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THE STRONG COUPLING CONSTANTS $g_{D_{s1}D^*K}$ AND $g_{D_{s1}D^*K_0^*}$ IN QCD SUM RULES

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The structure of the $D_{s1}(2460)$ meson has not yet been exactly known in the quark model. Considering the D_{s1} meson as a conventional $c\bar{s}$ meson, we investigate the strong form factors and coupling constants $g_{D_{s1}D^*K}$ and $g_{D_{s1}D^*K_0^*}$ in the framework of the three point QCD sum rules. Any future experimental measurement on these form factors as well as coupling constants $g_{D_{s1}D^*K}$ and $g_{D_{s1}D^*K_0^*}$ and their comparison with the obtained results in the present work can give considerable information about the structure of this meson.

Keywords: Coupling constant; QCD sum rules; mesons.

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1. Introduction

There are various application for the strong form factors and coupling constants associated with vertices involving mesons in QCD. They are important for explanation of hadronic processes in the strong interaction. Also determination of strong coupling constants can provide a real possibility for studying the nature of the bottomed and charmed pseudoscalar and axial vector mesons. Therefore they have received wide attention for the new researches in QCD. Some are as follows.

- In the hadronic decays of B meson, the strong coupling constants among the charmed meson such as $g_{D^*D^*P}$, g_{D^*DP}, g_{DDV} and $g_{D^*D^*V}$, where P and V stand for pseudoscalar and vector mesons respectively, play an important role in understanding the final state interaction.[1]

- In production of charmonium ψ/J, $\psi(2s), \dots$, which are useful sources of information in heavy ion collisions, the vertices involving charmed meson, namely $g_{DD\psi/J}$, $g_{D^*D\psi/J}$ and $g_{D^*D^*\psi/J}$ appear.[2]
- To recognize the structure of the new hadron states such as B_{s0}, B_{s1}, D_{s0} and D_{s1}, one can estimate the strong coupling constants $g_{B_{s0}BK}$, $g_{B_{s1}BK}$, $g_{D_{s0}DK}$ and $g_{D_{s1}D^*K}$.[4,3,5]

The structure of the charmed-strange meson $D_{s1}(2460)$ with the spin-parity ($J^P = 1^+$) has not been resolved, yet and has been debated in the quark model. Therefore different theoretical efforts are applied to the understanding of D_{s1} meson structure and quark content via various hadron states.[6–15] However, some physicists presumed that this discovered state is conventional $c\bar{s}$ meson.[16–26] Analysis of the $D_{s1}(2460) \to D_s^*\gamma$ and $D_{s_1}(2460) \to D_{s_0}(2317)\gamma$ show that the quark content of this meson is probably $c\bar{s}$.[27]

In this work, we focus on charmed-strange meson D_{s1} and consider the strong form factor and coupling constant $g_{D_{s1}D^*K}$. We investigate D_{s1} meson as a conventional $c\bar{s}$ state and estimate the value of the strong coupling constant between this state and virtual meson loops D^*K via the three point QCD sum rules method (3PSR) based on Shifman, Vainshtein and Zakharov works.[28] The QCD sum rule has been successfully applied to calculation of the strong coupling constant in hadron physics, for example in Ref. 29. The strong coupling constant $g_{D_{s1}D^*K}$ has been calculated with other approaches such as light cone QCD sum rules[4] and heavy chiral unitary approach,[30] before. In this work, we also compute the strong form factor and coupling constant $g_{D_{s1}D^*K_0^*}$, where $K_0^*(1430)$ is a scalar meson. Finally, we compare our results with the predictions of the other approaches.

This paper is organized as follow. In Sec. 2, we calculate the form factors and strong coupling constants $g_{D_{s1}D^*K}$ and $g_{D_{s1}D^*K_0^*}$ within 3PSR method. Finally, Sec. 3 is devoted to the numeric results and discussions.

