QCD AND HQET APPROACHES FOR ANALYSIS OF THE $B_c \to D^0 l\nu$ TRANSITION

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The form factors of the semileptonic $B_c \to D^0 l\nu$ transition with $l = e, \tau$ are calculated in the framework of the three point QCD sum rules. In this case, the two gluon condensate contributions as the important correction on the non-perturbative part of the correlation function, are taken into account. The heavy quark effective theory limits of the form factors are also computed. The branching fractions of these decays are also evaluated and compared with the predictions of the other non-perturbative approaches.

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1. Introduction

The discovery of the B_c meson by the CDF detector at the Fermi Lab in $p\bar{p}$ collisions via the decay mode $B_c \to j/\psi\ l\nu$ at $\sqrt{s} = 1.8$ TeV [1] has illustrated the possibility of experimental study of the charm-beauty systems and has produced considerable interest in its spectroscopy. The B_c, is the only meson containing two heavy quarks with different charge and flavors and it is the lowest bound state of b and c quarks, so its decay modes properties are expected to be different than flavor neutral mesons. Since the excited levels of $b\bar{c}$ lie below the threshold of decay into the pair of heavy B and D mesons, such states decay weakly and they have no annihilation decay modes due to the electromagnetic and strong interactions (for more about the physics of the B_c meson see for example [2]). The study of the B_c transitions are useful for more precise determination of the Cabibbo, Kabayashi, Maskawa (CKM) matrix elements in the weak decays.

A large set of exclusive nonleptonic and semileptonic decays of the B_c meson have been studied within the potential model (PM) (see [3–12]), and also operator product expansion in inverse powers of the heavy quark masses [13–15]. In this work, considering the gluon corrections to the relevant form

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factors, the $B_c \to D^0 l\nu$ mode is investigated in the framework of the three-point QCD sum rules (3PSR) and also in the heavy quark effective theory (HQET). This decay mode has been discussed in different methods (for instance see [9–12, 16, 17]). This transition has also been investigated in the QCD sum rule approach, for example in [17, 18], but without considering the gluon corrections. In [17], the Coulomb-like corrections were considered to decrease the uncertainties in the calculations. The main goals of the present work are to calculate the gluon corrections and to check whether their contributions guarantee the convergence of the sum rules for the form factors or not, and also to compare between the form factors and their HQET limits. For this purpose, we plot the dependence of both form factors and their HQET limits on the transferred momentum square (q^2) and compare them at high and low q^2 values.

This paper includes three sections. The calculation of the sum rules for the relevant form factors are presented in Section 2. As a first correction on the non-perturbative part of the correlation function, the two gluon condensate contributions are taken into the sum rules expressions for the form factors. Also in this section, the HQET limits of the form factors are derived. The next section depicts our numerical analysis of the form factors and their comparison with he HQET limits of them. This section also contains the calculation of the total decay widths as well as the branching ratios of the $B_c \to D^0 l\nu$, $(l = e, \tau)$ decays via the 3PSR and HQET and their comparison with the predictions of other approaches.

2. Sum rules for $B_c \to D^0 l\nu$ transition form factors

In this section we study the transition form factors of the semileptonic $B_c \to D^0 l\nu$ decay by the QCD sum rules mechanism. The $B_c \to D^0 l\nu$ process is governed by the tree level $b \to ul\nu$ transition and c quark is the spectator, at quark level (see Fig. 1). The three-point correlation function

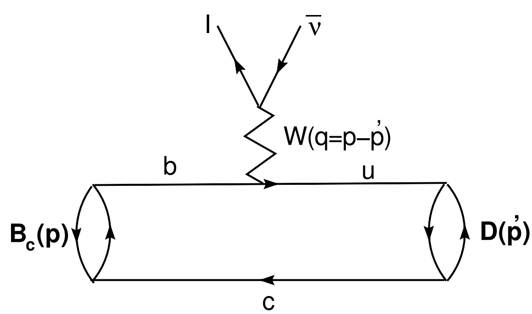

Fig. 1. The bare loop diagram for $B_c \to D^0 l\nu$ transition.

is considered for the evaluation of the transition form factors in the framework of the 3PSR. The three-point correlation function is constructed from the vacuum expectation value of time ordered product of three currents as follows:

$$\Pi_\mu\left(p^2, p'^2, q^2\right) = i^2 \int d^4x d^4y \; e^{+ip'x - ipy} \left\langle 0 \left| \mathrm{T}\left\{ J_{D^0}(x) J_\mu^W(0) J_{B_c}^\dagger(y) \right\} \right| 0 \right\rangle , \quad (1)$$

where $J_{D^0}(x) = \bar{c}\gamma_5 u$ and $J_{B_c}(y) = \bar{c}\gamma_5 b$ are the interpolating currents of the D^0 and B_c mesons. $J_\mu^W = \bar{u}\gamma_\mu(1-\gamma_5)b$ is the current of the weak transition.

We can obtain the correlation function of Eq. (1) in two sides. The phenomenological or physical part is calculated saturating the correlation by a tower of hadrons with the same quantum numbers as interpolating currents. The QCD or theoretical part, on the other side is obtained in terms of the quarks and gluons interacting in the QCD vacuum. To derive the phenomenological part of the correlation function given in Eq. (1), two complete sets of intermediate states with the same quantum numbers as the currents J_{D^0} and J_{B_c} are inserted. This procedure leads to the following representation of the above-mentioned correlation:

$$\begin{aligned}\Pi_\mu\left(p^2, p'^2, q^2\right) &= -\frac{\langle 0 \mid J_D \mid D^0(p')\rangle \langle D^0(p') \mid J_\mu \mid B_c(p)\rangle \langle B_c(p) \mid J_{B_c}^\dagger \mid 0\rangle}{(p'^2 - m_{D^0}^2)(p^2 - m_{Bc}^2)} \\ &\quad + \text{higher resonances and continuum states}\,. \end{aligned} \quad (2)$$

The general expression for the hadronic matrix element of the weak current with definition of the transition form factors is given by the formula:

$$\langle D(p') \mid \bar{u}\gamma_\mu(1-\gamma_5)b \mid B_c(p)\rangle = f_1\left(q^2\right) P_\mu + f_2\left(q^2\right) q_\mu\,, \quad (3)$$

and the $f_1(q^2)$ and $f_2(q^2)$ are the transition form factors, $P_\mu = (p+p')_\mu$ and $q_\mu = (p-p')_\mu$. Also the following matrix elements are defined in the standard way in terms of the leptonic decay constants of the D^0 and B_c mesons as:

$$\langle 0 \mid J_{D^0} \mid D^0(p')\rangle = i\frac{f_{D^0} m_{D^0}^2}{m_c + m_u}\,, \qquad \langle 0 \mid J_{B_c} \mid B_c(p)\rangle = i\frac{f_{B_c} m_{B_c}^2}{m_b + m_c}\,, \quad (4)$$

where f_{D^0} and f_{B_c} are the leptonic decay constants of D^0 and B_c mesons, respectively. Using Eqs. (3) and (4) in Eq. (2), we get the following result for the physical part:

$$\begin{aligned}\Pi_\mu\left(p^2, p'^2, q^2\right) &= -\frac{f_{B_c} f_{D^0}}{(m_b + m_c)\,(m_c + m_u)} \times \frac{m_{B_c}^2 m_{D^0}^2}{\left(p'^2 - m_D^2\right)\left(p^2 - m_{B_c}^2\right)} \\ &\quad \times \left[f_1\left(q^2\right) P_\mu + f_2\left(q^2\right) q_\mu\right] + \text{excited states}\,. \end{aligned} \quad (5)$$

The coefficients of Lorentz structures P_μ and q_μ in the correlation function Π_μ will be chosen in determination of the form factors $f_1(q^2)$ and $f_2(q^2)$, respectively.

