this paper it is attempted to study $\psi$-decay process theoretically via Feynman diagrams and it was concluded that the only effective way that $\psi$ can decay into $\pi^+$ and $\pi^-$ mesons is via one photon emission in an electromagnetic process. Such conclusion is based upon the fact that color conservation is violated in one-gluon mechanism and charge conjugation is violated in two-gluon mechanism and G-parity and isospin is violated in three-gluon mechanism. By presenting the Feynman diagram, the branching ratio of $\psi$-decay is calculated. The obtained value from our calculations is in good agreement with the recently reported experimental value.

* email address: ghahramany @ susc.ac.ir

## I. INTRODUCTION

The bound state of a heavy non relativistic quark and its anti quark is called quarkoniom. Therefore the bound state of a charm and anti-charm form charmonium. The charmonium spectrum is denoted by $n^{2S+1}L_J$ in which $n = 1, 2, \cdots$ is the principal quantum number and $L = S, P, D, \cdots$ is the orbital angular momentum, $S$ is the total spin and $J$ is the total angular momentum. Parity and charge conjugation parity are defined as:

$$P = (-1)^{L+1}, C = (-1)^{L+S}. \tag{1}$$

If $\hat{C}$ is charge conjugation operator then:

$$\hat{C}|M, p, J, \lambda; B, Q, L_e, N_\mu >= \eta_c|M, p, J, \lambda; -B, -Q, -L_e, -N_\mu >, \tag{2}$$

where the symbols in the ket stand respectively for mass, momentum, angular momentum quantum number, helicity, baryon number, charge, lepton and muon numbers. It is further assumed that $\hat{C}$ operator commutes with both strong and electromagnetic Hamiltonian. In other words $\hat{C}$ is a symmetry operator for strong and electromagnetic interactions [5].

Isospin and G-parity are also used to specify charmonium state. For example for $\psi(2s)$ or $J/\psi$ we write $J^G(J^{PC}) = 0^-(1^{--})$.

The isospin group play a significant role in particle strong interactions. let us consider the QCD lagrangian density:

$$\mathcal{L} = \sum_{f=1}^{N} \bar{\psi}_f(i\gamma^\mu D_\mu - m_f)\psi_f - \frac{1}{4}F^j_{\mu\nu}F^{\mu\nu}_j, \tag{3}$$

$$D_\mu = \partial_\mu + \frac{i}{2}g\lambda_l A^l_\mu, \tag{4}$$

$$F^j_{\mu\nu} = \partial_\mu A^j_\nu - \partial_\nu A^j_\mu - g f_{jkl} A^k_\mu A^l_\nu, \tag{5}$$

where $f$ is for flavor index, $N$ is the number of flavor, $g$ is the gauge coupling constant, $A^l_\mu$ is the space-time component of the $l^{th}$ gluon vector potential, $\lambda_l$ are the eight Gell-Mann matrices and $f_{jkl}$ are the SU(3) group structure constants. If the mass difference of quark could be ignored, then the given lagrangian is invariant under $\psi_j = U_{jk}\psi_k$ , where $jk$ are flavor indices and $U_{jk}$ is a unitary matrix; for the $(u, d, s)$ quarks this implies approximate flavor SU(3) symmetry, and isospin invariance for the lightest quarks $(u, d)$ [6]. In strong interactions isospin conservation is assumed but the mass difference of light quarks $u$ and $d$

is considered to be the cause of isospin non conservation in strong interactions [7]. Therefore as has been known, the strong interaction amplitude is reduced by $(m_u^0 - m_d^0)/Q$ factor due to isospin break down [8], where $Q$ is a proper energy scale in strong interaction.

G-parity, as a transformation in isospin space is an internal quantum number, which is useful in explaining some processes such as $\omega, \rho \to \pi\pi$ or $\pi\pi\pi$ , by inverting the unit vectors in isospin space. The internal symmetry is expressed in terms of G-parity; therefore is conserved in strong interactions [9]. Each meson possess either positive or negative G-parity, therefore it can decay into other meson combinations that have an overall positive or negative G-parity. The concept of isospin and G-parity will be used in $\psi(2s) \to \pi^+\pi^-$ process in the next argument.

