PHYSICAL REVIEW D **78,** 036005 (2008)

Analysis of the rare semileptonic $B_c \to P(D, D_s)l^+l^-/\nu\bar{\nu}$ decays within QCD sum rules

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(Received 3 June 2008; revised manuscript received 14 July 2008; published 15 August 2008)

Considering the gluon condensate corrections, the form factors relevant to the semileptonic rare $B_c \to D$, $D_s(J^P = 0^-)l^+l^-$ with $l = \tau$, μ, e and $B_c \to D$, $D_s(J^P = 0^-)\nu\bar{\nu}$ transitions are calculated in the framework of the three point QCD sum rules. The heavy quark effective theory limit of the form factors is computed. The branching fraction of these decays is also evaluated and compared with the predictions of the relativistic constituent quark model. Analyzing such transitions could give useful information about the strong interactions inside the pseudoscalar D_s meson and its structure.

DOI: 10.1103/PhysRevD.78.036005 PACS numbers: 11.55.Hx, 13.20.He

I. INTRODUCTION

With the chances that in the future a large amount of B_c mesons will be produced at LHC (with the luminosity values of $\mathcal{L} = 10^{34}$ cm^{-2} s^{-1} and $\sqrt{s} = 14$ TeV, the number of $B_c^{\pm}$ mesons is expected to be about 10^8–10^{10} per year [1,2]), one might explore the rare B_c decays to pseudoscalar (D, D_s) and $l^+l^-/\nu\bar{\nu}$. Such types of transitions could be useful because of the following reasons: (1) Analyzing of such transitions could give valuable information about the nature of the pseudoscalar D_s meson and the strong interactions inside it. (2) The form factors of these transitions could be used in the study of the polarization asymmetries, CP and T violations. (3) These will provide a new framework for more precise calculation of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{tq} ($q = d$, s, b) and leptonic decay constants of $D_{s,d}$ and B_c mesons. (4) These transitions occur at loop level in standard model (SM) via the flavor changing neutral current (FCNC) transitions of $b \to s, d$, which are sensitive to the new physics beyond the SM, so these decays are useful to constrain the parameters beyond the SM. (5) A possible fourth generation, SUSY particles [3] and light dark matter [4] might contribute to the loop transitions of $b \to s, d$.

The B_c is the only meson containing two heavy quarks with different charge and flavors and it is the lowest bound state of b and c quarks, so its decay mode properties are expected to be different than flavor neutral mesons. Since the excited levels of $\bar{b}c$ lie below the threshold of decay into the pair of heavy B and D mesons, such states decay weakly and they have no annihilation decay modes due to the electromagnetic and strong interactions (for more about the physics of the B_c meson, see for example [5]). This paper describes the annihilation of the B_c into the pseudoscalar $(D, D_s)l^+l^-/\nu\bar{\nu}$ in the framework of the three point QCD sum rules as a nonperturbative approach based on the fundamental QCD Lagrangian. These transitions are parametrized in terms of some form factor calculation which plays a crucial role in analyzing those decay channels. These decays at quark level proceed by the loop $b \to s$, d in the SM with the intermediate u, c, and t quarks and the main contribution comes from the intermediate top quark. These decay modes have also been studied in the relativistic constituent quark model (RCQM) [6]. Some other possible channels such as $B_c \to l\bar{\nu}\gamma$, $B_c \to \rho^+\gamma$, $B_c \to K^{*+}\gamma$, $B_c \to B_u^* l^+l^-$, $B_c \to B_u^*\gamma$, $B_c \to D_{s,d}^*\gamma$, $B_c \to D_{s,d}^* l^+l^-$, and $B_c \to X\nu\bar{\nu}$ where X is the axial vector particle, $D_{s1}(2460)$, and vector particles, D^*, D_s^* are studied in the light cone or traditional QCD sum rule methods in [7–13], respectively. For a set of exclusive nonleptonic and semileptonic decays of the B_c meson, which have been studied in the relativistic constituent quark model, see [14].

The content of paper is as follows: In Sec. II, we calculate the sum rules for the related form factors considering the gluon correction contributions to the correlation function. The light quark condensate contributions are killed applying the double borel transformations with respect to momentum of the initial and final states. The heavy quark effective theory (HQET) limit of the form factors is presented in Sec. III. Section IV depicts our numerical analysis of the form factors and their comparison with the HQET limit of them, results, discussions, and comparison of our results with the prediction of the RCQM model.

II. QCD SUM RULES FOR TRANSITION FORM FACTORS OF THE $B_c \to (D, D_s)l^+l^-/\nu\bar{\nu}$

At quark level, the processes $B \to Pl^+l^-/\nu\bar{\nu}(P = D, D_s)$ are described by the loop $b \to q_i l^+l^-/\nu\bar{\nu}$ transitions, $(q_1 = d, q_2 = s)$ in the SM (see Fig. 1), and receive contributions from photon and Z-penguin and box diagrams for l^+l^- and only Z-penguin and box diagrams for

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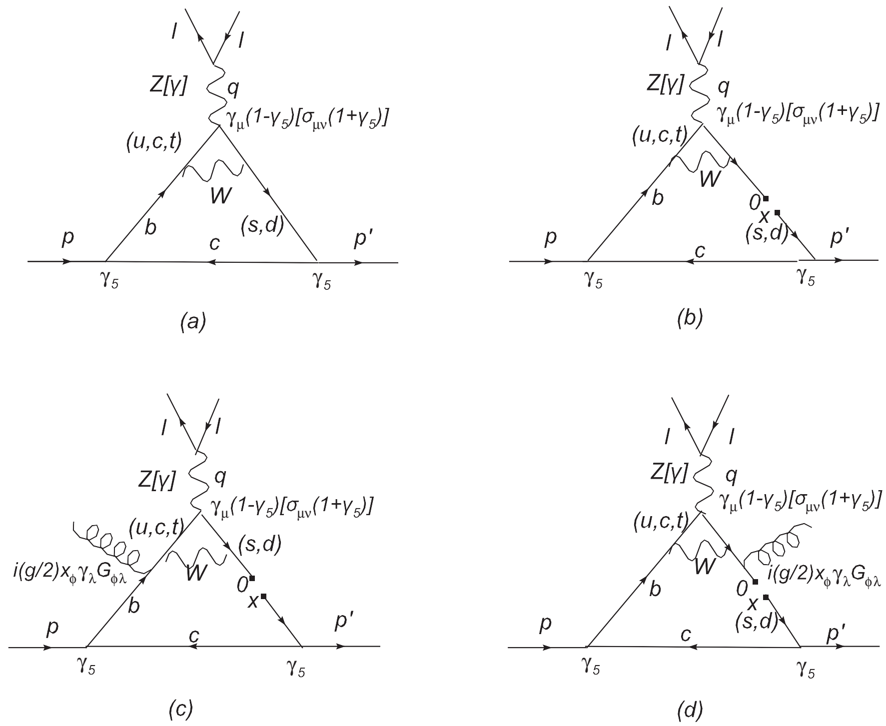

FIG. 1. Loop diagrams for $B_c \to (D, D_s)l^+l^-/\nu\bar{\nu}$ transitions, bare loop [diagram (a)] and light quark condensates [without any gluon, diagram (b), and with one gluon emission, diagrams (c), (d)].

$\nu\bar{\nu}$. These loop transitions occur via the intermediate u, c, t quarks, where the dominant contribution comes from the intermediate top quark. The effective Hamiltonian responsible for $b \to q_i l^+ l^-$ decays is described in terms of the Wilson coefficients, $C_7^{\rm eff}$, $C_9^{\rm eff}$, and C_{10} as

$$\mathcal{H}_{\rm eff} = \frac{G_F\alpha}{2\sqrt{2}\pi} V_{tb}V_{tq_i}^*\bigg[C_9^{\rm eff}\bar{q}_i\gamma_\mu(1-\gamma_5)b\bar{\ell}\gamma_\mu\ell + C_{10}\bar{q}_i\gamma_\mu(1-\gamma_5)b\bar{\ell}\gamma_\mu\gamma_5\ell - 2C_7^{\rm eff}\frac{m_b}{q^2}\bar{q}_i i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\bar{\ell}\gamma_\mu\ell\bigg], \qquad (1)$$

where G_F is the Fermi constant, α is the fine structure constant at Z mass scale, and V_{ij} are elements of the CKM matrix. For the $\nu\bar{\nu}$ case, only the term containing C_{10} is considered. The amplitudes for $B_c \to Pl^+l^-/\nu\bar{\nu}$ decays are obtained by sandwiching Eq. (1) between initial and final meson states:

$$\mathcal{M} = \frac{G_F\alpha}{2\sqrt{2}\pi} V_{tb}V_{tq_i}^*\bigg[C_9^{\rm eff} < P(p') \mid \bar{q}_i\gamma_\mu(1-\gamma_5)b \times \mid B_c(p) > \bar{\ell}\gamma_\mu\ell + C_{10} < P(p') \times \mid \bar{q}_i\gamma_\mu(1-\gamma_5)b \mid B_c(p) > \bar{\ell}\gamma_\mu\gamma_5\ell - 2C_7^{\rm eff}\frac{m_b}{q^2} < P(p') \mid \bar{q}_i i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b \times \mid B_c(p) > \bar{\ell}\gamma_\mu\ell\bigg]. \qquad (2)$$

Next, we calculate the matrix elements $\langle P(p') \mid \bar{q}_i\gamma_\mu(1-\gamma_5)b \mid B_c(p)\rangle$ and $\langle P(p') \mid \bar{q}_i i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b \mid B_c(p)\rangle$ appearing in the above equation. The parts of the transition currents containing γ_5 do not contribute, so we consider only $\bar{q}_i\gamma_\mu b$ and also $\bar{q}_i\sigma_{\mu\nu}q^\nu b$ parts. Considering Lorentz and parity invariances, these matrix elements can be parametrized in terms of the form factors as

$$\langle P(p') \mid \bar{q}_i\gamma_\mu b \mid B_c(p)\rangle = -(\mathcal{P}_\mu f_+(q^2) + q_\mu f_-(q^2)), \qquad (3)$$

$$\langle P(p')|\bar{q}_i i\sigma_{\mu\nu}q^\nu b|B_c(p)\rangle = \frac{f_T(q^2)}{m_B + m_P}[\mathcal{P}_\mu q^2 - q_\mu(m_{B_c}^2 - m_P^2)], \qquad (4)$$

where $f_+(q^2)$, $f_-(q^2)$ and $f_T(q^2)$ are the transition form factors, $\mathcal{P}_\mu = (p + p')_\mu$ and $q_\mu = (p - p')_\mu$. Here, we should mention that for the $\nu\bar{\nu}$ case the form factor $f_T(q^2)$ does not contribute since it is related to the photon vertex $(\sigma_{\mu\nu}q^\nu)$. To calculate the form factors $f_+(q^2)$, $f_-(q^2)$, and $f_T(q^2)$, we start with the following correlation function:

$$\Pi_\mu^{V,T} = i^2\int d^4xd^4ye^{-ipx}e^{ip'y}\langle 0|\mathcal{T}\{J_P(y)J_\mu^{V,T}(0)J_{B_c}^\dagger(x)\}|0\rangle, \qquad (5)$$

where $J_P(y) = \bar{c}\gamma_5 q_i$ ($q_i = s$ or d) and $J_{B_c}(x) = \bar{c}\gamma_5 b$ are the interpolating currents of the P and B_c mesons and $J_\mu^V = \bar{q}_i\gamma_\mu b$ and $J_\mu^T = \bar{q}_i i\sigma_{\mu\nu}q^\nu b$ are transition currents. From the general philosophy of the QCD sum rules, we can calculate the above-mentioned correlator in two languages: (1) the hadron language called the physical or phenomenological side, 2) the quark gluon language which is the QCD or theoretical side. Equating two sides and applying the double Borel transformations with respect to the momentum of the initial and final states to suppress the contribution of the higher states and continuum, we get sum rule expressions for our form factors. The phenomenological part can be obtained by inserting the complete set of intermediate states with the same quantum numbers as the currents J_P and J_{B_c}. As a result of this procedure,

$$\Pi_\mu^{V,T}(p^2, p'^2, q^2) = \frac{\langle 0|J_P|P(p')\rangle\langle P(p')|J_\mu^{V,T}|B_c(p)\rangle\langle B_c(p)|J_{B_c}^\dagger|0\rangle}{(m_P^2 - p'^2)(m_{B_c}^2 - p^2)} \quad (6)$$

is obtained. The following matrix elements are defined in terms of the leptonic decay constants of the P and B_c mesons as

$$\langle 0|J_P|P\rangle = -i\frac{f_P m_P^2}{m_c + m_{q_i}}, \qquad \langle 0|J_{B_c}|B_c\rangle = -i\frac{f_{B_c} m_{B_c}^2}{m_b + m_c}. \quad (7)$$

Using Eqs. (3), (4), and (7) in Eq. (6), we obtain

$$\Pi_\mu^V(p^2, p'^2, q^2) = -\frac{f_{B_c} m_{B_c}^2}{(m_b + m_c)(m_c + m_{q_i})} \times \frac{f_P m_P^2}{(m_P^2 - p'^2)(m_{B_c}^2 - p^2)} \times [f_+ \mathcal{P}_\mu + f_- q_\mu] + \cdots, \quad (8)$$

$$\Pi_\mu^T(p^2, p'^2, q^2) = \frac{f_{B_c} m_{B_c}^2}{(m_b + m_c)(m_c + m_{q_i})} \times \frac{f_P m_P^2}{(m_P^2 - p'^2)(m_{B_c}^2 - p^2)} \left[\frac{f_T}{(m_{B_c} + m_P)} \times [q^2 \mathcal{P}_\mu - (m_{B_c}^2 - m_P^2) q_\mu]\right] + \cdots. \quad (9)$$

For extracting the expressions for form factors $f_+(q^2)$ and $f_-(q^2)$, we choose the coefficients of the structures $\mathcal{P}_\mu$ and q_μ from $\Pi_\mu^V(p^2, p'^2, q^2)$, respectively, and the structure q_μ from $\Pi_\mu^T(p^2, p'^2, q^2)$ is considered for the form factor $f_T(q^2)$. Therefore, the correlation functions are written in terms of the selected structures as

$$\Pi_\mu^V(p^2, p'^2, q^2) = \Pi_+ \mathcal{P}_\mu + \Pi_- q_\mu + \cdots, \quad (10)$$

$$\Pi_\mu^T(p^2, p'^2, q^2) = \Pi_T q_\mu + \cdots. \quad (11)$$

On the other side, to calculate the QCD part of correlation function, we evaluate the three-point correlator with the help of the operator product expansion (OPE) in the deep Euclidean region, where $p^2 \ll (m_b + m_c)^2$ and $p'^2 \ll (m_c + m_{q_i})^2$. For this aim, we write each Π_i function in terms of the perturbative and nonperturbative parts as

$$\Pi_i(p^2, p'^2, q^2) = \Pi_i^{\text{per}}(p_1^2, p_2^2, q^2) + \Pi_i^{\text{nonper}}(p^2, p'^2, q^2), \quad (12)$$

where i stands for $+$, $-$, and T and the nonperturbative part contains the light quark ($\langle \bar{q}q\rangle$) and gluon ($\langle G^2\rangle$) condensates. For the perturbative part, the bare loop diagram [Fig. 1(a)] is considered, however, diagrams (b), (c), and (d) in Fig. 1 are correspond to the light quark condensates contributing to the correlator. In principle, the light quark condensate diagrams give contributions to the correlation function, but applying double Borel transformations omits their contributions, hence as the first nonperturbative correction, we consider the gluon condensate diagrams [see Fig. 2(a)–2(f)].