With the help of the operator product expansion (OPE), in the deep Euclidean region where $p^2 \ll (m_b+m_c)^2$ and $p'^2 \ll m_c^2$, the vacuum expectation value of the expansion of the correlation function in terms of the local operators, is written as follows

$$\begin{aligned}\Pi_\mu\left(p^2,p'^2,q^2\right) &= (C_0)_\mu + (C_3)_\mu\langle\bar{q}q\rangle + (C_4)_\mu\left\langle G^2\right\rangle + (C_5)_\mu\left\langle\bar{q}\sigma_{\alpha\beta}G^{\alpha\beta}q\right\rangle \\ &\quad + (C_6)_\mu\left\langle\bar{q}\Gamma q\bar{q}\Gamma' q\right\rangle\,, \end{aligned} \tag{6}$$

where $(C_i)_\mu$ are the Wilson coefficients, $G_{\alpha\beta}$ is the gluon field strength tensor, Γ and Γ' are the matrices appearing in the calculations. We consider the condensate terms of dimension $3,4$ and 5. It is found that the heavy quark condensate contributions are suppressed by inverse of the heavy quark mass and can be safely omitted. The light u quark condensate contribution is zero after applying the double Borel transformation with respect to the both variables p^2 and p'^2, because only one variable appears in the denominator. Therefore in this case, we consider the two gluon condensate contributions with mass dimension 4 as a first correction on the non-perturbative part of correlation function, only *i.e.*,

$$\Pi_i\left(p^2,p'^2,q^2\right) = \Pi_i^{\rm per}\left(p^2,p'^2,q^2\right) + \Pi_i^{\langle G^2\rangle}\left(p^2,p'^2,q^2\right)\frac{\alpha_{\rm s}}{\pi}\left\langle G^2\right\rangle\,. \tag{7}$$

To obtain the contributions of this term, the Fock–Schwinger fixed-point gauge, $x^\mu A_\mu^a = 0$, are used; where A_μ^a is the gluon field. The procedure of the evaluation of the gluon condensate contributions has been discussed in Ref. [19], completely.

Using the double dispersion representation, the bare-loop contribution is determined:

$$\Pi_i^{\rm per} = -\frac{1}{(2\pi)^2}\int\int\frac{\rho_i^{\rm per}\left(s,s',q^2\right)}{\left(s-p^2\right)\left(s'-p'^2\right)}ds\,ds' + \text{subtraction terms}\,, \tag{8}$$

the spectral densities $\rho_i^{\rm per}\left(s,s',q^2\right)$ are found as:

$$\begin{aligned}\rho_1^{\rm per}\left(s,s',q^2\right) &= I_0\, N_c\Big\{\Delta+\Delta' \\ &\quad -2\,m_c\left[(2+E_1+E_2)m_c-(1+E_1+E_2)m_u\right] \\ &\quad +2\,m_b[(1+E_1+E_2)m_c-(E_1+E_2)m_u] \\ &\quad +(E_1+E_2)u\Big\}\,,\end{aligned}$$

$$\begin{aligned}\rho_2^{\rm per}\left(s,s',q^2\right)=I_0\, N_c \Big\{ & -\Delta+\Delta' \\ & -2\, m_c\,[(E_2-E_1-1)m_u+(E_1-E_2)m_c] \\ & -2m_b[(1-E_1+E_2)m_c+(E_1-E_2)m_u] \\ & +(E_1-E_2)u\Big\}\,,\end{aligned}$$

where

$$\begin{aligned} I_0\left(s,s',q^2\right) &= \frac{1}{4\lambda^{1/2}\left(s,s',q^2\right)}\,, \\ \lambda\left(s,s',q^2\right) &= s^2+s'^2+q^4-2sq^2-2s'q^2-2ss'\,, \\ E_1 &= \frac{1}{\lambda\left(s,s',q^2\right)}\left[2s'\Delta-\Delta' u\right]\,, \\ E_2 &= \frac{1}{\lambda\left(s,s',q^2\right)}\left[2s\Delta'-\Delta u\right]\,, \\ u &= s+s'-q^2\,, \\ \Delta &= s+m_c^2-m_b^2\,, \\ \Delta' &= s'+m_c^2-m_{q_i}^2\,, \end{aligned} \tag{9}$$

and the $N_c = 3$ is the color factor. The following inequalities are utilized to find the integration limits of the Eq. (8).

$$-1 \le \frac{2ss'+\left(s+s'-q^2\right)\left(m_b^2-m_c^2-s\right)+2s\left(m_c^2-m_u^2\right)}{\lambda^{1/2}\left(s,s',q^2\right)\lambda^{1/2}\left(m_b^2,m_c^2,s\right)} \le +1\,. \tag{10}$$

For the heavy quarkonium $b\bar{c}$, where the relative velocity of quark movement is small, an essential role is taken by the Coulomb-like α_s/v-corrections [17]. It leads to the finite renormalization for $\rho_i^{\rm per}$, so that:

$$\rho_i^c = \mathcal{C}\rho_i^{\rm per}\,, \tag{11}$$

with

$$\mathcal{C}^2 = \frac{4\pi\alpha_s^{\mathcal{C}}}{3v}\,\frac{1}{1-\exp\left(-\dfrac{4\pi\alpha_s^{\mathcal{C}}}{3v}\right)}\,, \tag{12}$$

where $\alpha_s^{\mathcal{C}}$ is the coupling constant of effective Coulomb interactions. Also v is the relative velocity of quarks in the $b\bar{c}$-system,

$$v = \sqrt{1-\frac{4m_b m_c}{p^2-(m_b-m_c)^2}}\,. \tag{13}$$

The value of the $\alpha_s^{\mathcal{C}}$ for the B_c meson is [17]:

$$\alpha_s^{\mathcal{C}}[b\bar{c}] = 0.45\,.$$

By performing the double Borel transformations over the variables p^2 and p'^2 on the physical parts of the correlation functions and bare-loop diagrams and also equating two representations of the correlation functions, the sum rules for the $f_i(q^2)$ are obtained:

