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PHYSICS OF ELEMENTARY PARTICLES AND ATOMIC NUCLEI. THEORY

$D_{s0}DK$ Vertex in QCD Sum Rules[1]

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Abstract—We calculate the form factors and the coupling constant in the $D_{s0}DK$ vertex in the framework of QCD sum rules. We evaluate the three point correlation functions of the vertex considering both D and K mesons off-shell. The form factors obtained are very different but give the same coupling constant.

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1. INTRODUCTION

The meson D_{s0} with the spin—parity ($J^P = 0^+$) is one of the famous heavy flavor hadrons. Determination of the D_{s0} meson width is limited by experimental resolution to a value of less than 4.6 MeV/c^2. The small width of D_{s0} meson is not surprising as its mass is below the threshold of DK system [1].

There are various applications for the strong form factors and coupling constants associated with vertices involving mesons in QCD. The standard procedure of QCDSR is followed in this work. We calculate the Operator Product Expansion (OPE) and the phenomenological contributions for the correlation function of $D_{s0}DK$ vertex and equate both contributions, following the principle of quark–hadron duality. In order to suppress higher order contributions from the OPE side as well as higher resonances (and continuum) from the phenomenological side, we use the Borel transform in both sides of the equation, obtaining the sum rule. The numerical integration of the sum rule, to estimate the coupling constant is performed. This coupling constant is a function not only of the transferred momentum Q^2 but also of the Borel masses. In general one considers the dependence of decay constants (f_D and f_K) with Borel mass to improve the stability of the coupling constant with respect to the variation of the Borel masses [2]. The outline of this paper is as follow: In Section 2, the general formalism of QCD sum rules is presented for $D_{s0}DK$ vertex. Numerical calculations and discussions are given in Section 3. Finally in Section 4, conclusion is presented.

2. THE SUM RULE FOR THE $D_{s0}DK$ VERTEX

The coupling at the $D_{s0}DK$ vertex can be evaluated by using the three-point function QCDSR. The three-point function associated with the $D_{s0}DK$ vertex, for an off-shell D meson, is given by

$$\Pi_\mu^{(D)}(p, p') = i^2 \int d^4x d^4y e^{i(p'.x - py)} \langle 0| T\{j_\mu^k(x) j^{D'}(y) j^{D'_{s0}}(0)\}|0\rangle.$$

And for an off-shell K meson:

$$\Pi_\mu^{(k)}(p, p') = i^2 \int d^4x d^4y e^{ip'.x} e^{-i(p'.x).y} \langle 0| T\{j^D(x) j_\mu^{k'}(y) j^{D'_{s0}}(0)\}|0\rangle, \tag{2}$$

where the interpolating currents are $j_\mu^K = \bar{u}\gamma_\mu\gamma_5 s$, $j^D = ic\gamma_5\bar{u}$, $j^{D_{s_0}} = \bar{c}s$, with u, s, c are *up*, *strange*, *charm* quark fields respectively. In both cases, each one of these currents has the same quantum numbers as the corresponding mesons.

We can write each Π_μ in terms of the invariant amplitudes associated with each one of these structures in the following form:

$$\Pi_\mu(p, p') = F_1(p^2, p'^2, q^2)p_\mu + F_2(p^2, p'^2, q^2)p'_\mu. \tag{3}$$

Where $q = p - p$.

Equations (1) and (2) can be calculated in two different ways: using quark degrees of freedom—the theoretical or OPE side—or using hadronic degree of freedom—the phenomenological side.

Using the above currents to evaluate the correlation functions (1) and (2), the theoretical or QCD side is obtained.

The framework to calculate the correlators in the QCD side is the Wilson operator product expansion (OPE).

$$\Pi_\mu^{(D)}(x, y) = \langle 0| T\left\{ j_\mu^k(x) j^{D'}(y) j^{D'_{s0}}(0) \right\}|0\rangle, \tag{4}$$

$$\Pi_\mu^{(D)}(x, y) = \overline{A}_\mu.1 + \overline{B}_\mu.\langle q\bar{q}\rangle + \cdots, \tag{5}$$

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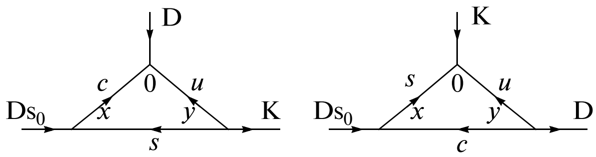

Fig. 1. Perturbative diagrams for the D off-shell (left) and K off-shell (right) correlators.

where 1 is the identity operator, $\overline{A_\mu}(x, y)$ is the perturbative contribution, $\langle q\bar{q}\rangle$ is the quark condensate and $\overline{B_\mu}(x, y)$ is the respective coefficient.