With the help of the double dispersion representation, the bare-loop contribution is written as

$$\Pi_i^{\text{per}} = -\frac{1}{(2\pi)^2}\int ds' \int ds \frac{\rho_i^{\text{per}}(s, s', Q^2)}{(s - p^2)(s' - p'^2)} + \text{subtraction terms}, \quad (13)$$

where $Q^2 = -q^2$. The spectral densities $\rho_i^{\text{per}}(s, s', Q^2)$ are calculated with the help of the Gutkovsky rule, i.e., the propagators are replaced by Dirac-delta functions

$$\frac{1}{p^2 - m^2} \rightarrow -2i\pi\delta(p^2 - m^2), \quad (14)$$

expressing that all quarks are real. The integration region in Eq. (13) is obtained by requiring that the argument of three deltas vanish, simultaneously. This condition results in the following inequality:

$$-1 \le \frac{2ss' + (s + s' + Q^2)(m_b^2 - m_c^2 - s) + 2s(m_c^2 - m_{q_i}^2)}{\lambda^{1/2}(s, s', -Q^2)\lambda^{1/2}(m_b^2, m_c^2, s)} \le +1, \quad (15)$$

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. From this inequality, to use in the lower and upper limit of the integration over s in subtractions, it is easy to express s in terms of s' i.e. $f_\pm(s')$ in the $s - s'$ plane.

Straightforward calculations end up in the following results for the spectral densities:

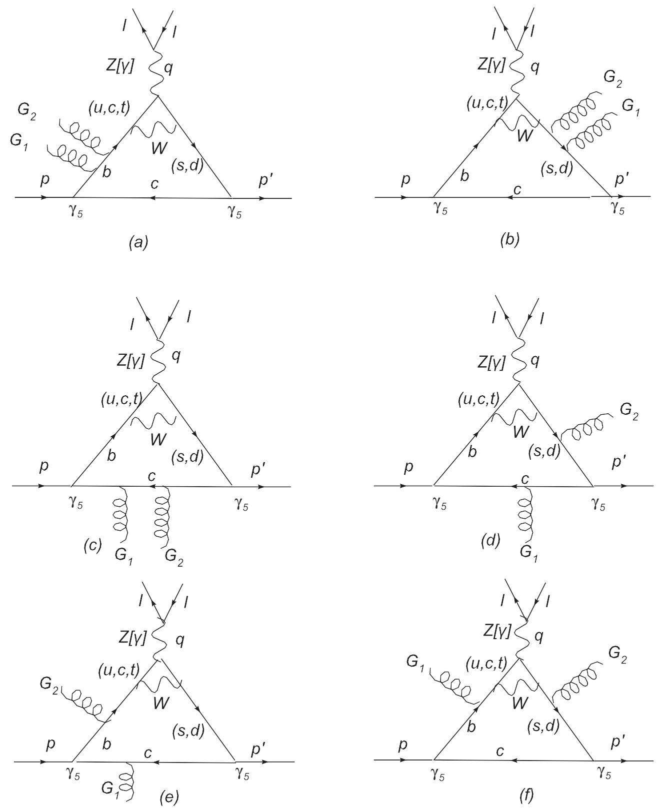

FIG. 2. Gluon condensate contributions to $B_c \to (D, D_s)l^+l^-/\nu\bar{\nu}$ transitions.

$$\begin{aligned}
\rho_+^V(s,s',q^2) &= I_0 Nc\{\Delta + \Delta' + -2m_c[(+2 + E_1 + E_2)m_c - (1 + E_1 + E_2)m_{q_i}] \\
&\quad + 2m_b[(1 + E_1 + E_2)m_c - (E_1 + E_2)m_{q_i}] + (E_1 + E_2)u\}, \\
\rho_-^V(s,s',q^2) &= I_0 Nc\{-\Delta + \Delta' - 2m_c[(E_2 - E_1 - 1)m_{q_i} + (E_1 - E_2)m_c] \\
&\quad - 2m_b[(1 - E_1 + E_2)m_c + (E_1 - E_2)m_{q_i}] + (E_1 - E_2)u\}, \\
\rho_T^T(s,s',q^2) &= -I_0 N_c\{\Delta(2m_c - m_b - m_{q_i}) + \Delta'(m_b - 2m_c + m_{q_i}) + 2[m_c(E_1 - E_2 - 1) + m_{q_i}(E_2 - E_1)]s \\
&\quad - 2[m_b(E_1 - E_2) - m_c(E_1 - E_2 + 1)]s' + (E_1 - E_2)(m_b - 2m_c + m_{q_i})u\},
\end{aligned}$$

where

$$\begin{aligned}
I_0(s,s',Q^2) &= \frac{1}{4\lambda^{1/2}(s,s',Q^2)}, \qquad \lambda(s,s',Q^2) = s^2 + s'^2 + Q^4 + 2sQ^2 + 2s'Q^2 - 2ss', \qquad E_1 = \frac{1}{\lambda(s,s',Q^2)}[2s'\Delta - \Delta' u], \\
E_2 &= \frac{1}{\lambda(s,s',Q^2)}[2s\Delta' - \Delta u], \qquad u = s + s' + Q^2, \qquad \Delta = s + m_c^2 - m_b^2, \qquad \Delta' = s' + m_c^2 - m_{q_i}^2,
\end{aligned} \tag{16}$$

and $N_c = 3$ is the color factor.

Now as the first correction to the nonperturbative part of the correlator, we calculate the gluon condensate contributions (see diagrams in Fig. 2). The calculations proceed the same as [13] (see also [9,11,12,15]) and the Fock-Schwinger fixed-point gauge [16–18], $x^\mu G_\mu^a = 0$, where G_μ^a is the gluon field. In calculations, the following type of integrals are encountered:

$$\begin{aligned}
I_0[a,b,c] &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 - m_b^2]^a[(p+k)^2 - m_c^2]^b[(p'+k)^2 - m_{q_i}^2]^c}, \\
I_\mu[a,b,c] &= \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu}{[k^2 - m_b^2]^a[(p+k)^2 - m_c^2]^b[(p'+k)^2 - m_{q_i}^2]^c}.
\end{aligned} \tag{17}$$

Performing integration over loop momentum and applying double Borel transformations with respect to the p^2 and p'^2, we obtain the Borel transformed form of the integrals as follows:

$$\begin{aligned}\hat{I}_0(a,b,c) &= \frac{(-1)^{a+b+c}}{16\pi^2\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{2-a-b} \\ &\quad\times (M_2^2)^{2-a-c}\mathcal{U}_0(a+b+c-4, \\ &\quad 1-c-b), \\ \hat{I}_\mu(a,b,c) &= \frac{1}{2}[\hat{I}_1(a,b,c)+\hat{I}_2(a,b,c)]\mathcal{P}_\mu \\ &\quad + \frac{1}{2}[\hat{I}_1(a,b,c)-\hat{I}_2(a,b,c)]q_\mu, \end{aligned} \tag{18}$$

where

$$\begin{aligned}\hat{I}_1(a,b,c) &= i\frac{(-1)^{a+b+c+1}}{16\pi^2\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{2-a-b} \\ &\quad\times (M_2^2)^{3-a-c}\mathcal{U}_0(a+b+c-5, \\ &\quad 1-c-b), \\ \hat{I}_2(a,b,c) &= i\frac{(-1)^{a+b+c+1}}{16\pi^2\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{3-a-b} \\ &\quad\times (M_2^2)^{2-a-c}\mathcal{U}_0(a+b+c-5, \\ &\quad 1-c-b),\end{aligned} \tag{19}$$

The hat in Eq. (18) denotes the double Borel transformed form of integrals. M_1^2 and M_2^2 are the Borel parameters in the s and s' channels, respectively, and the function $\mathcal{U}_0(a,b)$ is defined as

$$\mathcal{U}_0(a,b) = \int_0^\infty dy(y+M_1^2+M_2^2)^a y^b \times \exp\left[-\frac{B_{-1}}{y}-B_0-B_1 y\right],$$

where

$$\begin{aligned}B_{-1} &= \frac{1}{M_1^2M_2^2}[m_{q_i}^2M_1^4+m_b^2M_2^4+M_2^2M_1^2(m_b^2+m_{q_i}^2+Q^2)], \\ B_0 &= \frac{1}{M_1^2M_2^2}[(m_{q_i}^2+m_c^2)M_1^2+M_2^2(m_b^2+m_c^2)], \\ B_1 &= \frac{m_c^2}{M_1^2M_2^2}.\end{aligned} \tag{20}$$

After straightforward but lengthy calculations, we get the following results for the gluon condensate contributions:

$$\Pi_i^{\langle G^2\rangle} = i\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{C_i}{6}, \tag{21}$$

where the explicit expressions for C_i are given in Appendix A.

The next step is to apply the Borel transformations with respect to the p^2 $(p^2\to M_1^2)$ and p'^2 $(p'^2\to M_2^2)$ on the phenomenological as well as the perturbative parts of the correlation function, continuum subtraction and equate these two representations of the correlator. The following sum rules for the form factors f_+, f_-, and f_T are derived:

$$\begin{aligned}f_+ &= \frac{(m_b+m_c)(m_c+m_{q_i})}{f_{B_c}m_{B_c}^2f_Pm_P^2}e^{m_{B_c}^2/M_1^2}e^{m_P^2/M_2^2}\frac{1}{4\pi^2} \\ &\quad\times\left\{\int_{(m_c+m_q)^2}^{s_0'}ds'\int_{f_-(s')}^{\min(s_0,f_+(s'))}ds\rho_+^V(s,s',Q^2)\right. \\ &\quad\left.\times e^{-s/M_1^2}e^{-s'/M_2^2}-iM_1^2M_2^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{C_+}{6}\right\},\end{aligned} \tag{22}$$

$$\begin{aligned}f_- &= \frac{(m_b+m_c)(m_c+m_{q_i})}{f_{B_c}m_{B_c}^2f_Pm_P^2}e^{m_{B_c}^2/M_1^2}e^{m_P^2/M_2^2}\frac{1}{4\pi^2} \\ &\quad\times\left\{\int_{(m_c+m_q)^2}^{s_0'}ds'\int_{f_-(s')}^{\min(s_0,f_+(s'))}ds\rho_-^V(s,s',Q^2)\right. \\ &\quad\left.\times e^{-s/M_1^2}e^{-s'/M_2^2}-iM_1^2M_2^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{C_-}{6}\right\},\end{aligned} \tag{23}$$

$$\begin{aligned}f_T &= \frac{(m_b+m_c)(m_c+m_{q_i})}{f_{B_c}m_{B_c}^2f_Pm_P^2(m_{B_c}-m_P)}e^{m_{B_c}^2/M_1^2}e^{m_P^2/M_2^2}\frac{1}{4\pi^2} \\ &\quad\times\left\{\int_{(m_c+m_q)^2}^{s_0'}ds'\int_{f_-(s')}^{\min(s_0,f_+(s'))}ds\rho_T^T(s,s',Q^2)\right. \\ &\quad\left.\times e^{-s/M_1^2}e^{-s'/M_2^2}-iM_1^2M_2^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{C_T}{6}\right\},\end{aligned} \tag{24}$$

where s_0 and s_0' are the continuum thresholds and $s=f_\pm(s')$ in the lower and upper limit of the integral over s are obtained from inequality (15). The $\min(s_0,f_+(s'))$ means that for each value of the q^2, the smaller one between s_0 and f_+ is selected. In the above equations, in order to subtract the contributions of the higher states and the continuum, the quark-hadron duality assumption is also used:

$$\rho^{\text{higher states}}(s,s') = \rho^{\text{OPE}}(s,s')\theta(s-s_0)\theta(s'-s_0'). \tag{25}$$

At the end of this section, we would like to present the differential decay width of $B_c\to Pl^+l^-/\nu\bar{\nu}$ decays. Using the parametrization of these transitions in terms of form factors and amplitude in Eq. (2), we get

$$\begin{aligned}\frac{d\Gamma}{dQ^2}(B_c^\pm\to P^\pm\nu\bar{\nu}) &= \frac{G_F^2\alpha^2}{2^8\pi^5}|V_{tq_i}V_{tb}^*|^2\,\phi_P^{3/2}(1,r_P,s) \\ &\quad\times m_{B_c}^3|C_{10}|^2|f_+(Q^2)|^2,\end{aligned} \tag{26}$$

where, $\phi_P(1,r_P,s)$ is the usual triangle function

$$\phi_P(1,r_P,s) = 1+r_P^2+s^2-2r_P-2s-2r_Ps,$$

with

$$r_P = \frac{m_P^2}{m_{B_c}^2}, \qquad s = -\frac{Q^2}{m_{B_c}^2},$$

and

$$\frac{d\Gamma}{dQ^2}(B_c^\pm \to P^\pm l^+ l^-) = \frac{G_F^2 \mid V_{tq_i} V_{tb}^* \mid^2 m_{B_c}^3 \alpha^2}{3 \cdot 2^9 \pi^5} \times \upsilon \phi_P^{1/2}(1, r_P, s)\left[\left(1 + \frac{2t}{s}\right) \times \phi_P(1, r_P, s)\alpha_1 + 12t\beta_1\right], \quad (27)$$

where $t = m_l^2/m_{B_c}^2$ and the expressions of α_1 and β_1 and υ are given as

$$\upsilon = \sqrt{1 + \frac{4m_l^2}{Q^2}},$$

$$\alpha_1 = \left| C_9^{\mathrm{eff}} f_+(Q^2) + \frac{2\hat{m}_b C_7^{\mathrm{eff}} f_T(Q^2)}{1 + \sqrt{r_P}} \right|^2 + |C_{10} f_+(Q^2)|^2,$$

$$\beta_1 = |C_{10}|^2\left[\left(1 + r_P - \frac{s}{2}\right)|f_+(Q^2)|^2 + (1 - r_P)\,\mathrm{Re}(f_+(Q^2) f_-^*(Q^2)) + \frac{1}{2}s|f_-(Q^2)|^2\right],$$

where $\hat{m}_b = m_b/m_{B_c}$.