$$f_i\left(q^2\right)=\frac{(m_b+m_c)(m_c+m_u)}{f_{B_c}m_{B_c}^2 f_{D^0}m_{D^0}^2}e^{m_{B_c}^2/M_1^2}e^{m_{D^0}^2/M_2^2}\frac{1}{4\pi^2}\left\{\int\limits_{(m_c+m_u)^2}^{s_0'}ds'\int\limits_{f_-(s')}^{\min(s_0,f_+(s'))}\right.$$
$$\left.\times\, ds\rho_i^c\left(s,s',q^2\right)e^{-s/M_1^2}e^{-s'/M_2^2}-iM_1^2M_2^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{C_i^4}{6}\right\}, \qquad (14)$$

where $i=1$ and 2, s_0 and s_0' are the continuum thresholds in pseudoscalars B_c and D^0 channels, respectively, and $s=f_\pm(s')$ in the lower and upper limit of the integral over s are obtained from inequality Eq. (10). The $\min(s_0,f_+(s'))$ means that for each value of the q^2, the smaller one between s_0 and f_+ is selected. The explicit expressions for C_i^4 are presented in Appendix A.

In the above equations, in order to subtract the contributions of the higher states and the continuum the quark–hadron duality assumption is also used

$$\rho^{\text{higher states}}(s,s')=\rho^{\text{OPE}}(s,s')\theta(s-s_0)\theta(s'-s_0')\,. \qquad (15)$$

Now, we apply the heavy quark effective theory (HQET) to analyze the form factors of $B_c\to D^0 l\nu$ calculated by 3PSR. To this aim, we use the following parametrization (see also [20, 21]):

$$y=\nu\nu'=\frac{m_{B_c}^2+m_{D^0}^2-q^2}{2m_{B_c}m_{D^0}}\,, \qquad (16)$$

where ν and ν' are the four-velocities of the initial and final meson states, respectively and $y=1$ are so-called zero recoil limit. After some complicated calculations, the y-dependent expressions of the $f_i^{\text{HQET}}(y)$ are obtained as follows:

$$f_1^{\text{HQET}}(y)=\frac{1}{\hat{f}_{B_c}\hat{f}_{D^0}}e^{\frac{\Lambda}{T_1}}e^{\frac{\bar{\Lambda}}{T_2}}\left\{-\frac{3(1+\sqrt{Z})^2(2y\sqrt{Z}-1)(1-(1+2y)\sqrt{Z}+(1+y)Z)}{8\left(y^2-1\right)^{3/2}Z^{9/4}}\right.$$
$$\times\frac{-1}{4\pi^2}\int\limits_0^{\nu_0}d\nu\int\limits_0^{\nu_0'}d\nu' e^{-\frac{\nu}{2T_1}}e^{-\frac{\nu'}{2T_2}}\theta\left(2y\nu\nu'-\nu^2-\nu'^2\right)$$

$$+\left(i\,\frac{2\left(\sqrt{Z}+1\right)^2}{3Z^{5/4}}\; T_1T_2\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle \right)\times \lim_{m_b\to\infty} C_1^{\rm HQET}\Bigg\}, \tag{17}$$

$$f_2^{\rm HQET}(y) = \frac{1}{\hat{f}_{B_c}\hat{f}_{D^0}} e^{\frac{\Lambda}{T_1}} e^{\frac{\bar{\Lambda}}{T_2}} \Bigg\{ \frac{3(1+\sqrt{Z})^2(2y\sqrt{Z}-1)(-1+\sqrt{Z}+(1+y)Z)}{8\left(y^2-1\right)^{3/2} Z^{9/4}}$$
$$\times\frac{-1}{4\pi^2}\int_0^{\nu_0} d\nu \int_0^{\nu_0'} d\nu' e^{-\frac{\nu}{2T_1}} e^{-\frac{\nu'}{2T_2}} \theta\left(2y\nu\nu' - \nu^2 - \nu'^2\right)$$
$$+\left(i\,\frac{2\left(\sqrt{Z}+1\right)^2}{3Z^{5/4}}\; T_1T_2\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle \right)\times \lim_{m_b\to\infty} C_2^{\rm HQET}\Bigg\}. \tag{18}$$

In these heavy quark limit expressions $\Lambda = m_{B_c} - m_b$, $\bar{\Lambda} = m_{D^0} - m_c$, $\sqrt{Z} = y + \sqrt{y^2-1}$, $\hat{f}_{B_c} = \sqrt{m_b} f_{B_c}$, $\hat{f}_{D^0} = \sqrt{m_c} f_{D^0}$. The continuum thresholds ν_0, ν_0' and integration variables ν, ν' are defined as:

$$\nu_0 = \frac{s_0 - m_b^2}{m_b}, \qquad \nu_0' = \frac{s_0' - m_c^2}{m_c}, \tag{19}$$

$$\nu = \frac{s - m_b^2}{m_b}, \qquad \nu' = \frac{s' - m_c^2}{m_c}. \tag{20}$$

Also we apply $T_1 = M_1^2/2m_b$, $T_2 = M_2^2/2m_c$ and $m_c = m_b/\sqrt{Z}$.

The explicit expressions of the coefficients $C_i^{\rm HQET}$are given in Appendix B. In the expressions of the $C_i^{\rm HQET}$, $\bar{I}_0(a,b,c)$ and $\bar{I}_{1(2)}(a,b,c)$ are defined as:

$$\bar{I}_0(a,b,c) = \frac{(-1)^{a+b+c}}{16\pi^2\Gamma(a)\Gamma(b)\Gamma(c)}\left(\frac{1}{\sqrt{Z}}\right)^{2-a-c} (2m_b)^{4-2a-b-c} T_1^{2-a-b} T_2^{2-a-c}$$
$$\mathcal{U}_0^{\rm HQET}(a+b+c-4, 1-c-b),$$
$$\bar{I}_{1(2)}(a,b,c) = i\frac{(-1)^{a+b+c+1}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)}\left(\frac{1}{\sqrt{Z}}\right)^{4-a-c-1(2)} (2m_b)^{5-2a-b-c} T_1^{1-a-b+1(2)} T_2^{4-a-c-1(2)}$$
$$\mathcal{U}_0^{\rm HQET}(a+b+c-5, 1-c-b). \tag{21}$$

The function $\mathcal{U}_0^{\rm HQET}(m,n)$ takes the following form

$$\mathcal{U}_0^{\rm HQET}(m,n) = \int_0^\infty (2m_b)^m \left(\frac{x}{2m_b} + T_1 + \frac{T_2}{\sqrt{Z}}\right)^m x^n \left[-\frac{\overline{B}_{-1}}{x} - \overline{B}_0 - \overline{B}_1 x\right] dx\,, \tag{22}$$

with

$$\overline{B}_{-1} = \frac{\sqrt{Z}}{T_1T_2}\left[\frac{mb^2}{Z}\,T_2^2 + \frac{1}{\sqrt{Z}}T_1T_2\left(m_b^2 - q^2\right)\right],$$
$$\overline{B}_0 = \frac{\sqrt{Z}}{2m_bT_1T_2}\left[m_c^2T_1 + \frac{T_2}{\sqrt{Z}}\left(m_b^2 + m_c^2\right)\right],$$
$$\overline{B}_1 = \frac{1}{4\sqrt{Z}T_1T_2}\,. \tag{23}$$