### A. Charmonium transitions

Stability for charmonium means that $c\bar{c}$ can be explained by experimental based OZI-rule, that has also been accepted on the basis of QCD, state that [10,11,12]: The quarks contained in the incoming particles are distributed over the particles in the final state of the reaction. In other words this rule state that a continuous quark diagram is more probable to occur than a discontinuous one .

The decay of a spin-1 meson such as $\psi(2s)$ in strong interaction via OZI-rule breaking mode by quark-antiquark annihilation and 3 gluon production is explained. In $\psi(2s) \to \pi^+\pi^-$ decay the process proceed via OZI-rule breaking mode because pion does not contain quark $c$. In a diagram such as in Fig.1 Up, the initial and final states, are connected by one gluon exchange; but due to non conservation of color does not occur. Two gluon exchange is possible with proper choice of color but is not likely due to charge conjugation parity violation for $\psi(2s)$ with $J^{PC} = 1^{--}$. Therefore, the simplest possible case for $\psi(2s)$ decay in strong mode is via three gluon exchange. Each virtual gluon corresponds to a $\alpha_s$ factor in Feynman diagrams. So for 3 gluon exchange a factor of $\alpha_s^3$ appears. In energy range of nearly 3.5 GeV ($M_{\psi(2s)} \simeq 3.5$GeV) the value of $\alpha_s = 0.2$ . As energy increases, $\alpha_s$ decreases continuously. Therefore QCD help us to understand this suppression mechanism (in OZI-rule breaking decays). The charmonium decay mechanisms include annihilation processes, radiative transitions and hadronic transitions. Here we study the annihilation and the radiative transition modes. Therefore a charmonium in state $n^3S_1$ can decay via

3 gluon or 2 gluon and one photon or one photon or 3 photon in to a psudoscalar meson and the corresponding anti-meson. The decay by emitting one or 3 photon in QED and by emitting 3 gluon in QCD interaction is shown in Fig. 1.

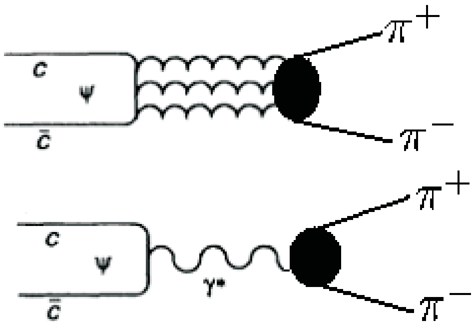


FIG. 1: Effective Feynman diagrams $\psi$ decay to $\pi^+$ and $\pi^-$. Up: Three gluons contribution. Down: One photon contribution.

Therefore the decay amplitude of $\psi(2s) \to \pi^+\pi^-$ is:

$$A_{\psi(2s)} \longrightarrow A_\gamma + A_{ggg} + A_{\gamma\gamma\gamma} + A_{\gamma gg}. \tag{6}$$

### B. The dominant process in $\psi(2s) \to \pi^+\pi^-$ decay

Following reasons are stated to convince one that the dominant process in $\psi(2s) \to \pi^+\pi^-$ decay is via one photon exchange:

**I)** In QED each virtual photon corresponds to a $\alpha_E$ factor in Feynman diagrams. So for 3 photon exchange a factor of $\alpha_E^3$ appears where the value of $\alpha_E = 1/137$. So it can safely be ignored compare to one photon mode.

**II)** The decay rate of each of the above modes (Eq.(6)) including the QCD radiation corrections are summarized in table 1.

Considering the perturbative QCD prediction, the comparison of $\Gamma_{gg\gamma}$ and $\Gamma_{ggg}$ up to $1^{st}$ order corrections indicate that:

$$\frac{\Gamma_{gg\gamma}}{\Gamma_{ggg}} = (\frac{51\alpha}{16\alpha_s})(\frac{1 - \frac{0.9\alpha_s}{\pi}}{1 + \frac{4.9\alpha_s}{\pi}}) \simeq 0.049, \tag{7}$$

we have taken $0.25 < \alpha_s < 0.35$ . Therefore on the basis of comparison $\Gamma_{gg\gamma}$ can be ignored compare to three gluon exchange which is a strong one.