III. HQET LIMIT OF THE FORM FACTORS

In this section, we present the infinite heavy quark mass limit of the form factors for $B_c \to (D, D_s)l^+l^-/\nu\bar{\nu}$ transitions. To this aim, we use the following parametrization (see also [19–23]):

$$y = \nu\nu' = \frac{m_{B_c}^2 + m_{D_{q_i}}^2 - q^2}{2m_{B_c} m_{Dq_i}}, \quad (28)$$

where ν and ν' are the four-velocities of the initial and final meson states, respectively, and $y = 1$ are so-called zero recoil limit. Now, to obtain the y dependent expressions of the form factors we define $m_b \to \infty$, $m_c = \frac{m_b}{\sqrt{z}}$, where z is given by $\sqrt{z} = y + \sqrt{y^2 - 1}$ and we also set the mass of light quarks to zero. In this limit the new Borel parameters T_1 and T_2 take the form $T_1 = M_1^2/2m_b$ and $T_2 = M_2^2/2m_c$.

The new continuum thresholds ν_0, and ν_0' are defined as

$$\nu_0 = \frac{s_0 - m_b^2}{m_b}, \qquad \nu_0' = \frac{s_0' - m_c^2}{m_c}, \quad (29)$$

and the new integration variables become

$$\nu = \frac{s - m_b^2}{m_b}, \qquad \nu' = \frac{s' - m_c^2}{m_c}. \quad (30)$$

The leptonic decay constants are rescaled:

$$\hat{f}_{B_c} = \sqrt{m_b} f_{B_c}, \qquad \hat{f}_{D_{q_i}} = \sqrt{m_c} f_{D_{q_i}}. \quad (31)$$

The corresponding expressions for $\hat{I}_0(a, b, c)$, $\hat{I}_1(a, b, c)$, and $\hat{I}_2(a, b, c)$ in this limit are defined as

$$\begin{aligned}
\hat{I}_0(a, b, c)^{\mathrm{HQET}} &= \frac{(-1)^{a+b+c}}{16\pi^2\Gamma(a)\Gamma(b)\Gamma(c)}(T_1)^{2-a-b}(T_2)^{2-a-c}\mathcal{U}_0^{\mathrm{HQET}}(a + b + c - 4, 1 - c - b),\\
\hat{I}_1(a, b, c)^{\mathrm{HQET}} &= i\frac{(-1)^{a+b+c+1}}{16\pi^2\Gamma(a)\Gamma(b)\Gamma(c)}(T_1)^{2-a-b}(T_2)^{3-a-c}\mathcal{U}_0^{\mathrm{HQET}}(a + b + c - 5, 1 - c - b),\\
\hat{I}_2(a, b, c)^{\mathrm{HQET}} &= i\frac{(-1)^{a+b+c+1}}{16\pi^2\Gamma(a)\Gamma(b)\Gamma(c)}(T_1)^{3-a-b}(T_2)^{2-a-c}\mathcal{U}_0^{\mathrm{HQET}}(a + b + c - 5, 1 - c - b),
\end{aligned} \quad (32)$$

where T_1 and T_2 are the Borel parameters in the s and s' channel, respectively, and the function $\mathcal{U}_0^{\mathrm{HQET}}(m, n)$ is defined as

$$\mathcal{U}_0^{\mathrm{HQET}}(m, n) = \int_0^\infty (x + T_1 + T_2)^m x^n \exp\{\hat{A} + \hat{B} + \hat{C}\}dx, \quad (33)$$

with

$$A = -\frac{m_b^2 T_2^2 + T_1 T_2(m_b^2 + y)}{T_1 T_2 x} \qquad B = -\frac{T_2(m_b^2 + \frac{m_b^2}{z}) + \frac{T_1 m_b^2}{z}}{T_1 T_2} \qquad C = -\frac{m_b^2 x}{T_1 T_2 z}. \quad (34)$$

In order for the calculations to be easy, the following redefinitions for the form factors are applied:

$$\tilde{f}_i = f_i\{m_{B_c} + m_{D_{q_i}}\}. \quad (35)$$

After some calculations, we obtain the y-dependent expressions of the form factors as follows:

$$\tilde{f}_+^{\rm HQET}(y) = \frac{1}{32\pi^2 \hat{f}_{B_c}\hat{f}_P} e^{\Lambda/T_1} e^{\bar\Lambda/T_2}\bigg\{\frac{3(1+\sqrt{z})^2}{(-1+\sqrt{z})F(y,z)}[-1+(1+3y)\sqrt{z}-(2+y+4y^2)z+(3+2y(2+y))z^{3/2} - 4yz^2+2z^{5/2}]\int_0^{\nu_0} d\nu \int_0^{\nu_0'} d\nu' e^{-(\nu/2T_1)} e^{-(\nu'/2T_2)}\theta(2y\nu\nu'-\nu^2-\nu'^2) + \lim_{m_b\to\infty}\bigg(i\frac{5z^2}{24m_b^5}(1+\sqrt{z})\bigg\langle\frac{\alpha_s}{\pi}G^2\bigg\rangle C_+^{\rm HQET}\bigg)\bigg\}, \tag{36}$$

$$\tilde{f}_-^{\rm HQET}(y) = \frac{1}{32\pi^2 \hat{f}_{B_c}\hat{f}_P} e^{\Lambda/T_1} e^{\bar\Lambda/T_2}\bigg\{\frac{-3(1+\sqrt{z})^2}{(-1+\sqrt{z})F(y,z)}[-1+(1+3y)\sqrt{z}-(2+5y)z+(7+2y(y-2))z^{3/2} - 4(y-1)z^2+2z^{5/2}]\int_0^{\nu_0} d\nu \int_0^{\nu_0'} d\nu' e^{-(\nu/2T_1)} e^{-(\nu'/2T_2)}\theta(2y\nu\nu'-\nu^2-\nu'^2) + \lim_{m_b\to\infty}\bigg(i\frac{5z^2}{24m_b^5}(1+\sqrt{z})\bigg\langle\frac{\alpha_s}{\pi}G^2\bigg\rangle C_-^{\rm HQET}\bigg)\bigg\}, \tag{37}$$

$$\tilde{f}_T^{\rm HQET}(y) = \frac{1}{32\pi^2 \hat{f}_{B_c}\hat{f}_P} e^{\Lambda/T_1} e^{\bar\Lambda/T_2}\bigg\{\frac{-3(1+\sqrt{z})^2}{(-1+\sqrt{z})F(y,z)}[3-(1+9y)\sqrt{z}+(4+y(3+8y))z-((2+y)(1+4y))z^{3/2} + (3+2y(y+2))z^2-4yz^{5/2}+z^3]\int_0^{\nu_0} d\nu \int_0^{\nu_0'} d\nu' e^{-(\nu/2T_1)} e^{-(\nu'/2T_2)}\theta(2y\nu\nu'-\nu^2-\nu'^2) + \lim_{m_b\to\infty}\bigg(i\frac{5}{48zm_b^5}(1+\sqrt{z})^2\bigg\langle\frac{\alpha_s}{\pi}G^2\bigg\rangle C_T^{\rm HQET}\bigg)\bigg\}, \tag{38}$$

where

$$F(y,z) = z^{3/4}[1+z+y^2z+z^2-2y\sqrt{z}(1+z)]^{3/2}. \tag{39}$$

In the heavy quark limit expressions of the form factors, $\Lambda = m_{B_q} - m_b$ and $\bar\Lambda = m_{D_q^*} - m_c$, and the explicit expressions of the coefficients $C_i^{\rm HQET}$ are given in Appendix B.

IV. NUMERICAL ANALYSIS

This section encompasses our numerical analysis of the form factors f_+, f_-, and f_T and their HQET limit, branching fractions, comparison of our results with the prediction of the RCQM, and discussion. The sum rule expressions of the form factors depict that the main input parameters entering the expressions are gluon condensate, Wilson coefficients $C_7^{\rm eff}$, $C_9^{\rm eff}$, and C_{10}, elements of the CKM matrix V_{tb}, V_{ts}, and V_{td}, leptonic decay constants; f_{B_c}, f_D, and f_{D_s}, Borel parameters M_1^2 and M_2^2, as well as the continuum thresholds s_0 and s_0'. In further numerical analysis, we choose the values of the Gluon condensate, leptonic decay constants, CKM matrix elements, Wilson coefficients, quark and meson masses as: $\langle\frac{\alpha_s}{\pi}G^2\rangle = 0.012\ \mathrm{GeV}^4$ [24], $C_7^{\rm eff} = -0.313$, $C_9^{\rm eff} = 4.344$, $C_{10} = -4.669$ [25,26], $|V_{tb}| = 0.77^{+0.18}_{-0.24}$, $|V_{ts}| = (40.6 \pm 2.7)\times 10^{-3}$, $|V_{td}| = (7.4 \pm 0.8)\times 10^{-3}$ [27], $f_{D_s} = 274 \pm 13 \pm 7$ MeV [28], $f_D = 222.6 \pm 16.7^{+2.8}_{-3.4}$ MeV, [29], $f_{B_c} = 350 \pm 25$ MeV [30–32], $m_c = 1.25 \pm 0.09$ GeV, $m_s = 95 \pm 25$ MeV, $m_b = (4.7 \pm 0.07)$ GeV, $m_d = (3\text{–}7)$ MeV, $m_{D_s} = 1.968$ GeV, $m_D = 1.869$ GeV, $m_{B_c} = 6.258$ GeV [33], $\Lambda = 0.62$ GeV [34], and $\bar\Lambda = 0.86$ GeV [35].

The expressions for the form factors contain also four auxiliary parameters: Borel mass squares M_1^2 and M_2^2 and continuum threshold s_0 and s_0'. These are not physical quantities, so the physical quantities, form factors, should be independent of them. The parameters s_0 and s_0', which are the continuum thresholds of B_c and P mesons, respectively, are determined from the conditions that guarantee the sum rules to have the best stability in the allowed M_1^2 and M_2^2 region. The values of continuum thresholds calculated from the two-point QCD sum rules are taken to be $s_0 = (45\text{–}50)\ \mathrm{GeV}^2$ and $s_0' = (6\text{–}8)\ \mathrm{GeV}^2$ [7,24,36]. The working regions for M_1^2 and M_2^2 are determined by requiring that not only contributions of the higher states and continuum are effectively suppressed, but the gluon condensate contributions are small, which guarantees that the contributions of higher dimensional operators are small. Both conditions are satisfied in the regions $10\ \mathrm{GeV}^2 \le M_1^2 \le 25\ \mathrm{GeV}^2$ and $4\ \mathrm{GeV}^2 \le M_2^2 \le 10\ \mathrm{GeV}^2$.

The dependence of the form factors f_+, f_-, and f_T on M_1^2 and M_2^2 for $B_c \to D_s l^+ l^-/\nu\bar\nu$ are shown in Figs. 3–5, respectively. Figures 6–8 also depict the dependence of the form factors on Borel mass parameters for $B_c \to D l^+ l^-/\nu\bar\nu$. This figures show a good stability of the form factors with respect to the Borel mass parameters in the working regions. Our numerical analysis shows that the

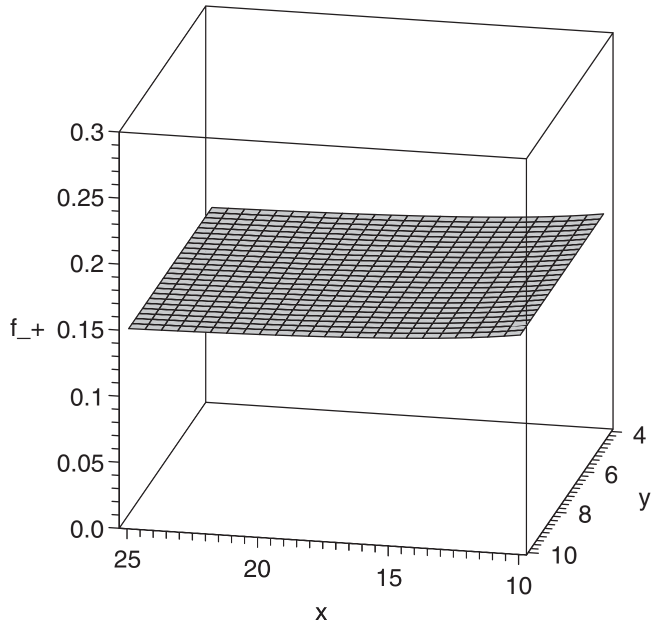

FIG. 3. The dependence of the form factor f_+ on Borel parameters $x = M_1^2$ (GeV^2) and $y = M_2^2$ (GeV^2) for $B_c \to D_s l^+ l^- / \nu \bar{\nu}$.