For each one of the invariant amplitudes appearing in Eq. (3), we can write a double dispersion relation over the virtualities p^2 and p'^2, holding $Q^2 = -q^2$ fixed:

$$\Pi_\mu^{(per)}(p^2, p'^2, Q^2) = -\frac{1}{4\pi^2}\int ds \int ds' \frac{\rho_\mu(p^2, p'^2, Q^2)}{(s-p^2)(s'-p'^2)} \quad (6)$$
$$+ \text{ subtraction terms,}$$

where $\rho_\mu(s, s, Q^2)$ equals the double discontinuity of the amplitude $\Pi_\mu(p^2, p'^2, Q^2)$, and is calculated using the Cutkosky's rules. The invariant amplitudes receive contributions from all terms in the OPE. The first one of those contributions comes from the perturbative term and it is represented in Fig. 1.

We can work with any structure appearing in Eq. (3), but those which have less ambiguities in the QCD sum rules approach is selected, which means, less influence from the higher dimension condensates and a better stability as a function of the Borel mass and any structure, appearing in phenomenological side. Because only the p'_μ structure appears in phenomenological side, the p'_μ structure is chosen. In this structure, the quark condensate (the condensate of lower dimension) contributes in the case of D meson off-shell. Using the following relations:

$$B_1 = \frac{1}{\lambda(s, s', q^2)}[2s'\Delta - \Delta' u],$$
$$B_2 = \frac{1}{\lambda(s, s', q^2)}[2s\Delta' - \Delta u].$$

And

$$I_0(s, s', q^2) = \frac{1}{4\lambda^{\frac{1}{2}}(s, s', q^2)},$$
$$\Delta = (s + m_3^2 - m_1^2),$$
$$\Delta' = (s' + m_3^2 - m_2^2),$$
$$u = s + s' - q^2,$$
$$\lambda(s, s', q^2) = s^2 + s'^2 + q^4 - 2sq^2 - 2s'^{q^2} - 2ss'.$$

The corresponding perturbative spectral densities which enter in Eq. (6) are:

$$\rho^{(D)}(s, s', Q^2) = \frac{3}{2[\lambda(s, s', Q^2)]^{1/2}}(m_c^2 + 2m_c m_s - s$$
$$+ [(2m_c^2 + 2m_c m_s - s + Q^2 + s')(m_c^2(s + Q^2 + s')$$
$$+ s(s' - Q^2 - s))][\lambda(s, s', Q^2)]^{-1}) \quad (9)$$

for D off-shell, and

$$\rho^{(K)}(s, s', Q^2) = \frac{3}{[\lambda(s, s', Q^2)]^{3/2}}[m_c^4(s + Q^2 + 3s')$$
$$+ s'(m_c m_s(s - Q^2 - s') + s(-s - Q^2 + s'))$$
$$+ m_c^2(-2s'(s + Q^2 + s') + m_c m_s(s + Q^2 + 3s'))]. \quad (8)$$

For K off-shell. Here

$$s = p^2, \quad s' = p'^2, \quad t = -Q^2,$$
$$\lambda(s, s', t) = s^2 + s'^2 + Q^2 - 2st - 2ss' - 2ts'.$$

3. NUMERICAL CALCULATIONS AND DISCUSSIONS

The non-perturbative contributions in the QCD side containing the quark–quark and quark–gluon condensate are calculated. The quark–quark condensate is considered for light quarks u, d and s. Contributions of the quark–gluon condensate are zero after applying the double Borel transformation with respect to both variables p^2 and p'^2, because only one variable appears in the calculations. Contributions of The quark–quark condensate are given by,

$$\Pi_\mu^{(\bar{s}s)D}(q) = -\langle\bar{s}s\rangle\left\{\frac{1}{4}Tr[F_\mu^{(a)}(p, p')]\right.$$
$$-\frac{m_s}{16}Tr\left[\left(\frac{\partial}{\partial p^\alpha}\frac{\partial}{\partial p'^\alpha}\right)F_\mu^{(\alpha)}(p, p')\gamma_\alpha\right] + \frac{1}{32}(m_s^2 - m_0^2) \quad (9)$$
$$\left.\times Tr\left[\left(\frac{\partial^2}{\partial p^\alpha \partial p'^\alpha} + \frac{\partial^2}{(\partial p^\alpha)^2} + \frac{\partial^2}{(\partial p'^\alpha)^2}F_\mu^{(\alpha)}(p, p')\right)\right]\right\}.$$