TABLE I: Lowest order width expressions and the first order QCD corrections for $c\bar{c}$ decay [13,14].

| *Process* | *Width* | $1^{st}$ *order QCD correction* |
|---|---|---|
| $^3S_1 \rightarrow e^+e^-$ | $\frac{64\pi\alpha^2}{9}\frac{\vert\Psi(0)\vert^2}{(2m_c)^2}$ | $1-\frac{16\alpha_s}{3\pi}$ |
| $^3S_1 \rightarrow \gamma\gamma\gamma$ | $\frac{4096\alpha^3(\pi^2-9)}{2187}\frac{\vert\Psi(0)\vert^2}{(2m_c)^2}$ | $1-\frac{12.6\alpha_s}{\pi}$ |
| $^3S_1 \rightarrow ggg$ | $\frac{160\alpha_s^3(\pi^2-9)}{81}\frac{\vert\Psi(0)\vert^2}{(2m_c)^2}$ | $1+\frac{4.9\alpha_s}{\pi}$ |
| $^3S_1 \rightarrow gg\gamma$ | $\frac{512\alpha_s^2\alpha(\pi^2-9)}{81}\frac{\vert\Psi(0)\vert^2}{(2m_c)^2}$ | $1-\frac{0.9\alpha_s}{\pi}$ |

**III)** As mentioned in previous section G-parity is considered to be conserved in strong interactions where as in $\psi(2s) \rightarrow \pi^+\pi^-$ decay via three gluon which is a strong interaction, G-parity conservation is violated ie:

$$\psi(0^-) \rightarrow \pi^+(1^-)\pi^-(1^-). \tag{8}$$

Therefore this decay either does not occur or it happens with ignorable probability [15].

**IV)** Because the $\psi(2s) \rightarrow \pi^+\pi^-$ violates isospin , this purely hadronic process can proceed only via the isospin breaking parameter $(m_d - m_u)/Q$ which appears explicity in the QCD Lagrangian[16]. Such an amplitude should therefore be suppressed by the small dimensionless factor $(m_d - m_u)/Q$ with $Q$ some typical momentum in the problem. Rather than relying on any explicit model dependent calculation, we present the following more general argument by comparing with the SU(3) analog process $\psi(2s) \rightarrow K\bar{K}$. Since the $\psi(2s) \rightarrow K\bar{K}$ decay violates SU(3) symmetry, because of the mass difference, the corresponding purely hadronic decay amplitude $A_\pi^{(ggg)}$ will have in this case the explicit small SU(3) breaking suppression factor $(m_s - m_{u,d})/Q$ [17]. Consequently we expect that:

$$\frac{A_\pi^{(ggg)}}{A_K^{(ggg)}} \approx \frac{m_d - m_u}{m_s - m_{u,d}} \approx 0.02 - 0.03, \tag{9}$$

where in the spirit of the Vafa-Witten theorem [16] we used the values of Lagrangian or "current" quark masses in estimating the above ratio. There are two $K\bar{K}$ decay modes, $\psi(2s) \to K^0\overline{K}^0$ (or$K^0_S K^0_L$ ) and $\psi(2s) \to K^+K^-$ . The amplitude $A_K^{(ggg)}$ is simply given in the form of:

$$A_K^{(ggg)} \approx A^{\psi(2s)\to K^0_S K^0_L}. \tag{10}$$

The point is that the one photon and $gg\gamma$ contributions to the $\psi(2s) \to K^0_S K^0_L$ decay also vanish in the SU(3) limit due to canceling contribution of $s, \bar{d}$ quarks of opposite charge [18]. Multiplying Eq.(9) with the observed branching rate

$$Br(\psi(2s) \to K^0_S K^0_L) = (5.2 \pm 0.7) \times 10^{-5}, \tag{11}$$

implies that

$$A_\pi^{(ggg)} \approx 0.06 A^{\varphi(2s)\to\pi^+\pi^-}, \tag{12}$$

so that it can be safely ignored [19].