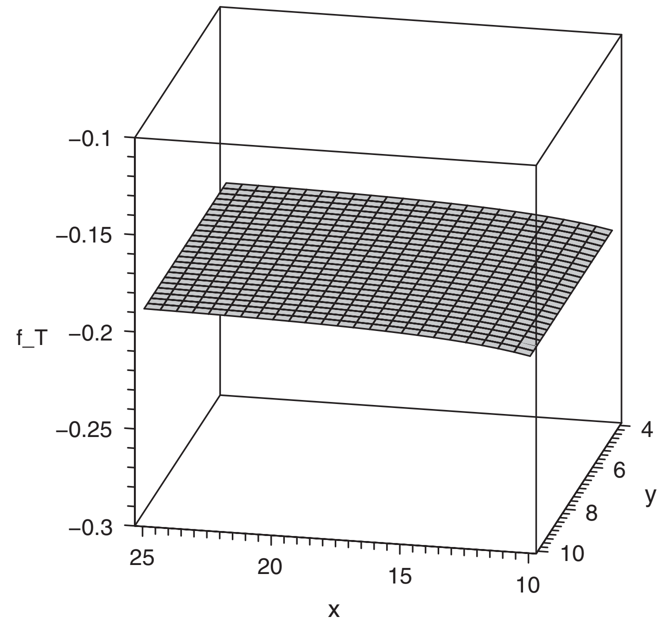

FIG. 5. The dependence of the form factor f_T on Borel parameters $x = M_1^2$ (GeV^2) and $y = M_2^2$ (GeV^2) for $B_c \to D_s l^+ l^-$.

contribution of the nonperturbative part (the gluon condensate diagrams) is about 8% of the total and the main contribution comes from the perturbative part of the form factors.

The values of the form factors at $q^2 = 0$ are shown in Table I: The sum rules for the form factors are truncated at about 2 GeV^2 below the perturbative cut, so to extend our results to the full physical region, we look for parametrization of the form factors in such a way that in the region $0 \leq q^2 \leq 19.26(18.41)$ GeV^2 for $D(D_s)$, this parametrization coincides with the sum rules prediction. Our numerical calculations show that the sufficient parametrization of the form factors with respect to q^2 is as follows:

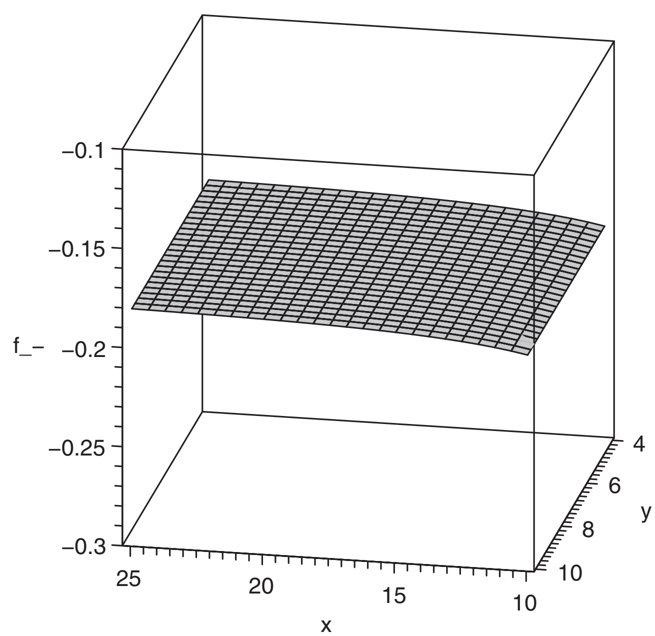

FIG. 4. The same as Fig. 3, but for f_-.

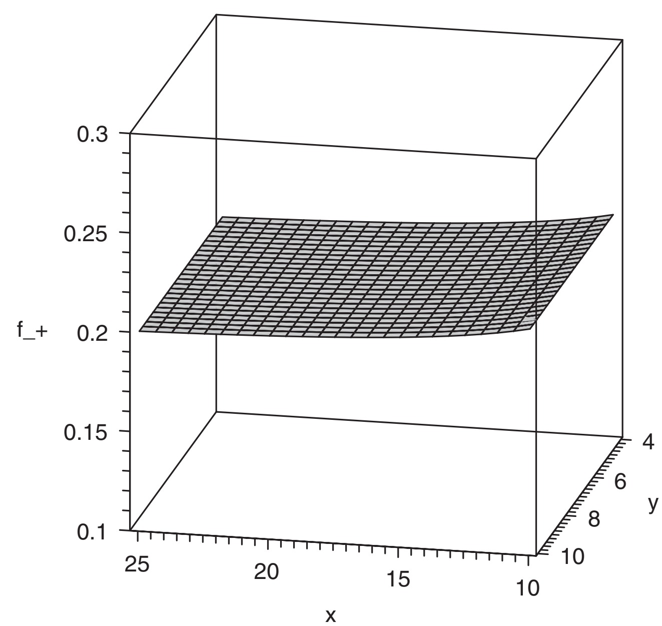

FIG. 6. The dependence of the form factor f_+ on Borel parameters $x = M_1^2$ (GeV^2) and $y = M_2^2$ (GeV^2) for $B_c \to D l^+ l^- / \nu \bar{\nu}$.

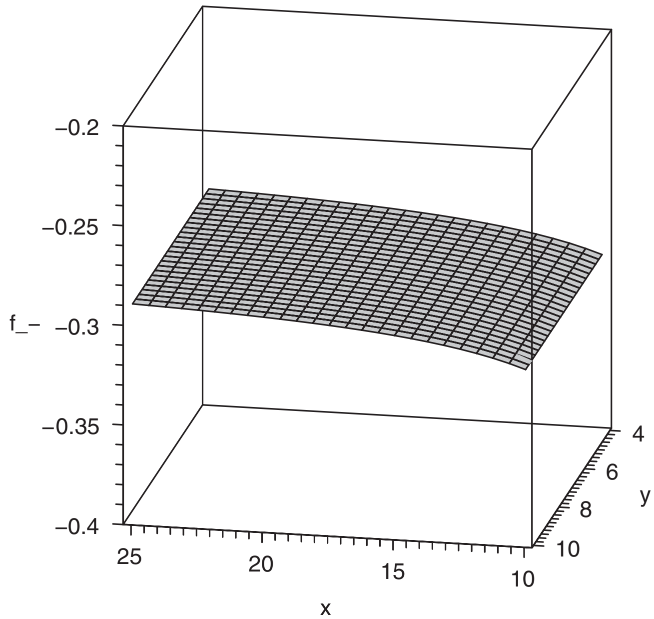

FIG. 7. The same as Fig. 6, but for f_-.

$$f_i(q^2) = \frac{f_i(0)}{1 + \alpha\hat{q} + \beta\hat{q}^2}, \tag{40}$$

where $\hat{q} = q^2/m_{B_c}^2$. The values of the parameters $f_i(0)$, α, and β are given in Tables II and III. The errors are estimated by the variation of the Borel parameters M_1^2 and M_2^2, the variation of the continuum thresholds s_0 and s_0', the variation of b and c quark masses, and leptonic decay constants f_{B_c} and $f_{D,(D_s)}$. The main uncertainty comes

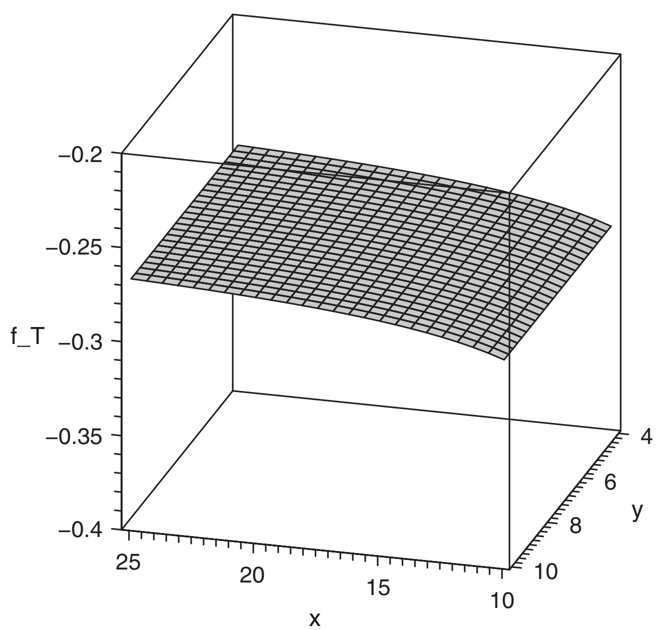

FIG. 8. The dependence of the form factor f_T on Borel parameters $x = M_1^2$ (GeV^2) and $y = M_2^2$ (GeV^2) for $B_c \to D l^+ l^-$.

TABLE I. The values of the form factors at $q^2 = 0$.

	$B_c \to D$	$B_c \to D_s$
$f_+(l^+l^-/\nu\bar{\nu})$	0.22 ± 0.045	0.16 ± 0.032
$f_-(l^+l^-/\nu\bar{\nu})$	-0.29 ± 0.056	-0.18 ± 0.038
$f_T(l^+l^-)$	-0.27 ± 0.054	-0.19 ± 0.040

from the thresholds and the decay constants, which is about $\sim$18% of the central value, while the other uncertainties are small, constituting a few percent.

Now, we compare the extrapolation values for the form factors and their HQET values obtained from Eqs. (36)–(38) in Tables IV and V for $B_c \to D l^+ l^-/\nu\bar{\nu}$ and $B_c \to D_s l^+ l^-/\nu\bar{\nu}$, respectively.

At $y = 1$, called the zero recoil limit, the HQET limit of the form factors is not finite, and at this value we can determine only the ratio of the form factors. For other values of y and corresponding q^2, the behavior of the form factors and their HQET values is the same, i.e., when y increases (q^2 decreases) both the form factors and their HQET values decrease. Moreover, at high q^2 values, the form factors and their HQET values are close to each other while at low q^2, the form factor values are about 2–3 times greater than that of their HQET limit.

At the end of this section we would like to present the values of the branching ratios. Integrating Eqs. (26) and (27) over q^2 in the whole physical region and using the total mean lifetime $\tau \simeq 0.46$ ps of B_c meson [37], the branching ratio of the $B_c \to P(D, D_s) l^+ l^-/\nu\bar{\nu}$ decays are obtained as Table VI. This table also includes a comparison of our results with the prediction of the RCQM. This table presents a good agreement between two models especially when the errors are taken into account. Any experimental measurements on the branching fractions of these decays and those comparisons with the results of the phenomenological models like QCD sum rules could give valuable

TABLE II. Parameters appearing in the form factors of the $B_c \to D l^+ l^-/\nu\bar{\nu}$ decay for $M_1^2 = 15$ GeV^2, $M_2^2 = 8$ GeV^2.

	$f(0)$	α	β
$f_+(l^+l^-/\nu\bar{\nu})$	0.22	−1.10	−2.48
$f_-(l^+l^-/\nu\bar{\nu})$	−0.29	−0.63	−4.06
$f_T(l^+l^-)$	−0.27	−0.72	−3.24

TABLE III. Parameters appearing in the form factors of the $B_c \to D_s l^+ l^-/\nu\bar{\nu}$ decay for $M_1^2 = 15$ GeV^2, $M_2^2 = 8$ GeV^2.

	$f(0)$	α	β
$f_+(l^+l^-/\nu\bar{\nu})$	0.16	−1.55	−2.80
$f_-(l^+l^-/\nu\bar{\nu})$	−0.18	−0.77	−6.71
$f_T(l^+l^-)$	−0.19	−1.43	−3.06

TABLE IV. The comparison of the extrapolation values for the form factors and their HQET limit for $B_c \to D l^+ l^-$ at $M_1^2 = 15\ \mathrm{GeV}^2$, $M_2^2 = 8\ \mathrm{GeV}^2$ and corresponding $T_1 = 1.6$ GeV, $T_2 = 3.2$ GeV.

y	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
q^2	19.26	16.93	14.59	12.25	9.91	7.57	5.23	2.89	0.55
$f_+(q^2)$	2.19	1.36	0.88	0.56	0.36	0.29	0.27	0.24	0.23
$f_-(q^2)$	−3.01	−1.93	−1.20	−0.75	−0.52	−0.39	−0.33	−0.32	−0.31
$f_T(q^2)$	−2.52	−1.53	−1.12	−0.70	−0.49	−0.37	−0.31	−0.29	−0.28
$f_+^{\mathrm{HQET}}(y)$	?	1.35	0.50	0.29	0.20	0.15	0.12	0.10	0.08
$f_-^{\mathrm{HQET}}(y)$	?	−1.90	−0.75	−0.44	−0.30	−0.22	−0.18	−0.15	−0.12
$f_T^{\mathrm{HQET}}(y)$	?	−1.51	−0.58	−0.33	−0.23	−0.17	−0.14	−0.11	−0.10

TABLE V. The comparison of the extrapolation values for the form factors and their HQET limit for $B_c \to D_s l^+ l^-$ at $M_1^2 = 15\ \mathrm{GeV}^2$, $M_2^2 = 8\ \mathrm{GeV}^2$ and corresponding $T_1 = 1.6$ GeV, $T_2 = 3.2$ GeV.

y	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7
q^2	18.41	15.94	13.48	11.02	8.55	6.09	3.63	1.16
$f_+(q^2)$	2.17	1.12	0.79	0.53	0.31	0.22	0.18	0.17
$f_-(q^2)$	−2.50	−1.53	−0.79	−0.43	−0.29	−0.24	−0.23	−0.22
$f_T(q^2)$	−2.25	−1.23	−0.70	−0.37	−0.27	−0.23	−0.21	−0.20
$f_+^{\mathrm{HQET}}(y)$	?	1.08	0.41	0.24	0.16	0.12	0.10	0.08
$f_-^{\mathrm{HQET}}(y)$	?	−1.52	−0.60	−0.35	−0.24	−0.18	−0.14	−0.12
$f_T^{\mathrm{HQET}}(y)$	?	−1.22	−0.46	−0.27	−0.18	−0.14	−0.11	−0.10

TABLE VI. Values for the branching fractions of the $B_c \to P(D, D_s) l^+ l^- / \nu\bar{\nu}$ decays and their comparison with the predictions of the RCQM [6].