Where $\langle s\bar{s}\rangle = 0.8\ \langle q\bar{q}\rangle$, $\langle q\bar{q}\rangle = (-0.245)^3$ [6], and,

$$F_\mu^{ij(\alpha)}(p, p') = \gamma_\mu\gamma_5 i s_u^{ik}(p)\gamma_5 i s_c^{kj}(-p'). \quad (10)$$

For the K off-shell, there is no quark–quark and quark–gluon condensate contribution. Our calculations show that for two cases D and K off-shell, the gluon condensate contributions are very small and we can easily ignore them in our calculations. The phenomenological side of the vertex functions are obtained considering the contributions of the D and D_{s0} mesons to the matrix element in Eq. (1) and the D and K mesons to the matrix element in Eq. (2). The meson decay constants f_K, $f_{D_{s_0}}$ and f_D, which are defined by the following matrix elements are introduced:

$$\langle D_{s_0}(p)|K(p')D(q)\rangle = g^{(D)}_{D_{s_0}Dk}(q^2),$$

$$\langle D_{s_0}(p)|D(p')K(q)\rangle = g^{(K)}_{D_{s_0}Dk}(q^2),$$

$$\langle 0|j^{D_{s_0}}|D_{s_0}(p)\rangle = m_{D_{s_0}Dk}f_{D_{s_0}},$$

$$\langle 0|j^D|D(q)\rangle = \frac{m_D^2}{m_C}f_D,$$

$$\langle 0|j^k_\mu|k(p')\rangle = ip'_\mu f_k.$$

The corresponding phenomenological amplitudes in these structures are

$$\Pi^{(D)phen}_\mu(p,p') = g^{(D)}_{D_{s_0}Dk}(q^2)\frac{f_{D_{s_0}}f_kf_D\frac{m_D^2}{m_C}m_{D_{S_0}}m_k}{(p^2-m^2_{D_{s_0}})(q^2-m_D^2)(p'^2-m_k^2)}p'_\mu. \quad (12)$$

For D off-shell, and,

$$\Pi^{(k)phen}_\mu(p,p') = g^{(k)}_{D_{s_0}Dk}(q^2)\frac{f_{D_{s_0}}f_Df_k\frac{m_D^2}{m_C}m_{D_{S_0}}m_k}{(p^2-m^2_{D_{s_0}})(q^2-m_k^2)(p'^2-m_D^2)}q_\mu. \quad (13)$$

for K off-shell.

After performing the Borel transformation [7] with respect to the variables p^2 and p'^2 on the physical (phenomenological) and QCD parts and equating these two representations of the correlations, the equation for the strong form factors is obtained as follows:

$$g^{(D)}_{D_{s0}DK}(Q^2) = (Q^2+m_D^2)\frac{m_c}{m_{D_{s0}}m_D^2f_Kf_Dd_{D_{s0}}}e^{m^2_{D_{s0}}/M^2}e^{m_K^2/M'^2}$$
$$\times\left[m_c\langle\bar{s}s\rangle e^{-m_c^2/M^2} - \frac{1}{4\pi^2}\int_{s_0}^{s_{\max}}ds\int_0^{s'}ds'\exp(-s/M^2)\right. \quad (14)$$
$$\left.\times\exp(-s'/M'^2)f(s,s',Q^2)\right].$$

Where,

$$f(s,s',Q^2) = \frac{3}{2[\lambda(s,s',Q^2)]^{1/2}}(m_c^2+2m_cm_s-s$$
$$+[(2m_c^2+2m_cm_s-s+Q^2+s')(m_c^2(s+Q^2+s') \quad (15)$$
$$+s(s'-Q^2-s))][\lambda(s,s',Q^2)]^{-1})$$