3. Numerical analysis

The sum rules expressions of the form factors show the main input parameters entering the expressions are gluon condensate, elements of the CKM matrix V_{ub}, leptonic decay constants, f_{B_c} and f_{D^0}, Borel parameters M_1^2 and M_2^2, as well as the continuum thresholds s_0 and s'_0. We choose the values of the gluon condensate, leptonic decay constants, CKM matrix elements and quark and meson masses as follows: $\langle\frac{\alpha_s}{\pi}G^2\rangle = 0.012$ GeV^4 [22], $\mid V_{ub} \mid= 0.0037$ [16], $f_{B_c} = (0.35 \pm 0.025)$ GeV [18, 23], $f_{D^0} = (0.22 \pm 0.016)$ GeV, $m_c(\mu = m_c) = (1.275 \pm 0.015)$ GeV, $m_u = (1.5 - 3)$ MeV, $m_b = (4.7 \pm 0.01)$ GeV, $m_{D^0} = 1.964$ GeV, $m_{B_C} = 6.258$ GeV [24].

The expressions for the form factors also contain four auxiliary parameters: Borel mass squares M_1^2 and M_2^2 and continuum thresholds s_0 and s'_0. These are mathematical objects, so the physical quantities, the form factors, should be independent of them. The parameters s_0 and s'_0, which are the continuum thresholds of B_c and D^0 mesons, respectively, are determined from the conditions that guarantee the sum rules to have the best stability in the allowed M_1^2 and M_2^2 regions. The values of the continuum thresholds calculated from the two-point QCD sum rules are taken to be $s_0 = (45–50)$ GeV^2 and $s'_0 = 4$ GeV^2 [22, 25, 26]. The working regions for M_1^2 and M_2^2 are determined by requiring that not only contributions of the higher states and continuum are effectively suppressed, but the gluon condensate contributions are small, which guarantees that the contributions of higher dimensional operators are small. Both conditions are satisfied in the regions 10 GeV$^2 \leq M_1^2 \leq 25$ GeV^2 and 6 GeV$^2 \leq M_2^2 \leq 12$ GeV^2.

The dependence of the form factors f_1 and f_2 on M_1^2 and M_2^2 for $B_c \to D^0 l\nu$ are shown in Figs. 2 and 3, respectively. This figures show a good stability of the form factors with respect to the Borel mass parameters in the working regions. Our numerical analysis shows that the contribution of the non-perturbative part (the gluon condensate diagrams) is about 7% of the total and the main contribution comes from the perturbative part of the form factors. This means that the contribution of the higher dimension operators is small. This guarantees the convergence of the sum rules expression of the form factors, and those sum rules are reliable.

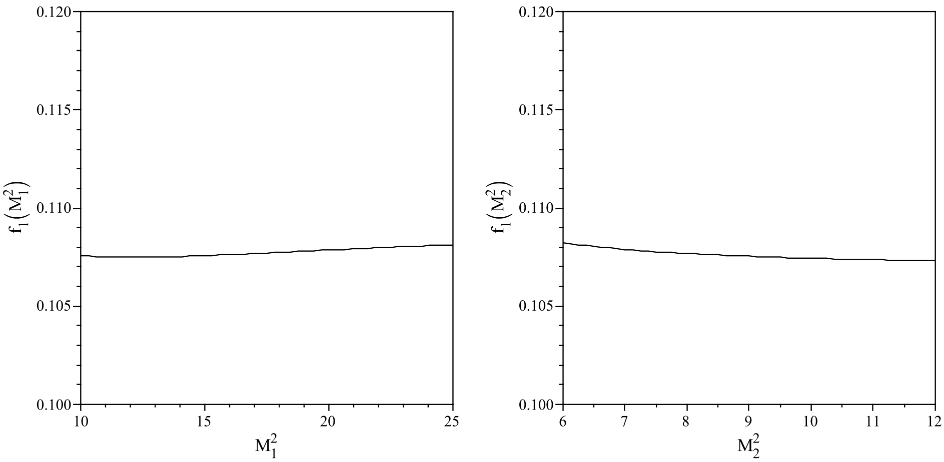

Fig. 2. The dependence of the f_1 form factor on M_1^2 and M_2^2.

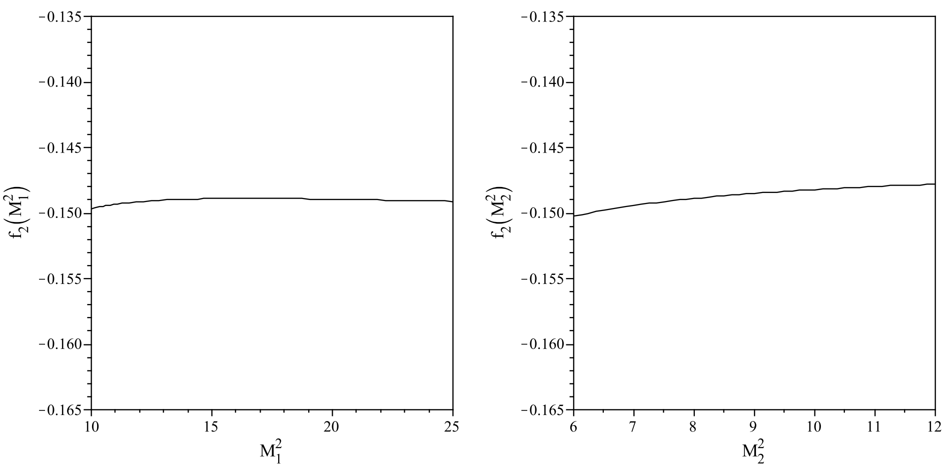

Fig. 3. The dependence of the f_2 form factor on M_1^2 and M_2^2.

The values of the form factors at $q^2 = 0$ are shown in Table I. In comparison, the predictions of the other approaches are also presented in this table.