Decay	Our results	RCQM [6]
$B_c \to D\nu\bar{\nu}$	$(3.48 \pm 0.71) \times 10^{-8}$	3.28×10^{-8}
$B_c \to D_s\nu\bar{\nu}$	$(0.49 \pm 0.12) \times 10^{-6}$	0.7×10^{-6}
$B_c \to D e^+ e^-$	$(1.34 \pm 0.25) \times 10^{-8}$	...
$B_c \to D_s e^+ e^-$	$(1.47 \pm 0.32) \times 10^{-7}$	...
$B_c \to D\mu^+\mu^-$	$(0.31 \pm 0.06) \times 10^{-8}$	0.44×10^{-8}
$B_c \to D_s\mu^+\mu^-$	$(0.61 \pm 0.15) \times 10^{-7}$	0.97×10^{-7}
$B_c \to D\tau^+\tau^-$	$(0.13 \pm 0.03) \times 10^{-8}$	0.11×10^{-8}
$B_c \to D_s\tau^+\tau^-$	$(0.23 \pm 0.05) \times 10^{-7}$	0.22×10^{-7}

information about the nature of the D_s meson and strong interactions inside it.

In summary, we investigated the $B_c \to P(D, D_s) l^+ l^- / \nu\bar{\nu}$ channels and computed the relevant form factors and their HQET limits considering the gluon condensate corrections. We also evaluated the total decay width and the branching fractions of those decays and compared our results with the predictions of the RCQM. Detection of these channels and their comparison with the phenomenological models like QCD sum rules could give useful information about the structure of the D_s meson.

ACKNOWLEDGMENTS

The authors would like to thank T. M. Aliev and A. Ozpineci for their useful discussions. One of the authors (K. Azizi) thanks Turkish Scientific and Research Council (TUBITAK) for their partial financial support.

APPENDIX A

In this Appendix, the explicit expressions of the coefficients of the gluon condensate entering the sum rules of the form factors f_+, f_-, and f_T are given:

$$
\begin{aligned}
C_+ = {} & -5I_1(3,2,2)m_c{}^6 - 5I_2(3,2,2)m_c{}^6 - 5I_0(3,2,2)m_c{}^6 + 5I_2(3,2,2)m_c{}^5m_b + 5I_1(3,2,2)m_c{}^5m_b + 5I_0(3,2,2)m_c{}^5m_b \\
& + 5I_2(3,2,2)m_c{}^4m_b{}^2 + 5I_1(3,2,2)m_c{}^4m_b{}^2 - 5I_1(3,2,2)m_c{}^3m_b{}^3 - 5I_0(3,2,2)m_c{}^3m_b{}^3 - 5I_2(3,2,2)m_c{}^3m_b{}^3 \\
& - 5I_2(3,2,1)m_c{}^4 + 15I_1^{[0,1]}(3,2,2)m_c{}^4 - 15I_1(2,2,2)m_c{}^4 + 15I_2^{[0,1]}(3,2,2)m_c{}^4 - 5I_2(3,1,2)m_c{}^4 \\
& - 5I_1(3,1,2)m_c{}^4 - 15I_2(4,1,1)m_c{}^4 - 5I_1(3,2,1)m_c{}^4 - 15I_0(4,1,1)m_c{}^4 - 15I_0(2,2,2)m_c{}^4 + 15I_0^{[0,1]}(3,2,2)m_c{}^4 \\
& - 15I_1(4,1,1)m_c{}^4 - 15I_2(2,2,2)m_c{}^4 + 5I_1(3,1,2)m_c{}^3m_b - 10I_0^{[0,1]}(3,2,2)m_c{}^3m_b + 10I_1(2,2,2)m_c{}^3m_b \\
& + 10I_0(3,2,1)m_c{}^3m_b - 10I_2^{[0,1]}(3,2,2)m_c{}^3m_b + 5I_0(3,1,2)m_c{}^3m_b - 10I_2(2,3,1)m_c{}^3m_b + 15I_0(4,1,1)m_c{}^3m_b \\
& + 5I_1(3,2,1)m_c{}^3m_b + 15I_1(4,1,1)m_c{}^3m_b - 10I_1(2,3,1)m_c{}^3m_b + 15I_2(4,1,1)m_c{}^3m_b + 10I_2(2,2,2)m_c{}^3m_b \\
& - 10I_0(2,3,1)m_c{}^3m_b - 10I_1^{[0,1]}(3,2,2)m_c{}^3m_b + 5I_2(3,1,2)m_c{}^3m_b + 10I_0(2,2,2)m_c{}^3m_b + 5I_2(3,2,1)m_c{}^3m_b \\
& + 5I_0^{[0,1]}(3,2,2)m_c{}^2m_b{}^2 + 30I_0(1,4,1)m_c{}^2m_b{}^2 - 5I_0(2,2,2)m_c{}^2m_b{}^2 - 10I_1(3,2,1)m_c{}^2m_b{}^2 \\
& + 30I_2(1,4,1)m_c{}^2m_b{}^2 - 10I_2(3,2,1)m_c{}^2m_b{}^2 + 30I_1(1,4,1)m_c{}^2m_b{}^2 - 5I_0^{[0,1]}(3,2,2)m_cm_b{}^3 - 30I_0(1,4,1)m_cm_b{}^3 \\
& + 10I_2(2,3,1)m_cm_b{}^3 + 10I_1(2,3,1)m_cm_b{}^3 - 5I_1^{[0,1]}(3,2,2)m_cm_b{}^3 - 5I_2^{[0,1]}(3,2,2)m_cm_b{}^3 + 10I_2(3,2,1)m_cm_b{}^3 \\
& - 30I_1(1,4,1)m_cm_b{}^3 + 10I_1(3,2,1)m_cm_b{}^3 - 30I_2(1,4,1)m_cm_b{}^3 + 15I_0(1,4,1)m_b{}^4 - 5I_0(3,2,1)m_b{}^4 \\
& + 15I_1^{[0,1]}(3,2,1)m_c{}^2 - 5I_0(3,1,1)m_c{}^2 - 5I_0(2,2,1)m_c{}^2 - 5I_0(2,1,2)m_c{}^2 + 30I_1^{[0,1]}(2,2,2)m_c{}^2 \\
& + 15I_0^{[0,1]}(4,1,1)m_c{}^2 + 15I_2^{[0,1]}(3,2,1)m_c{}^2 + 20I_0^{[0,1]}(3,2,1)m_c{}^2 - 15I_2^{[0,2]})(3,2,2)m_c{}^2 + 20I_0^{[0,1]}(3,1,2)m_c{}^2 \\
& + 15I_2^{[0,1]}(4,1,1)m_c{}^2 + 15I_1^{[0,1]}(4,1,1)m_c{}^2 - 15I_1^{[0,2]}(3,2,2)m_c{}^2 + 15I_2^{[0,1]}(3,1,2)m_c{}^2 - 10I_1(1,2,2)m_c{}^2 \\
& - 10I_0(1,2,2)m_c{}^2 + 30I_0^{[0,1]}(2,2,2)m_c{}^2 + 30(I_2^{[0,1]}(2,2,2)m_c{}^2 - 10I_2(1,2,2)m_c{}^2 + 15I_1^{[0,1]}(3,1,2)m_c{}^2 \\
& - 15I_0^{[0,2]}(3,2,2)m_c{}^2 - 25I_0(2,2,1)m_cm_b - 5I_2(2,1,2)m_cm_b - 10I_0^{[0,1]}(3,2,1)m_cm_b - 20I_1(2,2,1)m_cm_b \\
& - 40I_0(1,3,1)m_cm_b - 20I_2(2,2,1)m_cm_b - 10I_1^{[0,1]}(2,2,2)m_cm_b - 10I_1(1,2,2)m_cm_b - 10I_2^{[0,1]}(2,2,2)m_cm_b \\
& - 40I_1(1,3,1)m_cm_b - 5I_1(2,1,2)m_cm_b - 5I_2^{[0,1]}(3,2,1)m_cm_b - 10I_0^{[0,1]}(2,2,2)m_cm_b - 15I_1^{[0,1]}(3,1,2)m_cm_b \\
& - 5I_0(2,1,2)m_cm_b + 5I_1^{[0,2]}(3,2,2)m_cm_b - 15I_0^{[0,1]}(3,1,2)m_cm_b + 10I_0^{[0,1]}(2,3,1)m_cm_b - 10I_2(1,2,2)m_cm_b \\
& - 10I_0(1,2,2)m_cm_b - 15I_2^{[0,1]}(3,1,2)m_cm_b + 5I_2^{[0,2]}(3,2,2)m_cm_b - 5I_1^{[0,1]}(3,2,1)m_cm_b + 5I_0^{[0,2]}(3,2,2)m_cm_b \\
& - 40I_2(1,3,1)m_cm_b + 10I_1^{[0,1]}(2,3,1)m_cm_b + 10I_2^{[0,1]}(2,3,1)m_cm_b + 10I_1^{[0,1]}(2,2,2)m_b{}^2 - 30I_2^{[0,1]}(1,4,1)m_b{}^2 \\
& + 15I_0(1,3,1)m_b{}^2 + 5I_0(2,2,1)m_b{}^2 + 5I_2(2,2,1)m_b{}^2 - 5I_2^{[0,2]}(3,2,2)m_b{}^2 - 5I_0(3,1,1)m_b{}^2 - 5I_0(1,2,2)m_b{}^2 \\
& + 10I_1^{[0,1]}(3,2,1)m_b{}^2 - 5I_1^{[0,2]}(3,2,2)m_b{}^2 - 30I_0^{[0,1]}(1,4,1)m_b{}^2 + 10I_2^{[0,1]}(3,2,1)m_b{}^2 + 10I_2^{[0,1]}(2,2,2)m_b{}^2 \\
& - 30I_1^{[0,1]}(1,4,1)m_b{}^2 - 5I_0(2,1,2)m_b{}^2 + 15I_0^{[0,1]}(2,2,2)m_b{}^2 + 5I_1(2,2,1)m_b{}^2 - 10I_1^{[0,2]}(3,1,2) + 10I_2(1,2,1) \\
& + 5I_0(2,1,1) + 10I_0^{[0,1]}(2,1,2) + 10I_1(1,1,2) + 10I_2(1,1,2) + 5I_0(1,2,1) + 10I_0^{[0,1]}(2,2,1) - 10I_2^{[0,2]}(3,2,1) \\
& - 15I_2^{[0,2]}(2,2,2) - 10I_1^{[0,2]}(3,2,1) + 5I_0(1,1,2) + 5I_1^{[0,1]}(2,2,1) + 5I_2^{[0,1]}(2,1,2) + 5I_1^{[0,1]}(2,1,2) + 5I_2^{[0,1]}(2,2,1) \\
& - 15I_0^{[0,2]}(2,2,2) - 15I_1^{[0,2]}(2,2,2) + 10I_1(1,2,1) + 10I_1(2,1,1) + 10I_2(2,1,1) + 10I_0^{[0,1]}(3,1,1) \\
& - 10I_2^{[0,2]}(3,1,2) + 10I_0^{[0,1]}(1,2,2) + 10I_1^{[0,1]}(1,2,2) + 10I_2^{[0,1]}(1,2,2),
\end{aligned}
$$