Table 1. The leptonic decay constants in GeV [5]

f_K	f_D	$f_{D_{s0}}$
0.160	0.240	0.225

for D off-shell. And,

$$g^{(K)}_{D_{s0}DK}(Q^2) = (Q^2+m_K^2)\frac{m_c}{m_{D_{s0}}m_D^2f_Kf_Dd_{D_{s0}}}e^{m^2_{D_{s0}}/M^2}e^{m_D^2/M'^2}$$
$$\times\left[-\frac{1}{4\pi^2}\int_{s_0}^{s_{\max}}ds\int_{s'_{\min}}^{s'_{\max}}ds'\exp(-s/M^2)\exp(-s'/M'^2)g(s,s',Q^2)\right]. \quad (16)$$

Where,

$$g(s,s',Q^2) = \frac{3}{[\lambda(s,s',Q^2)]^{3/2}}[m_c^4(s+Q^2+3s')$$
$$+s'(m_cm_s(s-Q^2-s')+s(-s-Q^2+s')) \quad (17)$$
$$+m_c^2(-2s'(s+Q^2+s')+m_cm_s(s+Q^2+3s'))]$$

for K off-shell.

Where $Q^2 = -q^2$, $s_{\max}$ and $s_{\max}$ are the continuum thresholds and $s_0 = m_c^2$, $s'_{\min} = m_c^2 + \frac{m_c^2t}{s-m_c^2}$, $s'_1 = s + t - m_c^2 - st/m_c^2$ and $m_s = 0.13$, $m_c = 1.2$, $m_{Ds0} = 2.317$, $m_K = 0.498$, $m_D = 2.01$ [5]. The following relations between the Borel masses: $M^2/M'^2 = \frac{m^2_{D_{s0}}}{m_D^2}$ for K off-shell and $M^2/M'^2 = \frac{m^2_{D_{s0}}}{m_K^2}$ for D off-shell, are used.

Table 1 shows the values of the parameters used in the present calculation. The expressions for the strong form factors and coupling constants contain also four auxiliary parameters namely, Borel mass parameters M and M' and continuum thresholds $s_{\max}$ and $s_{\max}$ These are mathematical objects, so the physical quantities i.e., strong form factors and coupling constants should be independent of them. The values of the continuum thresholds are $s_{\max} = (m_{D_{s0}}+\Delta_s)^2$ and $s'_{\max} = (m_K+\Delta_{s'})^2$, for D off-shell and $s'_{\max} = (m_D+\Delta_{s'})^2$, for K off-shell. Using $\Delta_s = \Delta_{s'} = 0.5$ GeV for the continuum thresholds and fixing $Q^2 = 1$ GeV^2, we found a good stability of the sum rule for $g^{(K)}_{D_{s0}DK}(Q^2)$, as a function of the Borel mass M^2, in the interval $15 < M^2 < 30$ GeV^2, as can be seen in Fig. 3.

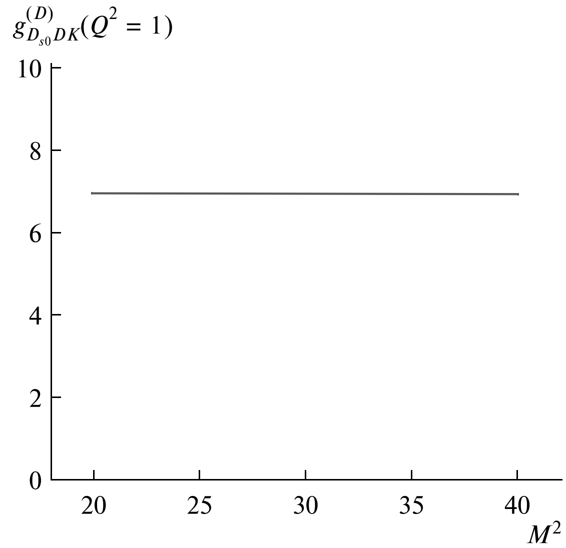

Fig. 2. $g^{(D)}_{D_{s0}DK}$ (Q^2 = 1 GeV^2) as a function of the Borel mass M^2.