2. The Three Point QCD Sum Rules Method

We start our discussions, considering the sufficient correlation functions responsible for the $D_{s1}D^*K(K_0^*)$ meson vertices when both D^* and $K(K_0^*)$ can be off-shell. We write the three-point correlation function associated with $D_{s1}D^*K$ and $D_{s1}D^*K_0^*$ vertex which is given by:

$$\Pi_{\nu\mu}^{D^*}(p,p') = i^2 \int d^4x\, d^4y\, e^{i(p'x-py)} \langle 0|\mathcal{T}\Big\{ j^{K(K_0^*)}(x) j_\nu^{D^*}(0) j_\mu^{D_{s1}\dagger}(y)\Big\}|0\rangle\,, \quad (1)$$

for off-shell D^* meson, and:

$$\Pi_{\nu\mu}^{K(K_0^*)}(p,p') = i^2 \int d^4x\, d^4y\, e^{i(p'x-py)} \langle 0|\mathcal{T}\Big\{ j_\nu^{D^*}(x) j^{K(K_0^*)}(0) j_\mu^{D_{s1}\dagger}(y)\Big\}|0\rangle\,, \quad (2)$$

for off-shell $K(K_0^*)$ meson. Here $j^K = \bar{s}\gamma_5 d$, $j^{K_0^*} = \bar{s}d$, $j_\nu^{D^*} = \bar{d}\gamma_\nu c$ and $j_\mu^{D_{s1}} = \bar{s}\gamma_\mu\gamma_5 c$ are interpolating currents of K, K_0^*, D^*, D_{s1} mesons, respectively and have the same quantum numbers of the associative mesons. Also $\mathcal{T}$ is time ordering

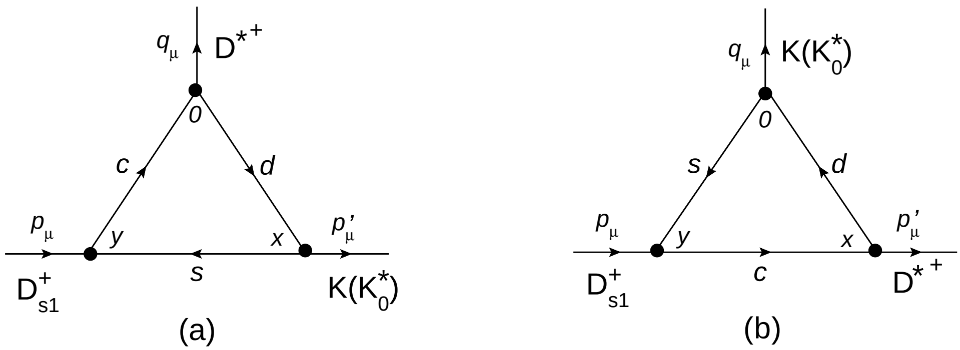

Fig. 1. Perturbative diagrams (a) for off-shell D^* and (b) off-shell $K(K_0^*)$.

product, p and p' are the four momentum of the initial and final mesons, respectively (see Fig. 1).

Equations (1) and (2) can be calculated in two different ways: in physical or phenomenological part, the representation is in terms of hadronic degrees of freedom which is responsible for the introduction of the form factors, decay constants and masses. In QCD or theoretical representation, we evaluate the correlation function in quark–gluon language and in terms of QCD degrees of freedom like quark condensate, gluon condensate, etc by the help of the Wilson operator product expansion (OPE).

In order to calculate the phenomenological part of the correlation function in Eq. (1), three complete sets of intermediate states with the same quantum number as the currents J^K, J^{D^*} and $J^{D_{s1}}$ are selected. Similarly, three complete sets of $J^{K_0^*}$, J^{D^*} and $J^{D_{s1}}$ are inserted in Eq. (2). After some calculations and using the following matrix elements:

$$\begin{aligned}
\langle D^*(p',\epsilon')K(q)|D_{s1}(p,\epsilon)\rangle &= -im_{D_{s1}}^2 g_{D_{s1}D^*K}(q^2)\epsilon\cdot\epsilon'\,,\\
\langle K_0^*(p')D^*(q,\epsilon')|D_{s1}(p,\epsilon)\rangle &= ig_{D_{s1}D^*K_0^*}(q^2)\epsilon^{\alpha\beta\gamma\sigma}\epsilon_\gamma(p)\epsilon'_\sigma(q)p_\alpha q_\beta\,,\\
\langle 0|j^K|K(p')\rangle &= \frac{m_K^2 f_K}{m_s+m_d}\,,\\
\langle 0|j^{K_0^*}|K(p')\rangle &= m_{K_0^*}f_{K_0^*}\,,\\
\langle 0|j_\mu^{D_{s1}}|D_{s1}(p,\epsilon)\rangle &= m_{D_{s1}}f_{D_{s1}}\epsilon_\mu(p)\,,\\
\langle 0|j_\nu^{D^*}|D^*(p',\epsilon')\rangle &= m_{D^*}f_{D^*}\epsilon'_\nu(p')\,,
\end{aligned} \tag{3}$$

where $q=p-p'$, $g_{D_{s1}D^*K}(q^2)$ and $g_{D_{s1}D^*K_0^*}(q^2)$ are the strong form factors, m_i and f_i, $i=K$, K_0^*, D_{s1} and D^* are the masses and decay constants of mesons, ϵ and ϵ' are the polarization vector of the D_{s1} and D^* mesons, respectively.