$$
\begin{aligned}
C_- = {} & 5I_2(3,2,2)m_c{}^6 - 5I_1(3,2,2)m_c{}^6 - 5I_0(3,2,2)m_c{}^5m_b + 5I_1(3,2,2)m_c{}^5m_b - 5I_2(3,2,2)m_c{}^5m_b + 5I_1(3,2,2)m_c{}^4m_b{}^2 \\
& - 5I_2(3,2,2)m_c{}^4m_b{}^2 - 5I_1(3,2,2)m_c{}^3m_b{}^3 + 5I_0(3,2,2)m_c{}^3m_b{}^3 + 5I_2(3,2,2)m_c{}^3m_b{}^3 - 15I_2^{[0,1]}(3,2,2)m_c{}^4 \\
& - 5I_1(3,1,2)m_c{}^4 + 15I_2(2,2,2)m_c{}^4 - 15I_1(4,1,1)m_c{}^4 + 5I_2(3,2,1)m_c{}^4 + 5I_2(3,1,2)m_c{}^4 + 15I_1^{[0,1]}(3,2,2)m_c{}^4 \\
& - 5I_1(3,2,1)m_c{}^4 - 15I_1(2,2,2)m_c{}^4 + 15I_2(4,1,1)m_c{}^4 + 5I_1(3,1,2)m_c{}^3m_b + 10I_2^{[0,1]}(3,2,2)m_c{}^3m_b \\
& + 10I_0^{[0,1]}(3,2,2)m_c{}^3m_b - 5I_2(3,1,2)m_c{}^3m_b - 10I_1(2,3,1)m_c{}^3m_b - 15I_0(4,1,1)m_c{}^3m_b + 10I_2(2,3,1)m_c{}^3m_b \\
& + 15I_1(4,1,1)m_c{}^3m_b - 15I_2(4,1,1)m_c{}^3m_b - 10I_0(2,3,1)m_c{}^3m_b + 5I_0(3,2,1)m_c{}^3m_b - 5I_0(3,1,2)m_c{}^3m_b \\
& - 10I_2(2,2,2)m_c{}^3m_b - 10I_0(2,2,2)m_c{}^3m_b - 10I_1^{[0,1]}(3,2,2)m_c{}^3m_b + 5I_1(3,2,1)m_c{}^3m_b - 5I_2(3,2,1)m_c{}^3m_b \\
& + 10I_1(2,2,2)m_c{}^3m_b - 5I_0^{[0,1]}(3,2,2)m_c{}^2m_b{}^2 + 5I_0(2,2,2)m_c{}^2m_b{}^2 + 30I_1(1,4,1)m_c{}^2m_b{}^2 + 10I_2(3,2,1)m_c{}^2m_b{}^2 \\
& - 10I_1(3,2,1)m_c{}^2m_b{}^2 - 30I_2(1,4,1)m_c{}^2m_b{}^2 + 5I_0^{[0,1]}(3,2,2)m_cm_b{}^3 + 30I_0(1,4,1)m_cm_b{}^3 - 30I_1(1,4,1)m_cm_b{}^3 \\
& + 10I_1(2,3,1)m_cm_b{}^3 + 10I_1(3,2,1)m_cm_b{}^3 - 5I_1^{[0,1]}(3,2,2)m_cm_b{}^3 + 30I_2(1,4,1)m_cm_b{}^3 - 10I_2(2,3,1)m_cm_b{}^3 \\
& - 10I_2(3,2,1)m_cm_b{}^3 + 5I_2^{[0,1]}(3,2,2)m_cm_b{}^3 + 5I_0(3,2,1)m_b{}^4 - 15I_0(1,4,1)m_b{}^4 + 15I_1^{[0,1]}(4,1,1)m_c{}^2 \\
& - 10I_0(2,2,1)m_c{}^2 + 10I_0^{[0,1]}(3,2,1)m_c{}^2 + 10I_0(2,1,2)m_c{}^2 + 30I_1^{[0,1]}(2,2,2)m_c{}^2 + 10I_2(1,2,2)m_c{}^2 - 10I_1(1,2,2)m_c{}^2 \\
& - 30I_2^{[0,1]}(2,2,2)m_c{}^2 - 15I_2^{[0,1]}(3,1,2)m_c{}^2 - 15I_2^{[0,1]}(4,1,1)m_c{}^2 + 15I_1^{[0,1]}(3,1,2)m_c{}^2 - 15I_2^{[0,1]}(3,2,1)m_c{}^2 \\
& + 15I_1^{[0,1]}(3,2,1)m_c{}^2 - 10I_0^{[0,1]}(3,1,2)m_c{}^2 - 15I_1^{[0,2]}(3,2,2)m_c{}^2 + 15I_2^{[0,2]}(3,2,2)m_c{}^2 + 40I_2(1,3,1)m_cm_b \\
& - 5I_0^{[0,2]}(3,2,2)m_cm_b - 5I_1^{[0,1]}(3,2,1)m_cm_b + 15I_0^{[0,1]}(3,1,2)m_cm_b - 10I_1(1,2,2)m_cm_b + 15I_2^{[0,1]}(3,1,2)m_cm_b \\
& - 15I_1^{[0,1]}(3,1,2)m_cm_b - 10I_1^{[0,1]}(2,2,2)m_cm_b + 5I_2^{[0,1]}(3,2,1)m_cm_b + 20I_0(1,3,1)m_cm_b + 5I_1^{[0,2]}(3,2,2)m_cm_b \\
& + 5I_2(2,1,2)m_cm_b + 20I_0(2,2,1)m_cm_b - 20I_1(2,2,1)m_cm_b + 10I_2(1,2,2)m_cm_b + 10I_0^{[0,1]}(2,2,2)m_cm_b \\
& - 10I_2^{[0,1]}(2,3,1)m_cm_b + 15I_0^{[0,1]}(3,2,1)m_cm_b - 5I_1(2,1,2)m_cm_b + 10I_2^{[0,1]}(2,2,2)m_cm_b + 10I_1^{[0,1]}(2,3,1)m_cm_b \\
& + 10I_0(1,2,2)m_cm_b + 5I_0(2,1,2)m_cm_b - 5I_2^{[0,2]}(3,2,2)m_cm_b + 20I_2(2,2,1)m_cm_b - 40I_1(1,3,1)m_cm_b \\
& + 10I_0^{[0,1]}(2,3,1)m_cm_b - 15I_0(1,3,1)m_b^2 - 30I_1^{[0,1]}(1,4,1)m_b^2 + 5I_0(2,1,2)m_b^2 - 10I_2^{[0,1]}(3,2,1)m_b{}^2 \\
& + 10I_1^{[0,1]}(3,2,1)m_b{}^2 - 5I_2(2,2,1)m_b{}^2 + 5I_1(2,2,1)m_b{}^2 - 5I_1^{[0,2]}(3,2,2)m_b{}^2 + 5I_0(3,1,1)m_b{}^2 + 5I_2^{[0,2]}(3,2,2)m_b{}^2 \\
& + 30I_2^{[0,1]}(1,4,1)m_b{}^2 - 5I_0^{[0,1]}(2,2,2)m_b{}^2 + 10I_1^{[0,1]}(2,2,2)m_b{}^2 - 10I_0(2,2,1)m_b{}^2 - 10I_2^{[0,1]}(2,2,2)m_b{}^2 - 10I_2(2,1,1) \\
& + 10I_1(1,1,2) - 10I_2(1,1,2) + 10I_1^{[0,1]}(1,2,2) - 10I_2(1,2,1) + 5I_0^{[0,1]}(2,2,1) + 10I_2^{[0,2]}(3,2,1) - 15I_1^{[0,2]}(2,2,2) \\
& + 15I_2^{[0,2]}(2,2,2) + 5I_1^{[0,1]}(2,1,2) - 10I_1^{[0,2]}(3,2,1) - 10I_1^{[0,2]}(3,1,2) + 10I_2^{[0,2]}(3,1,2) - 5I_0^{[0,1]}(2,1,2) - 10I_2^{[0,1]}(1,2,2) \\
& + 5I_1^{[0,1]}(2,2,1) - 5I_2^{[0,1]}(2,1,2) - 5I_2^{[0,1]}(2,2,1) + 10I_1(1,2,1) + 10I_1(2,1,1),
\end{aligned}
$$

$$
\begin{aligned}
C_T = {} & -5I_0(3,2,2)m_c{}^4m_b{}^3 + 5I_0(3,2,2)m_c{}^2m_b{}^5 - 5I_0(3,1,2)m_c{}^4m_b + 5I_0(3,2,1)m_c{}^4m_b - 5I_0(3,1,2)m_c{}^3m_b{}^2 \\
& + 5I_0(3,2,1)m_c{}^3m_b{}^2 + 10I_0^{[0,1]}(3,2,2)m_c{}^2m_b{}^3 - 15I_0(3,2,1)m_c{}^2m_b{}^3 - 15I_0(4,1,1)m_c{}^2m_b{}^3 - 10I_0(2,3,1)m_c{}^2m_b{}^3 \\
& + 5I_0(3,1,2)m_c{}^2m_b{}^3 - 10I_0(2,2,2)m_c{}^2m_b{}^3 + 5I_0^{[0,1]}(3,2,2)m_b{}^5 - 5I_0(2,2,2)m_b{}^5 - 10I_0(2,3,1)m_b{}^5 - 10I_0(3,2,1)m_b{}^5 \\
& + 30I_0(1,4,1)m_b{}^5 + 10I_0(3,1,1)m_c{}^3 - 10I_0(2,1,2)m_c{}^2m_b - 10I_0(3,1,1)m_c{}^2m_b - 10I_0^{[0,1]}(3,2,1)m_c{}^2m_b \\
& + 10I_0^{[0,1]}(3,1,2)m_c{}^2m_b - 5I_0^{[0,1]}(3,2,1)m_cm_b{}^2 + 5I_0(2,2,1)m_cm_b{}^2 - 5I_0(2,1,2)m_cm_b{}^2 + 5I_0^{[0,1]}(3,1,2)m_cm_b{}^2 \\
& - 10I_0(1,2,2)m_b{}^3 - 5I_0(2,2,1)m_b{}^3 - 5I_0^{[0,2]}(3,2,2)m_b{}^3 + 50I_0(1,3,1)m_b{}^3 + 15I_0^{[0,1]}(3,1,2)m_b{}^3 + 5I_0^{[0,1]}(3,2,1)m_b{}^3 \\
& - 10I_0(3,1,1)m_b{}^3 - 10I_0(2,1,2)m_b{}^3 + 10I_0^{[0,1]}(2,3,1)m_b{}^3 + 10I_0^{[0,1]}(2,2,2)m_b{}^3 - 10I_0^{[0,1]}(3,1,1)m_c + 10I_0(2,1,1)m_c \\
& + 30I_0(1,2,1)m_b - 10I_0(1,1,2)m_b + 5I_0(2,1,1)m_b - 5I_0^{[0,2]}(3,1,2)m_b + 5I_0^{[0,2]}(3,2,1)m_b + 10I_0^{[0,1]}(2,1,2)m_b,
\end{aligned}
$$

where

$$\hat{I}_n^{[i,j]}(a,b,c) = (M_1^2)^i (M_2^2)^j \frac{d^i}{d(M_1^2)^i}\frac{d^j}{d(M_2^2)^j}[(M_1^2)^i (M_2^2)^j \hat{I}_n(a,b,c)].$$

APPENDIX B

In this Appendix, the explicit expressions of the coefficients of the gluon condensate entering the HQET limit of the form factors $\tilde{f}_+^{\rm HQET}$, $\tilde{f}_-^{\rm HQET}$, and $\tilde{f}_T^{\rm HQET}$ are given. Note that only in this Appendix, by $\hat{I}_i(a,b,c)$ we mean $\hat{I}_i(a,b,c)^{\rm HQET}$ which are defined in Eq. (32):

$$\begin{aligned}
C_+^{\rm HQET} = {}& 2\frac{\hat{I}_2^{[0,1]}(3,2,2){m_b}^5}{\sqrt{z}} + \frac{\hat{I}_0^{[0,1]}(3,2,2){m_b}^4}{\sqrt{z}} - 32\frac{\hat{I}_2(2,1,1){m_b}^4}{\sqrt{z}} - 4\frac{\hat{I}_0(2,2,1){m_b}^4}{\sqrt{z}} - 8\frac{\hat{I}_0(2,1,1){m_b}^3}{\sqrt{z}} \\
&+ 16\frac{\hat{I}_2^{[0,2]}(3,1,2){m_b}^3}{\sqrt{z}} - 8\frac{\hat{I}_0^{[0,1]}(3,1,1){m_b}^2}{\sqrt{z}} + 4\frac{\hat{I}_0(2,1,2){m_b}^4}{\sqrt{z}} + 12\frac{\hat{I}_2^{[0,1]}(3,1,2){m_b}^4}{\sqrt{z}} - 16\frac{\hat{I}_0(1,1,2){m_b}^4}{\sqrt{z}} \\
&- 12\frac{\hat{I}_0^{[0,1]}(2,2,2){m_b}^4}{\sqrt{z}} - 8\frac{\hat{I}_2^{[0,1]}(3,2,1){m_b}^4}{\sqrt{z}} + 8\frac{\hat{I}_0(1,2,2){m_b}^5}{\sqrt{z}} + 8\frac{\hat{I}_2(2,1,2){m_b}^5}{\sqrt{z}} - 16\frac{\hat{I}_2^{[0,1]}(2,1,2){m_b}^4}{\sqrt{z}} \\
&+ 6\frac{\hat{I}_0^{[0,1]}(3,1,2){m_b}^3}{\sqrt{z}} - 16\frac{\hat{I}_0^{[0,1]}(2,1,2){m_b}^3}{\sqrt{z}} - 64\frac{\hat{I}_2(1,1,2){m_b}^5}{\sqrt{z}} + 4\frac{\hat{I}_2^{[0,2]}(3,2,2){m_b}^4}{\sqrt{z}} - 4\frac{\hat{I}_2(3,2,1){m_b}^5}{\sqrt{z}} \\
&- 8\frac{\hat{I}_2(2,2,1){m_b}^5}{\sqrt{z}} - 16\frac{\hat{I}_2^{[0,1]}(2,2,2){m_b}^5}{\sqrt{z}} + 16\frac{\hat{I}_1^{[0,2]}(3,1,2){m_b}^3}{z} - 2\frac{\hat{I}_0^{[0,2]}(3,2,2){m_b}^3}{z} + 24\frac{\hat{I}_0^{[0,2]}(2,2,2){m_b}^3}{z} \\
&- 6\frac{\hat{I}_2^{[0,1]}(4,1,1){m_b}^3}{z} + 4\frac{\hat{I}_0^{[0,1]}(3,2,1){m_b}^3}{z} - 64\frac{\hat{I}_2^{[0,1]}(1,2,2){m_b}^5}{z} - 16\frac{\hat{I}_0^{[0,1]}(2,2,1){m_b}^3}{z} - 64\frac{\hat{I}_1(1,1,2){m_b}^5}{z} \\
&+ 2\frac{\hat{I}_0(3,1,1){m_b}^3}{z} - 8\frac{\hat{I}_1(2,2,1){m_b}^5}{z} - 3/2\frac{\hat{I}_0(4,1,1){m_b}^3}{z} + 16\frac{\hat{I}_2^{[0,2]}(3,2,1){m_b}^3}{z} + 2\frac{\hat{I}_0(2,2,2){m_b}^5}{z} \\
&+ 1/2\frac{\hat{I}_0(3,2,2){m_b}^5}{z} - 24\frac{\hat{I}_0(1,3,1){m_b}^5}{z} + \frac{\hat{I}_2(3,2,2){m_b}^6}{z} + 16\frac{\hat{I}_0(1,2,2){m_b}^5}{z} - 8\frac{\hat{I}_0^{[0,1]}(3,1,2){m_b}^3}{z} \\
&- 3\frac{\hat{I}_0^{[0,1]}(4,1,1){m_b}^2}{z} + 2\frac{\hat{I}_1^{[0,1]}(3,2,2){m_b}^5}{z} - 4\frac{\hat{I}_1(3,2,1){m_b}^5}{z} - 2\frac{\hat{I}_2(3,1,2){m_b}^5}{z} - 16\frac{\hat{I}_2^{[0,1]}(2,2,1){m_b}^4}{z} \\
&+ 8\frac{\hat{I}_1(2,1,2){m_b}^5}{z} + 4\frac{\hat{I}_2(3,2,1){m_b}^5}{z} - 4\frac{\hat{I}_2^{[0,2]}(3,2,2){m_b}^4}{z} + 12\frac{\hat{I}_0^{[0,1]}(3,1,2){m_b}^4}{z} - 32\frac{\hat{I}_1(2,1,1){m_b}^4}{z} \\
&+ 16\frac{\hat{I}_2^{[0,1]}(2,2,2){m_b}^5}{z} - 12\frac{\hat{I}_2^{[0,1]}(3,1,2){m_b}^4}{z} + 32\frac{\hat{I}_2(2,2,1){m_b}^5}{z} - \frac{\hat{I}_0(3,1,2){m_b}^4}{z} + 20\frac{\hat{I}_0(2,2,1){m_b}^4}{z} \\
&- 8\frac{\hat{I}_1^{[0,1]}(3,2,1){m_b}^4}{z} + 8\frac{\hat{I}_0^{[0,1]}(2,2,2){m_b}^4}{z} - 3\frac{\hat{I}_2(4,1,1){m_b}^4}{z} - \frac{\hat{I}_0^{[0,1]}(3,2,2){m_b}^4}{z} + 48\frac{\hat{I}_2^{[0,2]}(2,2,2){m_b}^4}{z} \\
&- 8\frac{\hat{I}_2(2,3,1){m_b}^6}{z} - 16\frac{\hat{I}_0(1,2,1){m_b}^4}{z} - 16\frac{\hat{I}_1^{[0,1]}(2,2,2){m_b}^5}{z} + 4\frac{\hat{I}_0(2,1,2){m_b}^4}{z} - 64\frac{\hat{I}_2(1,2,1){m_b}^5}{z} \\
&- 32\frac{\hat{I}_0^{[0,1]}(1,2,2){m_b}^4}{z} - 16\frac{\hat{I}_0^{[0,1]}(2,1,2){m_b}^4}{z} + 32\frac{\hat{I}_2(1,2,2){m_b}^6}{z}
\end{aligned}$$