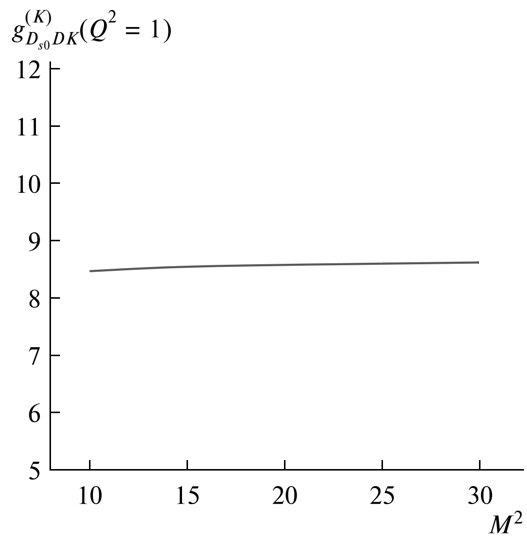

Fig. 3. $g^{(K)}_{D_{s0}DK}$ (Q^2 = 1 GeV^2) as a function of the Borel mass M^2.

In the case of $g^{(D)}_{D_{s0}DK}(Q^2)$, the interval for stability is also $20 < M^2 < 40$ GeV^2, as can be seen in Fig. 2.

Fixing $\Delta_s = \Delta_{s'} = 0.5$ GeV and $M^2 = 3$ GeV^2 in both cases, we calculate the momentum dependence of the

Table 2. value of our obtained and previously found [5] coupling constant

	LCQCD	THIS WORK(3PQCD)
$g_{D_{s0}DK}$	5.9 ± 1.7	6.5 ± 0.5

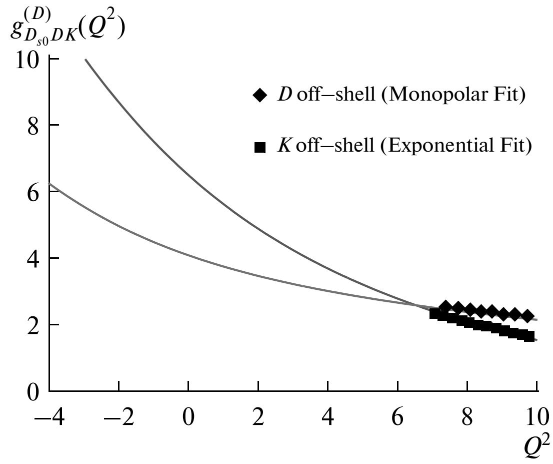

Fig. 4. $g^{(D)}_{D_{s0}DK}$ (circles) and $g^{(K)}_{D_{s0}DK}$ (squares) QCDSR form factors as a function of Q^2.

form factors which are shown in Fig. 4. The squares correspond to the $g^{(K)}_{D_{s0}DK}(Q^2)$ from factor in the interval where the sum rule is valid. The triangles are the result of the sum rule for the $g^{(D)}_{D_{s0}DK}(Q^2)$ from factor. In the case when the K meson is off-shell, our numerical results can be parameterized by an exponential function

$$g^{(K)}_{D_{s0}DK}(Q^2) = 6.56e^{-Q^2/7}. \tag{18}$$

The coupling constant was obtained as the value of the form factor at $Q^2 = -m_K^2$. In this case the resulting coupling constant is

$$g^{(K)}_{D_{s0}DK}(Q^2 = -m_K^2) = 6.8 \pm 0.4. \tag{19}$$

In the case when the D meson is off-shell, the sum rule result is represented by the triangles in Fig. 4, and they can be parameterized by a monopole formula,

$$g^{(D)}_{D_{s0}DK}(Q^2) = \frac{46.6}{Q^2 + 11.43} \tag{20}$$

giving the following coupling constant, obtained at the D pole:

$$g^{(D)}_{D_{s0}DK}(Q^2 = -m_D^2) = 6.2 \pm 0.6. \tag{21}$$

We can see that the two cases considered here, off-shell D or K, give compatible results for the coupling constant. Considering the uncertainties in the continuum thresholds and taking the average between the obtained values we have:

$$g_{D_{s0}DK} = 6.5 \pm 0.5 \text{ GeV}^{-1}. \tag{22}$$

which is given in Table 2.

CONCLUSIONS

From Table 2, we see that our result is in a fair agreement with the LCSR calculation in [5]. Determination of this strong form factors and the coupling constants and their comparison with the phenomenological models like QCD sum rules could provide useful information about the structure of the $D_{s0}(2317)$ meson.

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