The sum rules for the form factors are truncated at about 14 GeV^2, so to extend our results to the full physical region, we look for a parametrization of the form factors in such a way that in the region $0 \leq q^2 \leq 14\ \mathrm{GeV}^2$, this parametrization coincides with the sum rules predictions. Our numerical calculations show that the sufficient parametrization of the form factors with respect to q^2 is as follows:

$$f_i\left(q^2\right) = \frac{f_i(0)}{1 - \frac{q^2}{m_{\rm pol}^2}}\,. \quad (24)$$

The fitted pole masses are:

$$m_{\rm pol} = (5.42 \pm 0.07)\ {\rm GeV} \quad {\rm for} \quad f_1(q^2)\,,$$
$$m_{\rm pol} = (5.91 \pm 0.10)\ {\rm GeV} \quad {\rm for} \quad f_2(q^2)\,.$$

The dependence of the form factors $f_+(q^2)$ and $f_-(q^2)$ on q^2 are given in Fig. 4. This figure also contains the form factors obtained via 3PSR (see Eq. (14)). The form factors and their fit functions coincide well in the interval $0 \leq q^2 \leq 14$ GeV^2.

TABLE I

The form factors of the $B_c \to D^0 l\nu$ decay for $M_1^2 = 17$ GeV^2, $M_2^2 = 8$ GeV^2 at $q^2 = 0$ in different approaches: three-point sum rules (3PSR) with gluon condensate corrections, HQET, light-cone sum rules (LCSR), 3PSR without gluon condensate corrections, PM and quark model(QM).

Form factor	3PSR[Our]	HQET[Our]	LCSR [16]	3PSR [17]	PM [4]	QM [11]
$f_1(0)$	0.29 ± 0.08	0.57 ± 0.17	0.35	0.32	0.29	0.69
$f_2(0)$	-0.31 ± 0.09	-0.69 ± 0.20	-0.35	-0.34	-0.37	-0.64

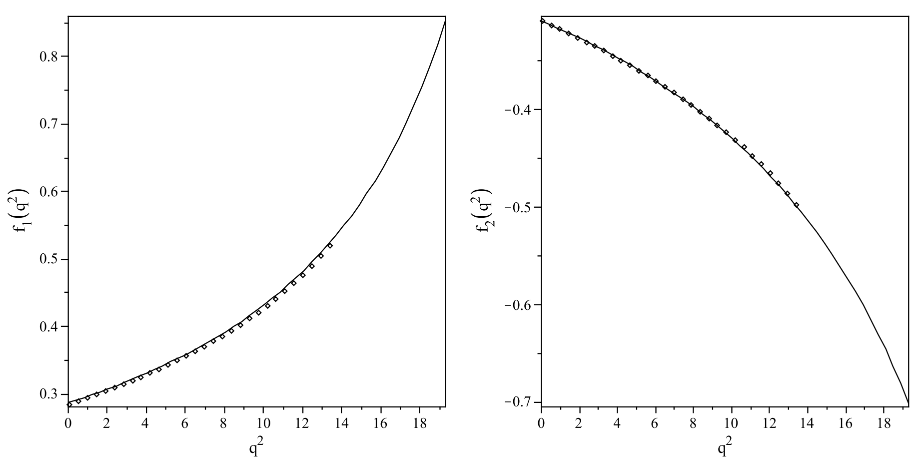

Fig. 4. The dependence of the form factors and the fit parametrization of them on q^2. The small boxes correspond to the form factors and the solid lines show the fit parametrization of them.

The errors are estimated by the variation of the Borel parameters M_1^2 and M_2^2, the variation of the continuum thresholds s_0 and s_0', the leptonic decay constants f_{B_c} and f_D and uncertainties in the values of the other input parameters. The main uncertainty comes from the continuum thresholds and the decay constants, which is about $\sim 25\%$ of the central value, while the other uncertainties are small, constituting a few percent.

The HQET form factors of the $B_c \to D^0$ transition were evaluated in Eqs. (17) and (18). Fig. 5 depict the f_i^{HQET} with respect to the q^2. At $y = 1$ in Eq. (16) called the zero recoil limit (corresponding to $q^2 = (m_{B_c} - m_{D^0})^2$), the HQET limit of the form factors is not finite. For other values of y and corresponding q^2, the behaviors of the $f_i(q^2)$ form factors shown in Fig. 4 and their HQET form factors f_i^{HQET} in Fig. 5 are the same, *i.e.*, when q^2 increases (y decreases) both, the form factors and their HQET values increase.

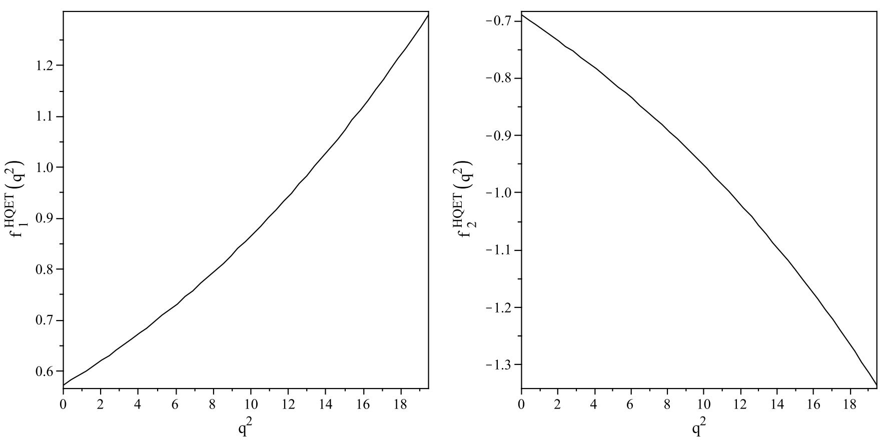

Fig. 5. The dependence of the HQET limits of the form factors on q^2.

At the end of this section, we would like to present the values of the branching ratio for $B_c \to D^0 l\nu$ decay. By using the expressions for the form factors, the differential decay width $d\Gamma/dq^2$ for the process $B_c \to D^0 l\nu$ is presented as follows [27]:

$$\begin{aligned}
\frac{d\Gamma\left(B_c \to D^0 l\nu\right)}{dq^2} &= \frac{1}{192\pi^3 m_{B_c}^3} G_{\mathrm{F}}^2 |V_{ub}|^2 \lambda^{1/2}\left(m_{B_c}^2, m_{D^0}^2, q^2\right)\left(\frac{q^2 - m_l^2}{q^2}\right)^2 \\
&\quad \times \Big\{ -\tfrac{1}{2}\left(2q^2 + m_l^2\right)\Big[\left|f_1\left(q^2\right)\right|^2\left(2m_{B_c}^2 + 2m_{D^0}^2 - q^2\right) \\
&\quad + 2\left(m_{B_c}^2 - m_{D^0}^2\right)\mathrm{Re}\left[f_1\left(q^2\right) f_2^*\left(q^2\right)\right] + \left|f_2\left(q^2\right)\right|^2 q^2\Big] \\
&\quad + \frac{\left(q^2 + 2m_l^2\right)}{q^2}\Big[\left|f_1\left(q^2\right)\right|^2\left(m_{B_c}^2 - m_{D^0}^2\right)^2 \\
&\quad + 2\left(m_{B_c}^2 - m_{D^0}^2\right) q^2 \mathrm{Re}\left[f_1\left(q^2\right) f_2^*\left(q^2\right)\right] + \left|f_2\left(q^2\right)\right|^2 q^4\Big]\Big\}\,. \qquad (25)
\end{aligned}$$

Integrating Eq. (25) over q^2 in the whole physical region and using the total mean life time $\tau \simeq 0.48 \pm 0.05$ ps of B_c meson [24], the branching ratios of the $B_c \to D^0 l\nu$, $(l = e, \tau)$ decays are obtained as presented in Table II. The branching ratios of these decays obtained using the HQET limits of the form factors Eqs. (17) and (18), are also shown in this table. This table also includes a comparison between our results via both SR and HQET and the predictions of the other approaches including the LCSR, 3PSR (without gluon condensate corrections), QM, BSE, PM and RM estimates.