The general expression for the $D_{s1}D^*K$ vertex has five independent Lorentz structures. In principle, we can work with any structure. But we must choose those which have less ambiguities in the QCD sum rules approach, which means, less influence of the condensates of higher dimension, and a better stability as a function of the Borel mass.

For the $D_{s1}D^*K$ vertex, we use the structure $p_\mu p'_\nu$, which presented a better behavior. When $K(D^*)$ is off-shell meson, the phenomenological part is

$$\Pi_{\nu\mu}^{K(D^*)} = i g_{D_{s1}D^*K}^{K(D^*)}(q^2) \frac{m_{D_{s1}} m_K^2 f_{D_{s1}} f_{D^*} f_K \left(m_{D_{s1}}^2 + m_{D^*(K)}^2 - q^2\right)}{\left(q^2 - m_{K(D^*)}^2\right)\left(p^2 - m_{D_{s1}}^2\right)\left(p'^2 - m_{D^*(K)}^2\right)(m_s + m_d)\, m_{D^*}} p_\mu p'_\nu + \cdots . \tag{4}$$

In the $D_{s1}D^*K_0^*$ vertex, we have only one structure $\epsilon^{\alpha\beta\mu\nu}p_\alpha p'_\beta$. For off-shell $K_0^*(D^*)$ meson, the phenomenological part is

$$\Pi_{\nu\mu}^{K_0^*(D^*)} = -i g_{D_{s1}D^*K_0^*}^{K_0^*(D^*)}(q^2) \frac{m_{Ds1} m_{K_0^*} m_{D^*} f_{D_{s1}} f_{D^*} f_{K_0^*}}{\left(q^2 - m_{K_0^*(D^*)}^2\right)\left(p^2 - m_{D_{s1}}^2\right)\left(p'^2 - m_{D^*(K_0^*)}^2\right)} \epsilon^{\alpha\beta\mu\nu} p_\alpha p'_\beta + \cdots . \tag{5}$$

In the Eqs. (4) and (5), $\ldots$ represents the contributions of the higher states and continuum.

With the help of the OPE in Euclidean region, where p^2, $p'^2 \to -\infty$, we calculate the QCD side of the correlation function containing perturbative and non-perturbative parts. For this aim, the correlation functions for the $D_{s1}D^*K$ and the $D_{s1}D^*K_0^*$ vertices are written as follows, respectively:

$$\begin{aligned} \Pi_{\nu\mu}^{K(D^*)}(p^2, p'^2, q^2) &= \big(\Pi^{(\text{per})K(D^*)}(p^2, p'^2, q^2) \\ &\quad + \Pi^{(\text{non-per})K(D^*)}(p^2, p'^2, q^2)) p_\mu p'_\nu + \cdots , \\ \Pi_{\nu\mu}^{K_0^*(D^*)}(p^2, p'^2, q^2) &= \big(\Pi^{(\text{per})K_0^*(D^*)}(p^2, p'^2, q^2) \\ &\quad + \Pi^{(\text{non-per})K_0^*(D^*)}(p^2, p'^2, q^2)) \epsilon^{\alpha\beta\mu\nu} p_\alpha p'_\beta + \cdots . \end{aligned} \tag{6}$$

Now, we calculate the perturbative part as shown in Fig. 1. Using the double dispersion relation for each coefficient of the Lorentz structures $p_\mu p'_\nu$ and $\epsilon^{\alpha\beta\mu\nu}p_\alpha p'_\beta$ appearing in correlation functions (Eq. (6)), we get

$$\begin{aligned} \Pi^{(\text{per})M}(p^2, p'^2, q^2) &= -\frac{1}{4\pi^2} \int ds \int ds' \frac{\rho^M(s, s', q^2)}{(s - p^2)(s' - p'^2)} \\ &\quad + \text{subtraction terms}\,, \end{aligned} \tag{7}$$

where $\rho^M(s, s', q^2)$ is spectral density and M stands for D^*, K or K_0^* off-shell meson. We calculate spectral densities in terms of the usual Feynman integrals by the help of the Cutkosky rules, where the quark propagators are replaced by Dirac delta functions $\frac{1}{p^2 - m^2} \to (-2\pi i)\delta(p^2 - m^2)$.