$$
\begin{aligned}
&+ 4\frac{\hat{I}_1^{[0,2]}(3,2,2)m_b^4}{z} - 12\frac{\hat{I}_0(1,4,1)m_b^6}{z} + 4\frac{\hat{I}_2^{[0,1]}(3,2,1)m_b^4}{z} + 3\frac{\hat{I}_2(4,1,1)m_b^4}{z^{3/2}} + 16\frac{\hat{I}_1^{[0,1]}(2,2,2)m_b^5}{z^{3/2}} - 3\frac{\hat{I}_1(4,1,1)m_b^4}{z^{3/2}} \\
&+ 4\frac{\hat{I}_1(3,2,1)m_b^5}{z^{3/2}} - 12\frac{\hat{I}_1^{[0,1]}(3,1,2)m_b^4}{z^{3/2}} - 16\frac{\hat{I}_1^{[0,1]}(2,2,1)m_b^4}{z^{3/2}} - 4\frac{\hat{I}_0(2,2,2)m_b^5}{z^{3/2}} + 6\frac{\hat{I}_0^{[0,2]}(3,2,2)m_b^3}{z^{3/2}} \\
&+ 48\frac{\hat{I}_0^{[0,1]}(1,4,1)m_b^5}{z^{3/2}} + 16\frac{\hat{I}_1^{[0,2]}(3,2,1)m_b^3}{z^{3/2}} - 64\frac{\hat{I}_1(1,2,1)m_b^5}{z^{3/2}} - 8\frac{\hat{I}_0^{[0,1]}(2,3,1)m_b^4}{z^{3/2}} - 4\frac{\hat{I}_1^{[0,2]}(3,2,2)m_b^4}{z^{3/2}} \\
&+ 4\frac{\hat{I}_1^{[0,1]}(3,2,1)m_b^4}{z^{3/2}} + 3/2\frac{\hat{I}_0(4,1,1)m_b^3}{z^{3/2}} + 2\frac{\hat{I}_2(3,1,2)m_b^5}{z^{3/2}} + 12\frac{\hat{I}_2^{[0,2]}(3,2,2)m_b^4}{z^{3/2}} - 48\frac{\hat{I}_2^{[0,1]}(2,2,2)m_b^5}{z^{3/2}} \\
&- 64\frac{\hat{I}_1^{[0,1]}(1,2,2)m_b^5}{z^{3/2}} - 64\frac{\hat{I}_1^{[0,1]}(1,2,2)m_b^5}{z^{3/2}} - 2\frac{\hat{I}_0(3,2,1)m_b^4}{z^{3/2}} + 4\frac{\hat{I}_0(2,2,1)m_b^4}{z^{3/2}} - 2\frac{\hat{I}_2(3,2,1)m_b^5}{z^{3/2}} + 16\frac{\hat{I}_0(1,2,2)m_b^5}{z^{3/2}} \\
&+ 32\frac{\hat{I}_1(1,2,2)m_b^6}{z^{3/2}} - 8\frac{\hat{I}_1(2,3,1)m_b^6}{z^{3/2}} + 32\frac{\hat{I}_2(1,2,2)m_b^6}{z^{3/2}} - 6\frac{\hat{I}_1^{[0,1]}(4,1,1)m_b^3}{z^{3/2}} + 128\frac{\hat{I}_2(1,3,1)m_b^6}{z^{3/2}} + 32\frac{\hat{I}_1(2,2,1)m_b^5}{z^{3/2}} \\
&+ 4\frac{\hat{I}_2^{[0,1]}(3,2,2)m_b^5}{z^{3/2}} - 24\frac{\hat{I}_0^{[0,1]}(2,2,2)m_b^4}{z^{3/2}} - 8\frac{\hat{I}_0^{[0,1]}(3,2,1)m_b^3}{z^{3/2}} + 64\frac{\hat{I}_0(1,3,1)m_b^5}{z^{3/2}} - 2\frac{\hat{I}_1(3,1,2)m_b^5}{z^{3/2}} \\
&- 16\frac{\hat{I}_2^{[0,1]}(2,3,1)m_b^5}{z^{3/2}} - 8\frac{\hat{I}_2(2,2,2)m_b^6}{z^{3/2}} + 24\frac{\hat{I}_0(1,4,1)m_b^6}{z^{3/2}} + \frac{\hat{I}_1(3,2,2)m_b^6}{z^{3/2}} + 48\frac{\hat{I}_1^{[0,2]}(2,2,2)m_b^4}{z^{3/2}} - \frac{\hat{I}_2(3,2,2)m_b^6}{z^{3/2}} \\
&+ 48\frac{\hat{I}_2(1,4,1)m_b^7}{z^{3/2}} + 96\frac{\hat{I}_2^{[0,1]}(1,4,1)m_b^6}{z^{3/2}} + 2\frac{\hat{I}_0^{[0,1]}(3,2,2)m_b^4}{z^{3/2}} - 12\frac{\hat{I}_2^{[0,1]}(3,2,1)m_b^4}{z^{3/2}} + 6\frac{\hat{I}_0(2,2,2)m_b^5}{z^2} \\
&- 2\frac{\hat{I}_1(3,2,1)m_b^5}{z^2} - 48\frac{\hat{I}_1^{[0,1]}(2,2,2)m_b^5}{z^2} - 12\frac{\hat{I}_1^{[0,1]}(3,2,1)m_b^4}{z^2} + 2\frac{\hat{I}_1(3,1,2)m_b^5}{z^2} - \frac{\hat{I}_2(3,2,2)m_b^6}{z^2} + 4\frac{\hat{I}_0(2,3,1)m_b^5}{z^2} \\
&+ 2\frac{\hat{I}_2(3,2,1)m_b^5}{z^2} - 1/2\frac{\hat{I}_0(3,2,2)m_b^5}{z^2} + 48\frac{\hat{I}_1(1,4,1)m_b^7}{z^2} - 3\frac{\hat{I}_0^{[0,1]}(3,2,2)m_b^4}{z^2} + 3\frac{\hat{I}_1(4,1,1)m_b^4}{z^2} - 48\frac{\hat{I}_2(1,4,1)m_b^7}{z^2} \\
&+ 12\frac{\hat{I}_1^{[0,2]}(3,2,2)m_b^4}{z^2} - 6\frac{\hat{I}_2^{[0,1]}(3,2,2)m_b^5}{z^2} + 12\frac{\hat{I}_2(2,2,2)m_b^6}{z^2} - 24\frac{\hat{I}_0(1,4,1)m_b^6}{z^2} - 8\frac{\hat{I}_1(2,2,2)m_b^6}{z^2} + 8\frac{\hat{I}_2(2,3,1)m_b^6}{z^2} \\
&+ 96\frac{\hat{I}_1^{[0,1]}(1,4,1)m_b^6}{z^2} + 4\frac{\hat{I}_1^{[0,1]}(3,2,2)m_b^5}{z^2} - 16\frac{\hat{I}_1^{[0,1]}(2,3,1)m_b^5}{z^2} - \frac{\hat{I}_1(3,2,2)m_b^6}{z^2} + 128\frac{\hat{I}_1(1,3,1)m_b^6}{z^2} \\
&+ 32\frac{\hat{I}_1(1,2,2)m_b^6}{z^2} + 2\frac{\hat{I}_1(3,2,1)m_b^5}{z^{5/2}} - 48\frac{\hat{I}_1(1,4,1)m_b^7}{z^{5/2}} + 12\frac{\hat{I}_1(2,2,2)m_b^6}{z^{5/2}} + 1/2\frac{\hat{I}_0(3,2,2)m_b^5}{z^{5/2}} - 6\frac{\hat{I}_1^{[0,1]}(3,2,2)m_b^5}{z^{5/2}} \\
&+ \frac{\hat{I}_2(3,2,2)m_b^6}{z^{5/2}} - \frac{\hat{I}_1(3,2,2)m_b^6}{z^{5/2}} + 8\frac{\hat{I}_1(2,3,1)m_b^6}{z^{5/2}} + \frac{\hat{I}_1(3,2,2)m_b^6}{z^3} + 4\hat{I}_0(2,1,2)m_b^4 + 2\hat{I}_0(3,1,1)m_b^3 \\
&+ \hat{I}_0(3,2,1)m_b^4,
\end{aligned}
$$