**V)** In paper [20] the writer selects a quite different approach to show that the dominant process in $\psi(2s)$ and $J/\psi$ decay to psudoscaler mesons occur only via one photon.

## II. CALCULATION OF BRANCHING RATIO $\psi(2S) \to \pi^+\pi^-$

The electromagnetic form factor of a spin-0 meson, studied with spacelike momentum transfers (Fig. 2), is related to the following matrix element

$$< m(p_2) \mid J_\mu^{em} \mid m(p_1) >= (p_2 + p_1)_\mu F(Q^2), \tag{13}$$

where the electromagnetic current $J_\mu^{em} = \sum_f \ e_f \bar{q}_f \gamma_\mu q_f$ is expressed in terms of quarks $q_f$ with flavor $f$ and electric charge $e_f$ ; the spacelike momentum transfer is defined as $Q^2 = -q^2 = t = (p_1 - p_2)^2$, where $p_1$ and $p_2$ are the initial and final momenta of the meson, respectively. The form factor $F(Q^2)$ measures the deviation of the meson from being a Dirac point particle. The matrix element for timelike momentum transfers (Fig. 2),is defined as

$$< m(p_1)\overline{m}(p_2) \mid J_\mu^{em} \mid 0 >= (p_1 - p_2)_\mu F(Q^2). \tag{14}$$

The timelike momentum transfer is defined as $-Q^2 = q^2 = s = (p_1 + p_2)^2$ , where s is the center of mass energy square of the system, and $p_1$ and $p_2$ are the momenta of the meson and the anti-meson, respectively. Electromagnetic form factors provide direct insight into the electromagnetic structure of a hadron, namely the distribution of charges and currents in the hadrons as they couple with the photon. Since targets of unstable hadrons are not possible, determination of their form factors for spacelike momentum transfers (positive $Q^2$ ) at large momentum transfers become impossible.

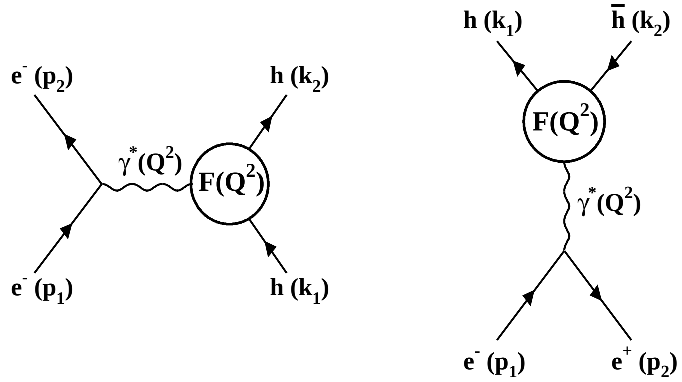


FIG. 2: Feymann diagrams for studying the electromagnetic form factors. Left: Spacelike momentum transfer from electron scattering .The initial and final four-momenta of the electron are $p_1$ and $p_2$, and the initial and final four-momenta of the hadron are $k_1$ and $k_2$, respectively. The four-momentum of the virtual photon is defined as $Q^2 = -q^2 = t = (p_1 - p_2)^2$. Right: Timelike momentum transfer from $e^+e^-$ annihilations. The initial four-momenta of the electron and positron are $p_1$ and $p_2$, and the final four-momenta of the hadron and 'anti'hadron are $k_1$ and $k_2$, respectively. The four-momentum of the virtual photon is defined as $-Q^2 = q^2 = s = (p_1 + p_2)^2$ .