- For the $p_\mu p'_\nu$ structure related to the $D_{s1}D^*K$ vertex; when D^* meson is off-shell:

$$\rho^{D^*}(s,s',q^2) = 4iN_cI_0[A(2m_s - 2m_d) + B_1(m_s - m_c) + B_2(m_s - m_d) + m_s]\,, \tag{8}$$

when K meson is off-shell:

$$\rho^{K}(s,s',q^2) = 4iN_cI_0[A(2m_d - 2m_s) + B_1(m_c - m_s) + B_2(m_c + m_d) + m_c]\,. \tag{9}$$

- For the $\epsilon^{\alpha\beta\mu\nu}p_\alpha p'_\beta$ structure associated to $D_{s1}D^*K_0^*$ vertex; when D^* meson is off-shell:

$$\rho^{D^*}(s,s',q^2) = 4iN_cI_0[B_1(m_s + m_c) + B_2(m_s + m_d) - m_s]\,, \tag{10}$$

when K_0^* meson is off-shell:

$$\rho^{K_0^*}(s,s',q^2) = 4iN_cI_0[-B_1(m_s + m_c) + B_2(-m_c + m_d) - m_c]\,, \tag{11}$$

where

$$\begin{aligned} A &= \frac{1}{\lambda^2(s,s',q^2)}\left[4ss'um_3^2 + 4ss'\Delta\Delta' - 3su\Delta'^2 \right.\\ &\quad \left. - 3u\Delta^2 s' - u^3m_3^2 + 2u^2\Delta\Delta'\right],\\ B_1 &= \frac{1}{\lambda(s,s',q^2)}[2s'\Delta - \Delta' u]\,,\\ B_2 &= \frac{1}{\lambda(s,s',q^2)}[2s\Delta' - \Delta u]\,, \end{aligned} \tag{12}$$

and

$$\begin{aligned} I_0(s,s',q^2) &= \frac{1}{4\lambda^{\frac{1}{2}}(s,s',q^2)}\,,\\ \Delta &= (s + m_3^2 - m_1^2)\,,\\ \Delta' &= (s' + m_3^2 - m_2^2)\,,\\ u &= s + s' - q^2\,,\\ \lambda(s,s',q^2) &= s^2 + s'^2 + q^4 - 2sq^2 - 2s'q^2 - 2ss'\,, \end{aligned} \tag{13}$$

for off-shell $D^*(K[K_0^*])$ case, m_1, m_2 and m_3 stand for the masses of the $c(s)$, d and $s(c)$ quarks, respectively. N_c represents the color factor.

We proceed to calculate the nonperturbative contributions in the QCD side that contain the quark–quark and quark–gluon condensate. The quark–quark condensate is considered for light quarks u, d and s. The corresponding diagrams of quark–quark and quark–gluon condensate for off-shell D^* are given in Fig. 2. Contributions of the diagrams (d), (e) and (f) in Fig. 2 are zero after applying the double Borel