TABLE II

The branching ratios of the $B_c \to D^0 l\nu$, $(l = e, \tau)$ decays (in %) in different approaches: 3PSR with gluon condensate corrections, HQET, LCSR [16], 3PSR without gluon corrections [17], QM [11], PM [10], the Bethe–Salpeter equation(BSE) [12] and a relativistic model with factorization to obtain the nonleptonic decay widths (RM) [9].

Mode	3PSR[Our]	HQET[Our]	LCSR [16]	3PSR [17]	QM [11]	PM [10]	BSE [12]	RM [9]
$D^0 e\nu$	$[(0.38 \pm 0.10) \times 10^{-2}$	$(0.52 \pm 0.21) \times 10^{-2}$	0.020	0.0043	0.0189	0.0018	0.0068	0.0036]%
$D^0 \tau\nu$	$[(0.15 \pm 0.04) \times 10^{-2}$	$(0.74 \pm 0.33) \times 10^{-2}$	0.015	0.0023	0.0102	—	—	—]%

In summary, considering the gluon corrections, we investigated the $B_c \to D^0 l\nu$, $(l = e, \tau)$ channels in the framework of the three-point QCD sum rules. We found that the gluon correction contributions to the sum rules expression of the form factors are small. This implies a small contribution of the higher dimension operators, and it also guarantees that the sum rules for the form factors are convergent and reliable. The HQET limits of the form factors, with their corresponding gluon condensate corrections are also computed. Finally, we evaluated the branching fractions of these decays and compared these with the predictions of the other approaches such as LCSR, 3PSR, QM, PM, BSE and RM.

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Appendix A

In this appendix, the explicit expressions of the coefficients of the gluon condensate entering the sum rules of the form factors $f_1(q^2)$ and $f_2(q^2)$ are given.

$$
\begin{aligned}
C_1^4 \;=\; & -5\,\hat{I}_1(3,2,2){m_c}^6 - 5\,\hat{I}_2(3,2,2){m_c}^6 - 5\,\hat{I}_0(3,2,2){m_c}^6 \\
& +5\,\hat{I}_2(3,2,2){m_c}^5 m_b + 5\,\hat{I}_1(3,2,2){m_c}^5 m_b + 5\,\hat{I}_0(3,2,2){m_c}^5 m_b \\
& +5\,\hat{I}_2(3,2,2){m_c}^4 {m_b}^2 + 5\,\hat{I}_1(3,2,2){m_c}^4 {m_b}^2 - 5\,\hat{I}_1(3,2,2){m_c}^3 {m_b}^3 \\
& -5\,\hat{I}_0(3,2,2){m_c}^3 {m_b}^3 - 5\,\hat{I}_2(3,2,2){m_c}^3 {m_b}^3 - 5\,\hat{I}_2(3,2,1){m_c}^4 \\
& +15\,\hat{I}_1^{[0,1]}(3,2,2){m_c}^4 - 15\,\hat{I}_1(2,2,2){m_c}^4 + 15\,\hat{I}_2^{[0,1]}(3,2,2){m_c}^4 \\
& -5\,\hat{I}_2(3,1,2){m_c}^4 - 5\,\hat{I}_1(3,1,2){m_c}^4 - 15\,\hat{I}_2(4,1,1){m_c}^4 \\
& -5\,\hat{I}_1(3,2,1){m_c}^4 - 15\,\hat{I}_0(4,1,1){m_c}^4 - 15\,\hat{I}_0(2,2,2){m_c}^4 \\
& +15\,\hat{I}_0^{[0,1]}(3,2,2){m_c}^4 - 15\,\hat{I}_1(4,1,1){m_c}^4 - 15\,\hat{I}_2(2,2,2){m_c}^4 \\
& +5\,\hat{I}_1(3,1,2){m_c}^3 m_b - 10\,\hat{I}_0^{[0,1]}(3,2,2){m_c}^3 m_b + 10\,\hat{I}_1(2,2,2){m_c}^3 m_b \\
& +10\,\hat{I}_0(3,2,1){m_c}^3 m_b - 10\,\hat{I}_2^{[0,1]}(3,2,2){m_c}^3 m_b + 5\,\hat{I}_0(3,1,2){m_c}^3 m_b \\
& -10\,\hat{I}_2(2,3,1){m_c}^3 m_b + 15\,\hat{I}_0(4,1,1){m_c}^3 m_b + 5\,\hat{I}_1(3,2,1){m_c}^3 m_b \\
& +15\,\hat{I}_1(4,1,1){m_c}^3 m_b - 10\,\hat{I}_1(2,3,1){m_c}^3 m_b + 15\,\hat{I}_2(4,1,1){m_c}^3 m_b \\
& +10\,\hat{I}_2(2,2,2){m_c}^3 m_b - 10\,\hat{I}_0(2,3,1){m_c}^3 m_b - 10\,\hat{I}_1^{[0,1]}(3,2,2){m_c}^3 m_b \\
& +5\,\hat{I}_2(3,1,2){m_c}^3 m_b + 10\,\hat{\hat{I}}_0(2,2,2){m_c}^3 m_b + 5\,\hat{I}_2(3,2,1){m_c}^3 m_b \\
& +5\,\hat{I}_0^{[0,1]}(3,2,2){m_c}^2 {m_b}^2 + 30\,\hat{I}_0(1,4,1){m_c}^2 {m_b}^2 - 5\,\hat{I}_0(2,2,2){m_c}^2 {m_b}^2 \\
& -10\,\hat{I}_1(3,2,1){m_c}^2 {m_b}^2 + 30\,\hat{I}_2(1,4,1){m_c}^2 {m_b}^2 - 10\,\hat{I}_2(3,2,1){m_c}^2 {m_b}^2 \\
& +30\,\hat{I}_1(1,4,1){m_c}^2 {m_b}^2 - 5\,\hat{I}_0^{[0,1]}(3,2,2)m_c\, {m_b}^3 - 30\,\hat{I}_0(1,4,1)m_c\, {m_b}^3 \\
& +10\,\hat{I}_2(2,3,1)m_c\, {m_b}^3 + 10\,\hat{I}_1(2,3,1)m_c\, {m_b}^3 - 5\,\hat{I}_1^{[0,1]}(3,2,2)m_c\, {m_b}^3 \\
& -5\,\hat{I}_2^{[0,1]}(3,2,2)m_c\, {m_b}^3 + 10\,\hat{I}_2(3,2,1)m_c\, {m_b}^3 - 30\,\hat{I}_1(1,4,1)m_c\, {m_b}^3 \\
& +10\,\hat{I}_1(3,2,1)m_c\, {m_b}^3 - 30\,\hat{I}_2(1,4,1)m_c\, {m_b}^3 + 15\,\hat{I}_0(1,4,1){m_b}^4 \\
& -5\,\hat{I}_0(3,2,1){m_b}^4 + 15\,\hat{I}_1^{[0,1]}(3,2,1){m_c}^2 - 5\,\hat{I}_0(3,1,1){m_c}^2 \\
& -5\,\hat{I}_0(2,2,1){m_c}^2 - 5\,\hat{I}_0(2,1,2){m_c}^2 + 30\,\hat{I}_1^{[0,1]}(2,2,2){m_c}^2 \\
& +15\,\hat{I}_0^{[0,1]}(4,1,1){m_c}^2 + 15\,\hat{I}_2^{[0,1]}(3,2,1){m_c}^2 + 20\,\hat{I}_0^{[0,1]}(3,2,1){m_c}^2 \\
& -15\,\hat{I}_2^{[0,2]})(3,2,2){m_c}^2 + 20\,\hat{I}_0^{[0,1]}(3,1,2){m_c}^2 + 15\,\hat{I}_2^{[0,1]}(4,1,1){m_c}^2\,,
\end{aligned}
$$