$$
\begin{aligned}
C_-^{\mathrm{HQET}} = & -8\frac{\hat{I}_2(2,1,2)m_b{}^5}{\sqrt{z}} + 32\frac{\hat{I}_2(2,1,1)m_b{}^4}{\sqrt{z}} - 4\frac{\hat{I}_0(2,1,2)m_b{}^4}{\sqrt{z}} + 4\frac{\hat{I}_0^{[0,1]}(2,2,2)m_b{}^4}{\sqrt{z}} - 12\frac{\hat{I}_2^{[0,1]}(3,1,2)m_b{}^4}{\sqrt{z}} \\
& + 64\frac{\hat{I}_2(1,1,2)m_b{}^5}{\sqrt{z}} - 16\frac{\hat{I}_2^{[0,2]}(3,1,2)m_b{}^3}{\sqrt{z}} - 6\frac{\hat{I}_0^{[0,1]}(3,1,2)m_b{}^3}{\sqrt{z}} + 8\frac{\hat{I}_0^{[0,1]}(2,1,2)m_b{}^3}{\sqrt{z}} + 8\frac{\hat{I}_2^{[0,1]}(3,2,1)m_b{}^4}{\sqrt{z}} \\
& + 16\frac{\hat{I}_2^{[0,1]}(2,2,2)m_b{}^5}{\sqrt{z}} + 8\frac{\hat{I}_0(2,2,1)m_b{}^4}{\sqrt{z}} + 16\frac{\hat{I}_2^{[0,1]}(2,1,2)m_b{}^4}{\sqrt{z}} - 2\frac{\hat{I}_2^{[0,1]}(3,2,2)m_b{}^5}{\sqrt{z}} + 4\frac{\hat{I}_2(3,2,1)m_b{}^5}{\sqrt{z}} \\
& - 4\frac{\hat{I}_2^{[0,2]}(3,2,2)m_b{}^4}{\sqrt{z}} + 8\frac{\hat{I}_2(2,2,1)m_b{}^5}{\sqrt{z}} - \frac{\hat{I}_0^{[0,1]}(3,2,2)m_b{}^4}{\sqrt{z}} - 4\frac{\hat{I}_2^{[0,1]}(3,2,1)m_b{}^4}{z} + 4\frac{\hat{I}_2^{[0,2]}(3,2,2)m_b{}^4}{z} \\
& - 16\frac{\hat{I}_0(1,2,2)m_b{}^5}{z} - 16\frac{\hat{I}_1^{[0,1]}(2,1,2)m_b{}^4}{z} - \frac{\hat{I}_2(3,2,2)m_b{}^6}{z} + 16\frac{\hat{I}_1^{[0,2]}(3,1,2)m_b{}^3}{z} - 8\frac{\hat{I}_1^{[0,1]}(3,2,1)m_b{}^4}{z} \\
& + 3\frac{\hat{I}_2(4,1,1)m_b{}^4}{z} - 32\frac{\hat{I}_2(2,2,1)m_b{}^5}{z} + 12\frac{\hat{I}_2^{[0,1]}(3,1,2)m_b{}^4}{z} + 16\frac{\hat{I}_2^{[0,1]}(2,2,1)m_b{}^4}{z} - 4\frac{\hat{I}_1(3,2,1)m_b{}^5}{z} \\
& - 2\frac{\hat{I}_0(2,2,2)m_b{}^5}{z} + \frac{\hat{I}_0(3,1,2)m_b{}^4}{z} - 16\frac{\hat{I}_1^{[0,1]}(2,2,2)m_b{}^5}{z} - 4\frac{\hat{I}_2(3,2,1)m_b{}^5}{z} + 4\frac{\hat{I}_0^{[0,1]}(3,1,2)m_b{}^3}{z} \\
& + 2\frac{\hat{I}_0^{[0,2]}(3,2,2)m_b{}^3}{z} - 64\frac{\hat{I}_1(1,1,2)m_b{}^5}{z} - 16\frac{\hat{I}_2^{[0,1]}(2,2,2)m_b{}^5}{z} - 6\frac{\hat{I}_0^{[0,1]}(3,2,1)m_b{}^3}{z} + 4\frac{\hat{I}_1^{[0,2]}(3,2,2)m_b{}^4}{z} \\
& - 8\frac{\hat{I}_0^{[0,1]}(2,2,1)m_b{}^3}{z} - 16\frac{\hat{I}_2^{[0,2]}(3,2,1)m_b{}^3}{z} + 12\frac{\hat{I}_0(1,4,1)m_b{}^6}{z} - 8\frac{\hat{I}_0(2,1,2)m_b{}^4}{z} + \frac{\hat{I}_0^{[0,1]}(3,2,2)m_b{}^4}{z} \\
& + 3/2\frac{\hat{I}_0(4,1,1)m_b{}^3}{z} + 8\frac{\hat{I}_1(2,1,2)m_b{}^5}{z} + 8\frac{\hat{I}_2(2,3,1)m_b{}^6}{z} - 1/2\frac{\hat{I}_0(3,2,2)m_b{}^5}{z} - 48\frac{\hat{I}_2^{[0,2]}(2,2,2)m_b{}^4}{z} \\
& - 8\frac{\hat{I}_0^{[0,1]}(2,2,2)m_b{}^4}{z} + 6\frac{\hat{I}_2^{[0,1]}(4,1,1)m_b{}^3}{z} + 64\frac{\hat{I}_2^{[0,1]}(1,2,2)m_b{}^5}{z} - 32\frac{\hat{I}_1(2,1,1)m_b{}^4}{z} + 2\frac{\hat{I}_2(3,1,2)m_b{}^5}{z} \\
& - 16\frac{\hat{I}_0(2,2,1)m_b{}^4}{z} - 32\frac{\hat{I}_2(1,2,2)m_b{}^6}{z} + 2\frac{\hat{I}_1^{[0,1]}(3,2,2)m_b{}^5}{z} + 24\frac{\hat{I}_0(1,3,1)m_b{}^5}{z} - 8\frac{\hat{I}_1(2,2,1)m_b{}^5}{z} \\
& + 12\frac{(C_{0,1})(3,1,2)m_b{}^4}{z} + 64\frac{\hat{I}_2(1,2,1)m_b{}^5}{z} + 48\frac{\hat{I}_2^{[0,1]}(2,2,2)m_b{}^5}{z^{3/2}} + 32\frac{\hat{I}_1(2,2,1)m_b{}^5}{z^{3/2}} - 8\frac{\hat{I}_1(2,3,1)m_b{}^6}{z^{3/2}} \\
& + 16\frac{\hat{I}_1^{[0,2]}(3,2,1)m_b{}^3}{z^{3/2}} - 96\frac{\hat{I}_2^{[0,1]}(1,4,1)m_b{}^6}{z^{3/2}} - 32\frac{\hat{I}_2(1,2,2)m_b{}^6}{z^{3/2}} - 64\frac{\hat{I}_1^{[0,1]}(1,2,2)m_b{}^5}{z^{3/2}} + 4\frac{\hat{I}_1^{[0,1]}(3,2,1)m_b{}^4}{z^{3/2}} \\
& + 16\frac{\hat{I}_2^{[0,1]}(2,3,1)m_b{}^5}{z^{3/2}} + 32\frac{\hat{I}_1(1,2,2)m_b{}^6}{z^{3/2}} + 2\frac{\hat{I}_2(3,2,1)m_b{}^5}{z^{3/2}} - 32\frac{\hat{I}_0(1,3,1)m_b{}^5}{z^{3/2}} + \frac{\hat{I}_2(3,2,2)m_b{}^6}{z^{3/2}} + 4\frac{\hat{I}_1(3,2,1)m_b{}^5}{z^{3/2}} \\
& - 2\frac{\hat{I}_2(3,1,2)m_b{}^5}{z^{3/2}} - 4\frac{\hat{I}_2^{[0,1]}(3,2,2)m_b{}^5}{z^{3/2}} - 4\frac{\hat{I}_1^{[0,2]}(3,2,2)m_b{}^4}{z^{3/2}} - 2\frac{\hat{I}_0^{[0,1]}(3,2,2)m_b{}^4}{z^{3/2}} + 12\frac{\hat{I}_2^{[0,1]}(3,2,1)m_b{}^4}{z^{3/2}} \\
& - 4\frac{\hat{I}_0^{[0,1]}(3,2,1)m_b{}^3}{z^{3/2}} - 3\frac{\hat{I}_2(4,1,1)m_b{}^4}{z^{3/2}} + 16\frac{\hat{I}_1^{[0,1]}(2,2,2)m_b{}^5}{z^{3/2}} - 16\frac{\hat{I}_1^{[0,1]}(2,2,1)m_b{}^4}{z^{3/2}} + 8\frac{\hat{I}_0(2,2,1)m_b{}^4}{z^{3/2}} \\
& - 12\frac{\hat{I}_2^{[0,2]}(3,2,2)m_b{}^4}{z^{3/2}} + 8\frac{\hat{I}_2(2,2,2)m_b{}^6}{z^{3/2}} - 128\frac{\hat{I}_2(1,3,1)m_b{}^6}{z^{3/2}} - 3\frac{\hat{I}_1(4,1,1)m_b{}^4}{z^{3/2}} - 8\frac{\hat{I}_0^{[0,1]}(2,3,1)m_b{}^4}{z^{3/2}} \\
& - 6\frac{\hat{I}_1^{[0,1]}(4,1,1)m_b{}^3}{z^{3/2}} - 12\frac{\hat{I}_1^{[0,1]}(3,1,2)m_b{}^4}{z^{3/2}} + 4\frac{\hat{I}_0(2,2,2)m_b{}^5}{z^{3/2}} - 2\frac{\hat{I}_1(3,1,2)m_b{}^5}{z^{3/2}} + 48\frac{\hat{I}_1^{[0,2]}(2,2,2)m_b{}^4}{z^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&- 64\frac{\hat{I}_1(1,2,1)m_b{}^5}{z^{3/2}} - \frac{\hat{I}_0(3,2,1)m_b{}^4}{z^{3/2}} - 24\frac{\hat{I}_0(1,4,1)m_b{}^6}{z^{3/2}} - 48\frac{\hat{I}_2(1,4,1)m_b{}^7}{z^{3/2}} + \frac{\hat{I}_1(3,2,2)m_b{}^6}{z^{3/2}} - 12\frac{\hat{I}_1^{[0,1]}(3,2,1)m_b{}^4}{z^2} \\
&+ 4\frac{\hat{I}_1^{[0,1]}(3,2,2)m_b{}^5}{z^2} - 16\frac{\hat{I}_1^{[0,1]}(2,3,1)m_b{}^5}{z^2} + 4\frac{\hat{I}_0(2,3,1)m_b{}^5}{z^2} + 3\frac{\hat{I}_1(4,1,1)m_b{}^4}{z^2} + 128\frac{\hat{I}_1(1,3,1)m_b{}^6}{z^2} - 2\frac{\hat{I}_2(3,2,1)m_b{}^5}{z^2} \\
&- 8\frac{\hat{I}_1(2,2,2)m_b{}^6}{z^2} + 96\frac{\hat{I}_1^{[0,1]}(1,4,1)m_b{}^6}{z^2} - 8\frac{\hat{I}_2(2,3,1)m_b{}^6}{z^2} + 32\frac{\hat{I}_1(1,2,2)m_b{}^6}{z^2} - \frac{\hat{I}_1(3,2,2)m_b{}^6}{z^2} + 2\frac{\hat{I}_1(3,1,2)m_b{}^5}{z^2} \\
&+ 12\frac{\hat{I}_1^{[0,2]}(3,2,2)m_b{}^4}{z^2} + 1/2\frac{\hat{I}_0(3,2,2)m_b{}^5}{z^2} - 2\frac{\hat{I}_1(3,2,1)m_b{}^5}{z^2} + 6\frac{\hat{I}_2^{[0,1]}(3,2,2)m_b{}^5}{z^2} - 48\frac{\hat{I}_1^{[0,1]}(2,2,2)m_b{}^5}{z^2} \\
&+ \frac{\hat{I}_2(3,2,2)m_b{}^6}{z^2} - 12\frac{\hat{I}_2(2,2,2)m_b{}^6}{z^2} + 48\frac{\hat{I}_2(1,4,1)m_b{}^7}{z^2} + 48\frac{\hat{I}_1(1,4,1)m_b{}^7}{z^2} - \frac{\hat{I}_2(3,2,2)m_b{}^6}{z^{5/2}} - 48\frac{\hat{I}_1(1,4,1)m_b{}^7}{z^{5/2}} \\
&- \frac{\hat{I}_1(3,2,2)m_b{}^6}{z^{5/2}} - 6\frac{\hat{I}_1^{[0,1]}(3,2,2)m_b{}^5}{z^{5/2}} + 8\frac{\hat{I}_1(2,3,1)m_b{}^6}{z^{5/2}} + 12\frac{\hat{I}_1(2,2,2)m_b{}^6}{z^{5/2}} + 2\frac{\hat{I}_1(3,2,1)m_b{}^5}{z^{5/2}} + \frac{\hat{I}_1(3,2,2)m_b{}^6}{z^3} \\
&- 2\hat{I}_0(3,1,1)m_b{}^3 - \hat{I}_0(3,2,1)m_b{}^4 - 4\hat{I}_0(2,1,2)m_b{}^4,
\end{aligned}
$$

$$
\begin{aligned}
C_T^{\mathrm{HQET}} = {}& 32\frac{\hat{I}_0(1,2,2)m_b{}^5}{\sqrt{z}} - 4\frac{\hat{I}_0^{[0,1]}(3,1,2)m_b{}^3}{\sqrt{z}} - 4\frac{\hat{I}_0^{[0,1]}(3,2,1)m_b{}^3}{\sqrt{z}} + 4\frac{\hat{I}_0^{[0,2]}(3,2,2)m_b{}^3}{\sqrt{z}} + 8\frac{\hat{I}_0(2,3,1)m_b{}^5}{\sqrt{z}} \\
&- \frac{\hat{I}_0(3,2,2)m_b{}^5}{\sqrt{z}} + 8\frac{\hat{I}_0(2,2,1)m_b{}^4}{\sqrt{z}} - 32\frac{\hat{I}_0^{[0,1]}(2,1,2)m_b{}^3}{\sqrt{z}} + 3\frac{\hat{I}_0(4,1,1)m_b{}^3}{\sqrt{z}} + 8\frac{\hat{I}_0^{[0,2]}(3,1,2)m_b{}^2}{\sqrt{z}} \\
&- 2\frac{\hat{I}_0(3,1,2)m_b{}^4}{\sqrt{z}} - 16\frac{\hat{I}_0^{[0,1]}(2,2,2)m_b{}^4}{\sqrt{z}} - 16\frac{\hat{I}_0(2,1,1)m_b{}^3}{\sqrt{z}} + 64\frac{\hat{I}_0(1,1,2)m_b{}^4}{\sqrt{z}} + 8\frac{\hat{I}_0(2,1,2)m_b{}^4}{\sqrt{z}} \\
&- 16\frac{\hat{I}_0^{[0,1]}(2,3,1)m_b{}^4}{z} - 8\frac{\hat{I}_0^{[0,2]}(3,2,1)m_b{}^2}{z} + 2\frac{\hat{I}_0(3,1,2)m_b{}^4}{z} + 6\frac{\hat{I}_0(3,2,1)m_b{}^4}{z} - 8\frac{\hat{I}_0^{[0,1]}(3,1,2)m_b{}^3}{z} \\
&- 48\frac{\hat{I}_0(1,4,1)m_b{}^6}{z} - 4\frac{\hat{I}_0^{[0,1]}(3,2,2)m_b{}^4}{z} + 16\frac{\hat{I}_0(2,1,2)m_b{}^4}{z} + 16\frac{\hat{I}_0^{[0,1]}(3,1,1)m_b{}^2}{z} + 4\frac{\hat{I}_0^{[0,1]}(3,2,1)m_b{}^3}{z} \\
&+ 8\frac{\hat{I}_0(3,1,1)m_b{}^3}{z} - 32\frac{\hat{I}_0(2,1,1)m_b{}^3}{z} - 192\frac{\hat{I}_0(1,2,1)m_b{}^4}{z} - 8\frac{\hat{I}_0(2,2,1)m_b{}^4}{z} + 8\frac{\hat{I}_0(2,2,2)m_b{}^5}{z} \\
&- 160\frac{\hat{I}_0(1,3,1)m_b{}^5}{z} + 2\frac{\hat{I}_0(3,1,2)m_b{}^4}{z^{3/2}} - 2\frac{\hat{I}_0(3,2,1)m_b{}^4}{z^{3/2}} + 8\frac{\hat{I}_0(2,3,1)m_b{}^5}{z^{3/2}} + \frac{\hat{I}_0(3,2,2)m_b{}^5}{z^{3/2}} - 8\frac{\hat{I}_0(3,1,1)m_b{}^3}{z^{3/2}} \\
&+ 8\frac{\hat{I}_0^{[0,1]}(3,2,1)m_b{}^3}{z^{3/2}} - 2\frac{\hat{I}_0(3,2,1)m_b{}^4}{z^2} + 4\hat{I}_0(3,2,1)m_b{}^4 - 2\hat{I}_0^{[0,1]}(3,2,2)m_b{}^4 + 8\hat{I}_0(3,1,1)m_b{}^3 \\
&- 12\hat{I}_0^{[0,1]}(3,1,2)m_b{}^3 + 4\hat{I}_0(2,2,2)m_b{}^5 + 16\hat{I}_0(2,1,2)m_b{}^4,
\end{aligned}
$$

where

$$\hat{I}_n^{[i,j]}(a,b,c) = \frac{(2m_b)^{i+j}}{(\sqrt{z})^j}(T_1^2)^i(T_2^2)^j\frac{d^i}{d(T_1^2)^i}\frac{d^j}{d(T_2^2)^j}[(T_1^2)^i(T_2^2)^j\hat{I}_n(a,b,c)].$$

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