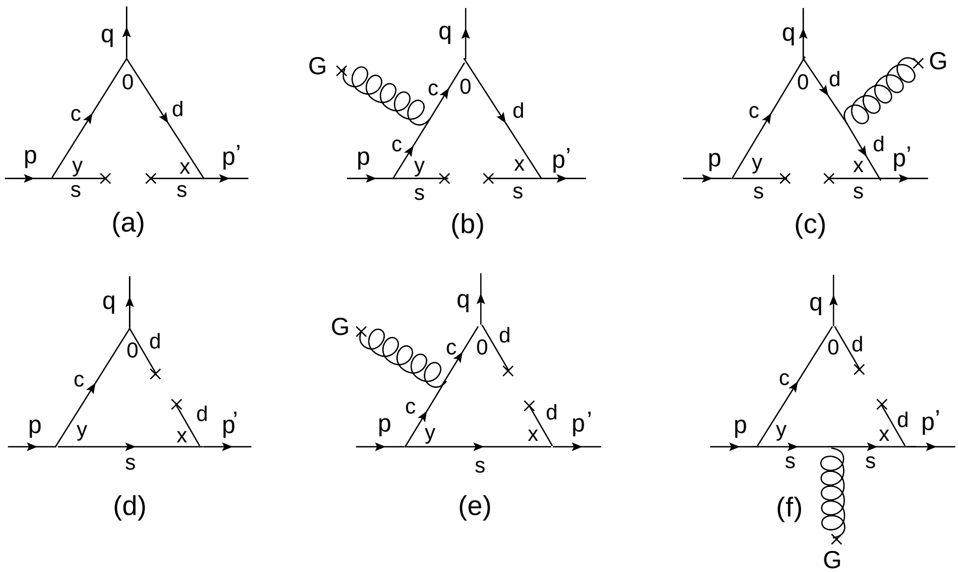

Fig. 2. (a) and (d) are the contributions of the quark–quark condensate and (b), (c), (e) and (f) are the contributions of the quark–gluon condensate for the D^* off-shell.

transformation with respect to the both variables p^2 and p'^2, because only one variable appears in calculations. For example, for the diagram (d), we obtain

$$\begin{aligned}\Pi_{\nu\mu}^{(\text{non-per})D^*} &= -\langle\bar{d}d\rangle\left\{-\frac{1}{4}\,\mathrm{Tr}\left[F_{\nu\mu}^{(d)}(p,k)\right]+\frac{m_d}{16}\right.\\ &\quad\times\mathrm{Tr}\left[\left(\frac{\partial}{\partial p^\alpha}+\frac{\partial}{\partial k^\alpha}\right)F_{\nu\mu}^{(d)}(p,k)\gamma_\alpha\right]+\frac{1}{32}\left(m_d^2-\frac{m_0^2}{2}\right)\\ &\quad\left.\times\mathrm{Tr}\left[\left(\frac{\partial^2}{\partial p^\alpha\partial k^\alpha}+\frac{\partial^2}{(\partial p^\alpha)^2}+\frac{\partial^2}{(\partial k^\alpha)^2}\right)F_{\nu\mu}^{(d)}(p,k)\right]\right\},\end{aligned}\tag{14}$$

where k is the four momentum of the s quark, $m_0^2 = 0.8 \pm 0.2$ GeV^2 (Refs. 31 and 32) and $F_{\nu\mu}^{(d)}(p,k)$ is

$$F_{\nu\mu}^{(d)}(p,k) = \gamma_\nu iS_c(p+k)\gamma_\mu\gamma_5 iS_s(k)i\gamma_5\,,\tag{15}$$

and for quark propagator with flavor f is

$$iS_f(p) = \frac{i}{\not{p}-m_f}\,.\tag{16}$$

As it is seen, there is no variable p' in the $F_{\nu\mu}^{(d)}(p,k)$ function. So the contribution of the diagram (d) is zero after the Borel transformation carried out over p^2 and p'^2.

Condensate contributions of the diagrams (a), (b) and (c) are given by:

$$\begin{aligned}\Pi_{\nu\mu}^{(\text{non-per})D^*} &= -\langle\bar{s}s\rangle\bigg\{\frac{1}{4}\,\mathrm{Tr}\left[F_{\nu\mu}^{(a)}(p,p')\right] - \frac{m_s}{16}\\ &\times \mathrm{Tr}\left[\left(\frac{\partial}{\partial p^\alpha}+\frac{\partial}{\partial p'^\alpha}\right)F_{\nu\mu}^{(a)}(p,p')\gamma_\alpha\right] + \frac{1}{32}\left(m_s^2-\frac{m_0^2}{2}\right)\\ &\times \mathrm{Tr}\left[\left(\frac{\partial^2}{\partial p^\alpha \partial p'^\alpha}+\frac{\partial^2}{(\partial p^\alpha)^2}+\frac{\partial^2}{(\partial p'^\alpha)^2}\right)F_{\nu\mu}^{(a)}(p,p')\right]\\ &+\frac{m_0^2}{96}\left(\frac{\partial}{\partial u'_\alpha}\right)\left[\mathrm{Tr}\left(F_{\nu\mu\alpha}^{(b)}(p,p')+F_{\nu\mu\alpha}^{(c)}(p,p')\right)\right]\bigg\}\,,\end{aligned} \tag{17}$$