$$
\begin{aligned}
C_2^4 \;=\; & 5\,\hat{I}_2(3,2,2){m_c}^6 - 5\,\hat{I}_1(3,2,2){m_c}^6 - 5\,\hat{I}_0(3,2,2){m_c}^5 m_b \\
& +5\,\hat{I}_1(3,2,2){m_c}^5 m_b - 5\,\hat{I}_2(3,2,2){m_c}^5 m_b + 5\,\hat{I}_1(3,2,2){m_c}^4 {m_b}^2 \\
& -5\,\hat{I}_2(3,2,2){m_c}^4 {m_b}^2 - 5\,\hat{I}_1(3,2,2){m_c}^3 {m_b}^3 + 5\,\hat{I}_0(3,2,2){m_c}^3 {m_b}^3 \\
& +5\,\hat{I}_2(3,2,2){m_c}^3 {m_b}^3 - 15\,\hat{I}_2^{[0,1]}(3,2,2){m_c}^4 - 5\,\hat{I}_1(3,1,2){m_c}^4 \\
& +15\,\hat{I}_2(2,2,2){m_c}^4 - 15\,\hat{I}_1(4,1,1){m_c}^4 + 5\,\hat{I}_2(3,2,1){m_c}^4 \\
& +5\,\hat{I}_2(3,1,2){m_c}^4 + 15\,\hat{I}_1^{[0,1]}(3,2,2){m_c}^4 - 5\,\hat{I}_1(3,2,1){m_c}^4
\end{aligned}
$$

$$
\begin{aligned}
&-15\,\hat{I}_1(2,2,2){m_c}^4+15\,\hat{I}_2(4,1,1){m_c}^4+5\,\hat{I}_1(3,1,2){m_c}^3m_b\\
&+10\,\hat{I}_2^{[0,1]}(3,2,2){m_c}^3m_b+10\,\hat{I}_0^{[0,1]}(3,2,2){m_c}^3m_b-5\,\hat{I}_2(3,1,2){m_c}^3m_b\\
&-10\,\hat{I}_1(2,3,1){m_c}^3m_b-15\,\hat{I}_0(4,1,1){m_c}^3m_b+10\,\hat{I}_2(2,3,1){m_c}^3m_b\\
&+15\,\hat{I}_1(4,1,1){m_c}^3m_b-15\,\hat{I}_2(4,1,1){m_c}^3m_b-10\,\hat{I}_0(2,3,1){m_c}^3m_b\\
&+5\,\hat{I}_0(3,2,1){m_c}^3m_b-5\,\hat{I}_0(3,1,2){m_c}^3m_b-10\,\hat{I}_2(2,2,2){m_c}^3m_b\\
&-10\,\hat{I}_0(2,2,2){m_c}^3m_b-10\,\hat{I}_1^{[0,1]}(3,2,2){m_c}^3m_b+5\,\hat{I}_1(3,2,1){m_c}^3m_b\\
&-5\,\hat{I}_2(3,2,1){m_c}^3m_b+10\,\hat{I}_1(2,2,2){m_c}^3m_b-5\,\hat{I}_0^{[0,1]}(3,2,2){m_c}^2{m_b}^2\\
&+5\,\hat{I}_0(2,2,2){m_c}^2{m_b}^2+30\,\hat{I}_1(1,4,1){m_c}^2{m_b}^2+10\,\hat{I}_2(3,2,1){m_c}^2{m_b}^2\\
&-10\,\hat{I}_1(3,2,1){m_c}^2{m_b}^2-30\,\hat{I}_2(1,4,1){m_c}^2{m_b}^2+5\,\hat{I}_0^{[0,1]}(3,2,2)m_c\,{m_b}^3\\
&+30\,\hat{I}_0(1,4,1)m_c\,{m_b}^3-30\,\hat{I}_1(1,4,1)m_c\,{m_b}^3+10\,\hat{I}_1(2,3,1)m_c\,{m_b}^3\\
&+10\,\hat{I}_1(3,2,1)m_c\,{m_b}^3-5\,\hat{I}_1^{[0,1]}(3,2,2)m_c\,{m_b}^3+30\,\hat{I}_2(1,4,1)m_c\,{m_b}^3\\
&-10\,\hat{I}_2(2,3,1)m_c\,{m_b}^3-10\,\hat{I}_2(3,2,1)m_c\,{m_b}^3+5\,\hat{I}_2^{[0,1]}(3,2,2)m_c\,{m_b}^3\\
&+5\,\hat{I}_0(3,2,1){m_b}^4-15\,\hat{I}_0(1,4,1){m_b}^4+15\,\hat{I}_1^{[0,1]}(4,1,1){m_c}^2\\
&-10\,\hat{I}_0(2,2,1){m_c}^2+10\,\hat{I}_0^{[0,1]}(3,2,1){m_c}^2+10\,\hat{I}_0(2,1,2){m_c}^2\\
&+30\,\hat{I}_1^{[0,1]}(2,2,2){m_c}^2+10\,\hat{I}_2(1,2,2){m_c}^2-10\,\hat{I}_1(1,2,2){m_c}^2\\
&-30\,\hat{I}_2^{[0,1]}(2,2,2){m_c}^2-15\,\hat{I}_2^{[0,1]}(3,1,2){m_c}^2-15\,\hat{I}_2^{[0,1]}(4,1,1){m_c}^2\\
&+15\,\hat{I}_1^{[0,1]}(3,1,2){m_c}^2-15\,\hat{I}_2^{[0,1]}(3,2,1){m_c}^2+15\,\hat{I}_1^{[0,1]}(3,2,1){m_c}^2\,,
\end{aligned}
$$

where

$$
\hat{I}_n^{[i,j]}(a,b,c)=\left(M_1^2\right)^i\left(M_2^2\right)^j\frac{d^i}{d\left(M_1^2\right)^i}\frac{d^j}{d\left(M_2^2\right)^j}\left[\left(M_1^2\right)^i\left(M_2^2\right)^j\hat{I}_n(a,b,c)\right].
$$