However, form factors of a meson (m) for timelike momentum transfers (negative $Q^2$ ) can be measured by $e^+e^- \to m^+m^-$ reactions and is given as follow [4]:

$$\sigma_0(e^+e^- \to m^+m^-) = \frac{\pi\alpha^2\beta_m^3}{3s}|F_m(s)|^2, \tag{15}$$

where $\beta_m$ is the pesudoscalar meson ($m = \pi, K$) velocity (in terms of c) measured in the laboratory system and given as $\beta_m = \sqrt{1-(4m_m^2)/s}$ where $m_m$ stands for meson mass; and $F_m(s)$ is electromagnetic form factor and $\alpha$ is the QED coupling constant, respectively. CLEO has recently measured $\sigma_0$ for pion and at $\sqrt{s} = 3.671$ GeV the value of $F_\pi$ is reported as [21]:

$$|F_\pi(13.48GeV^2)| = 0.075 \pm 0.008. \tag{16}$$

Let us the $\psi(2s) \to \pi^+\pi^-$ decay process occur as in Fig. 3:

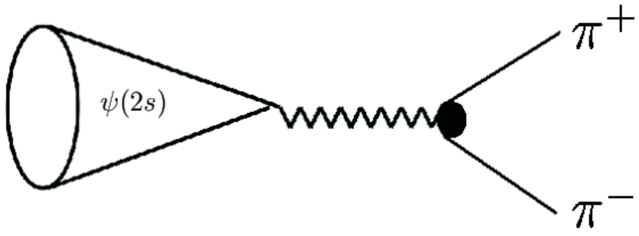


FIG. 3: Feynman diagram for $\psi$ electromagnetic decay to $\pi^+$ and $\pi^-$.

Considering:

$$< \pi^+(p_1)\pi^-(p_2) \mid J_\mu^{em} \mid 0 >= (p_1 - p_2)_\mu F_\pi(s), \tag{17}$$

$$< 0 \mid J_\mu^{em} \mid \psi(2s) >\equiv \sqrt{\alpha} g \varepsilon_\mu^\psi, \tag{18}$$

where $\alpha$ is the fine-structure constant, $g$ is the coupling constant of the $\psi$ to the virtual photon , $\varepsilon_\mu^\psi$ is the meson four vector polarization [22],and using the following relations

$$\Sigma_\lambda \varepsilon_\mu^{\lambda *} \varepsilon_\nu^\lambda = -g_{\mu\nu} + \frac{p_\mu p_\nu}{M^2_{\psi(2s)}}, \tag{19}$$

and in the approximation $m_e^2 << M_\psi^2$ :

$$\Gamma_{\psi \to e^+ e^-} = \frac{4\pi\alpha^2}{3} g^2 M_\psi \tag{20}$$

,where the decay $\Gamma_{\psi(2s)\to e^+e^-}$ is used to obtain $g$ [23], now by doing calculation in a frame which coincide with the $\psi$ meson, the following relation is obtained

$$\frac{\Gamma(\psi(2s) \to \pi^+\pi^-)}{\Gamma(\psi(2s) \to e^+e^-)} = 2F_\pi^2(M^2_{\psi(2s)})(\frac{p_\circ^\circ}{M_{\psi(2s)}})^3, \tag{21}$$

where

$$p_\circ^\circ \equiv \sqrt{\frac{M^2_{\psi(2s)} - 4m_\pi^2}{4}} \tag{22}$$

and $p_\circ^\circ$ has the value of 1.838.

Considering the value of $F_\pi$ ( Eq.(16)) and putting the value of $\Gamma(\psi(2s) \to e^+e^-)$ from PDG [1] the total value of $\Gamma(\psi(2s) \to \pi^+\pi^-)$ is calculated to be

$$\Gamma(\psi(2s) \to \pi^+\pi^-) = 1.03 \times 10^{-5}, \tag{23}$$

which is in an excellent agreement with the recent published data.

## III. CONCLUDING REMARKS

The decay mode of $\psi(2s) \rightarrow \pi^+\pi^-$ is investigated. It is found that the dominant mode of decay is one photon exchange via electromagnetic interaction. Using the recently measured form factor by CLEO, the corresponding branching ratio is calculated. Our finding for branching ratio is a factor $\sim 8$ smaller than in the PDG [1] and is in good agreement with the most resent published value given in references [2,3,4].

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