where $\langle\bar{s}s\rangle = -0.8\times(0.24\pm0.1)^3$,[33] and

$$\begin{aligned}F_{\nu\mu}^{(a)}(p,p') &= i\gamma_5 iS_d(p')\gamma_\nu iS_c(p)\gamma_\mu\gamma_5\,,\\ F_{\nu\mu\alpha}^{(b)}(p,p') &= i\gamma_5 iS_d(p')\gamma_\nu iS_c(p-u')\gamma_\beta iS_c(p)\gamma_\mu\gamma_5\sigma_{\alpha\beta}\,,\\ F_{\nu\mu\alpha}^{(c)}(p,p') &= i\gamma_5 iS_d(p')\gamma_\beta iS_d(p'-u')\gamma_\nu iS_c(p)\gamma_\mu\gamma_5\sigma_{\alpha\beta}\,,\end{aligned} \tag{18}$$

where u' is the four momentum of the gluon in this diagrams. For the K off-shell, there is no quark–quark and quark–gluon condensate condensate contribution. Our calculations show for two cases D^* and K off-shell, the gluon–gluon condensate contributions are very small in comparison with the quark–quark and quark–gluon condensate contributions and we can ignore their contributions in our calculations.

In the same way, the quark–quark and quark–gluon condensate contributions are calculated for $D_{s1}D^*K_0^*$ vertex.

Considering Eq. (6) for extracting the $\Pi^{(\text{non-per})M}(p^2,p'^2,q^2)$ and after performing the Borel transformation[34] with respect to the variables $p^2(B_{p^2}(M_1^2))$ and $p'^2(B_{p'}^2(M_2^2))$ on the physical (phenomenological) and QCD parts and equating these two representations of the correlations, we obtain the equation for the strong form factors as follows.

- For the $g_{D_{s1}D^*K}(Q^2)$ form factors; when D^* meson is off-shell:

$$\begin{aligned}g_{D_{s1}D^*K}^{D^*}(Q^2) &= C_1 e^{\frac{m_{D_{s1}}^2}{M_1^2}} e^{\frac{m_K^2}{M_2^2}}\bigg\{-\frac{1}{4\pi^2}\int_{(m_d+m_s)^2}^{s'_0} ds'\\ &\times\int_{s_1}^{s_0} ds\rho^{D^*}(s,s',q^2)e^{-\frac{s}{M_1^2}}e^{-\frac{s'}{M_2^2}}\\ &+B_{p^2}(M_1^2)B_{p'}^2(M_2^2)\Pi^{(\text{non-per})D^*}(p^2,p'^2,q^2)\bigg\}\,,\end{aligned} \tag{19}$$

when K meson is off-shell:

$$g^{K}_{D_{s1}D^*K}(Q^2) = C_2 e^{\frac{m^2_{D_{s1}}}{M_1^2}} e^{\frac{m^2_{D^*}}{M_2^2}} \Bigg\{ -\frac{1}{4\pi^2} \int_{(m_c+m_d)^2}^{s'_0} ds' \times \int_{s_2}^{s_0} ds\, \rho^{K}(s,s',q^2) e^{-\frac{s}{M_1^2}} e^{-\frac{s'}{M_2^2}} \Bigg\}, \tag{20}$$

where $Q^2 = -q^2$, s_0 and s'_0 are the continuum thresholds, s_1 and s_2 are the lower limits of the integrals over s as:

$$s_{1(2)} = \frac{\left(m^2_{s(c)} + q^2 - m^2_{c(s)} - s'\right)\left(m^2_{c(s)} s' - q^2 m^2_{s(c)}\right)}{\left(m^2_{c(s)} - q^2\right)\left(m^2_{s(c)} - s'\right)}, \tag{21}$$

and

$$C_{1(2)} = i \frac{m_{D^*}\left(q^2 - m^2_{D^*(K)}\right)(m_s + m_d)}{m_{Ds1} m_K^2 f_{D_{s1}} f_{D^*} f_K (m^2_{D_{s1}} + m^2_{D^*(K)} - q^2)}. \tag{22}$$