Appendix B

In this appendix, the explicit expressions of the coefficients of the gluon condensate entering the HQET limits of the form factors $f_1^{\rm HQET}$ and $f_2^{\rm HQET}$ are given.

$$
\begin{aligned}
C_1^{HQET}=&\,2\frac{\bar{I}_2^{[0,1]}(3,2,2)}{\sqrt{Z}}+\frac{\bar{I}_0^{[0,1]}(3,2,2)}{\sqrt{Z}}-32\frac{\bar{I}_2(2,1,1)}{\sqrt{Z}}\\
&-8\frac{\bar{I}_0^{[0,1]}(3,1,1)}{\sqrt{Z}}+4\frac{\bar{I}_0(2,1,2)}{\sqrt{Z}}+12\frac{\bar{I}_2^{[0,1]}(3,1,2)}{\sqrt{Z}}+8\frac{\bar{I}_0(1,2,2)}{\sqrt{Z}}\\
&+8\frac{\bar{I}_2(2,1,2)}{\sqrt{Z}}-16\frac{\bar{I}_2^{[0,1]}(2,1,2)}{\sqrt{Z}}+4\frac{\bar{I}_2^{[0,2]}(3,2,2)}{\sqrt{Z}}-4\frac{\bar{I}_2(3,2,1)}{\sqrt{Z}}\\
&-8\frac{\bar{I}_2(2,2,1)}{\sqrt{Z}}+24\frac{\bar{I}_0^{[0,2]}(2,2,2)}{Z}-6\frac{\bar{I}_2^{[0,1]}(4,1,1)}{Z}+4\frac{\bar{I}_0^{[0,1]}(3,2,1)}{Z}
\end{aligned}
$$

$$+2\frac{\bar{I}_0(3,1,1)}{Z} - 8\frac{\bar{I}_1(2,2,1)}{Z} - 3\frac{\bar{I}_0(4,1,1)}{Z} - 24\frac{\bar{I}_0(1,3,1)}{Z}$$
$$+\frac{\bar{I}_2(3,2,2)}{Z} + 16\frac{\bar{I}_0(1,2,2)}{Z} - 8\frac{\bar{I}_1(2,1,2)}{\sqrt{Z}} + 32\frac{\bar{I}_1(2,1,1)}{\sqrt{Z}}$$
$$-4\frac{\bar{I}_0(2,1,2)}{\sqrt{Z}} - 16\frac{\bar{I}_1^{[0,2]}(3,1,2)}{\sqrt{Z}} - 6\frac{\bar{I}_0^{[0,1]}(3,1,2)}{\sqrt{Z}} + 8\frac{\bar{I}_0^{[0,1]}(2,1,2)}{\sqrt{Z}}$$
$$+16\frac{\bar{I}_1^{[0,1]}(2,1,2)}{\sqrt{Z}} - 2\frac{\bar{I}_1^{[0,1]}(3,2,2)}{\sqrt{Z}} + 4\frac{\bar{I}_1(3,2,1)}{\sqrt{Z}},$$

$$C_2^{\rm HQET} = -8\frac{\bar{I}_2(2,1,2)}{\sqrt{Z}} + 32\frac{\bar{I}_2(2,1,1)}{\sqrt{Z}} - 4\frac{\bar{I}_0(2,1,2)}{\sqrt{Z}} - 16\frac{\bar{I}_2^{[0,2]}(3,1,2)}{\sqrt{Z}}$$
$$-6\frac{\bar{I}_0^{[0,1]}(3,1,2)}{\sqrt{Z}} + 8\frac{\bar{I}_0^{[0,1]}(2,1,2)}{\sqrt{Z}} + 16\frac{\bar{I}_2^{[0,1]}(2,1,2)}{\sqrt{Z}} - 2\frac{\bar{I}_2^{[0,1]}(3,2,2)}{\sqrt{Z}}$$
$$+4\frac{\bar{I}_2(3,2,1)}{\sqrt{Z}} - 4\frac{\bar{I}_2^{[0,1]}(3,2,1)}{Z} + 4\frac{\bar{I}_2^{[0,2]}(3,2,2)}{Z} - 16\frac{\bar{I}_0(1,2,2)}{Z}$$
$$-8\frac{\bar{I}_1^{[0,1]}(3,2,1)}{Z} + 3\frac{\bar{I}_2(4,1,1)}{Z} - 32\frac{\bar{I}_2(2,2,1)}{Z} - 2\frac{\bar{I}_0(2,2,2)}{Z}$$
$$+\frac{\bar{I}_0(3,1,2)}{Z} - 16\frac{\bar{I}_1^{[0,1]}(2,2,2)}{Z} - 64\frac{\bar{I}_1(1,1,2)}{Z} - 16\frac{\bar{I}_2^{[0,1]}(2,2,2)}{Z}$$
$$-6\frac{\bar{I}_0^{[0,1]}(3,2,1)}{Z} + 2\frac{\bar{I}_1^{[0,1]}(3,2,2)}{\sqrt{Z}} + \frac{\bar{I}_0^{[0,1]}(3,2,2)}{\sqrt{Z}} - 32\frac{\bar{I}_1(2,1,1)}{\sqrt{Z}}$$
$$-8\frac{\bar{I}_0^{[0,1]}(3,1,1)}{\sqrt{Z}} + 4\frac{\bar{I}_0(2,1,2)}{\sqrt{Z}} + 12\frac{\bar{I}_1^{[0,1]}(3,1,2)}{\sqrt{Z}} + 8\frac{\bar{I}_0(1,2,2)}{\sqrt{Z}}$$
$$+8\frac{\bar{I}_1(2,1,2)}{\sqrt{Z}} - 16\frac{\bar{I}_1^{[0,1]}(2,1,2)}{\sqrt{Z}},$$

where

$$\bar{I}_n^{[i,j]}(a,b,c) = \frac{2^{i+j}}{(\sqrt{Z})^j}\,(T_1)^i\,(T_2)^j\,\frac{d^i}{d\,(T_1)^i}\frac{d^j}{d\,(T_2)^j}\left[(T_1)^i\,(T_2)^j\,\bar{I}_n(a,b,c)\right].$$

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