- For the $g_{D_{s1}D^*K_0^*}(Q^2)$ form factors; when D^* meson is off-shell:

$$g^{D^*}_{D_{s1}D^*K_0^*}(Q^2) = C_3\, e^{\frac{m^2_{D_{s1}}}{M_1^2}} e^{\frac{m^2_{K_0^*}}{M_2^2}} \Bigg\{ -\frac{1}{4\pi^2} \int_{(m_d+m_s)^2}^{s'_0} ds' \times \int_{s_1}^{s_0} ds\, \rho^{D^*}(s,s',q^2) e^{-\frac{s}{M_1^2}} e^{-\frac{s'}{M_2^2}} + B_{p^2}(M_1^2) B^2_{p'}(M_2^2) \Pi^{(\text{non-per})D^*}(p^2, p'^2, q^2) \Bigg\}, \tag{23}$$

when K_0^* meson is off-shell:

$$g^{K_0^*}_{D_{s1}D^*K_0^*}(Q^2) = C_4 e^{\frac{m^2_{D_{s1}}}{M_1^2}} e^{\frac{m^2_{D^*}}{M_2^2}} \Bigg\{ -\frac{1}{4\pi^2} \int_{(m_c+m_d)^2}^{s'_0} ds' \times \int_{s_2}^{s_0} ds\, \rho^{K_0^*}(s,s',q^2) e^{-\frac{s}{M_1^2}} e^{-\frac{s'}{M_2^2}} \Bigg\}, \tag{24}$$

where

$$C_{3(4)} = -i \frac{\left(q^2 - m^2_{D^*(K_0^*)}\right)}{m_{D_{s1}} m_{D^*} m_{K_0^*} f_{D_{s1}} f_{D^*} f_{K_0^*}}. \tag{25}$$

3. Numerical Analysis

In this section, we analyze the strong form factors, and coupling constants for the $D_{s1}D^*K$ and $D_{s1}D^*K_0^*$ vertices. We choose the values of quark and meson masses as: $m_d = 5.6 \pm 1.6$ MeV, $m_s = 0.14 \pm 0.01$ GeV, $m_c = 1.3$ GeV, $m_K = 0.493$ GeV, $m_{K_0^*} = 1.425 \pm 0.05$ GeV, $m_{D^*} = 2.010$ GeV, $m_{D_{s1}} = 2.459$ GeV.[35] Also the

Table 1. The leptonic decay constants in MeV.

$f_{D_{s1}}$	f_{D^*}	f_K	$f_{K_0^*}$
225 ± 20	230 ± 20	159.8 ± 1.84	445 ± 50

leptonic decay constants used in the calculation of the QCD sum rule for $D_{s1}D^*K$ and $D_{s1}D^*K_0^*$ vertices are presented in Table 1.[35–38]

The expressions for the strong form factors and coupling constants contain also four auxiliary parameters: Borel mass parameters M_1 and M_2 and continuum thresholds s_0 and s_0'. These are mathematical objects, so the physical quantities i.e. strong form factors and coupling constants should be independent of them. The parameters of s_0 and s_0' are the continuum thresholds. The values of the continuum thresholds $s_0 = (m_i + r_1)^2$ and $s_0' = (m_o + r_2)^2$, where m_i and m_o are the masses of the incoming and outgoing meson, respectively.[39–41] We use $r_1 = 0.5$ GeV and $r_2 = 0.6$ GeV in the case of D^* off-shell and $r_1 = r_2 = 0.5$ GeV and for $K(K_0^*)$ off-shell in $Q^2 = 1$ GeV^2. For $D_{s1}D^*K$ vertex, we found a good stability of the sum rule in the interval 15 GeV$^2 \leq M_1 \leq 21$ GeV^2 and 7 GeV$^2 \leq M_2 \leq 15$ GeV^2 for two cases D^* and K off-shell. Also for $D_{s1}D^*K_0^*$ vertex, M_1^2 and M_2^2 are in the interval 14 GeV$^2 \leq M_1 \leq 20$ GeV^2 and 11 GeV$^2 \leq M_2 \leq 16$ GeV^2 for two cases D^* and K_0^* off-shell. The dependence of the strong form factors $g_{D_{s1}D^*K}$ and $g_{D_{s1}D^*K_0^*}$ on Borel mass parameters for off-shell D^* and K mesons are shown in Figs. 3 and 4, respectively.

For calculation of the strong form factors $g_{D_{s1}D^*K}(Q^2)$ and $g_{D_{s1}D^*K_0^*}(Q^2)$ within 3PSR, we have chosen the Borel masses parameters to be $M_1^2 = 17$ GeV^2, $M_2^2 = 8$ GeV^2 and $M_1^2 = 17$ GeV^2, $M_2^2 = 12$ GeV^2, respectively.

In Eqs. (19), (20) and Eqs. (23), (24), we calculated the Q^2 dependence of the strong coupling form factors in the region where the sum rule is valid. So to extend our results to the full physical region, we look for parametrization of the form factors

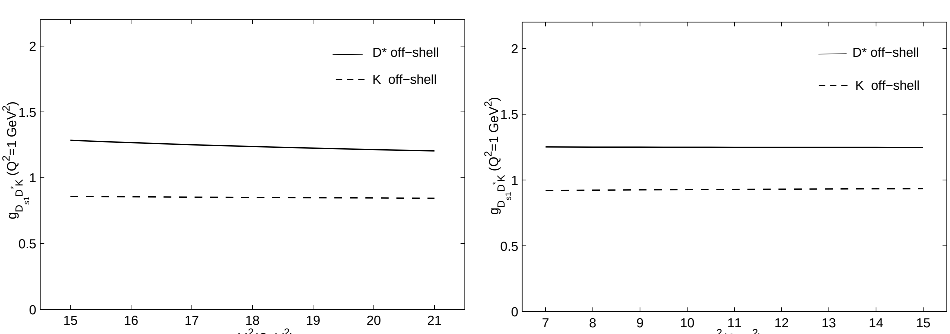

Fig. 3. The $g_{D_{s1}D^*K}(Q^2 = 1\text{ GeV}^2)$ as a function of the Borel mass M_1^2 with $M_2^2 = 8$ GeV^2 (left) and as a function of the Borel mass M_2^2 with $M_1^2 = 17$ GeV^2 (right) for two cases D^* and K off-shell meson.

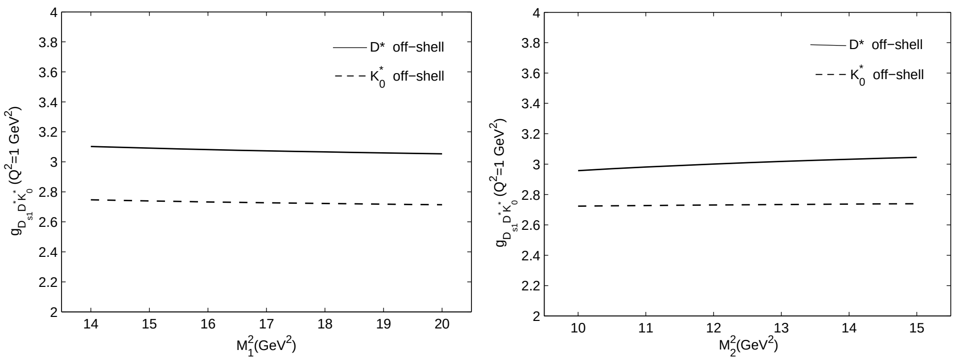

Fig. 4. The $g_{D_{s1}D^*K_0^*}(Q^2 = 1\ \text{GeV}^2)$ as a function of the Borel mass M_1^2 with $M_2^2 = 11\ \text{GeV}^2$ (left) and as a function of the Borel mass M_2^2 with $M_1^2 = 17\ \text{GeV}^2$ (right) for two cases D^* and K_0^* off-shell meson.

Table 2. Parameters appearing in the fit functions.

Form factor	A	B
$g^{K}_{D_{s1}D^*K}$	12.70	6.51
$g^{D^*}_{D_{s1}D^*K}$	1.32	56.92
$g^{K_0^*}_{D_{s1}D^*K_0^*}$	43.53	12.21
$g^{D^*}_{D_{s1}D^*K_0^*}$	3.24	15.61

in such a way that in the validity region of 3PSR, this parametrization coincides with the sum rules prediction. For off-shell K and K_0^* meson, our numerical calculations show that the satisfactory parametrization of the form factors with respect to Q^2 is:

$$g(Q^2) = \frac{A}{Q^2 + B}\,, \tag{26}$$

and for off-shell D^* meson the strong form factors can be fitted by the exponential fit function as given:

$$g(Q^2) = Ae^{-Q^2/B}\,. \tag{27}$$

The values of the parameters A and B are given in Table 2. The dependence of the strong form factors in Q^2 are shown in Figs. 5 and 6 for the $D_{s1}D^*K$ and the $D_{s1}D^*K_0^*$ vertices, respectively. These figures contain the strong form factors obtained via fit functions and 3PSR (see Eqs. (19)–(24)). In this figures, the small circles and boxes correspond to the form factors via 3PSR calculations. As it is seen, the form factors and their fit functions coincide together, well.

As in previous works,[39–41] we define the coupling constant as the value of the strong coupling form factor at $Q^2 = -m_m^2$ in Eqs. (26) and (27), where m_m is the mass of the off-shell meson.

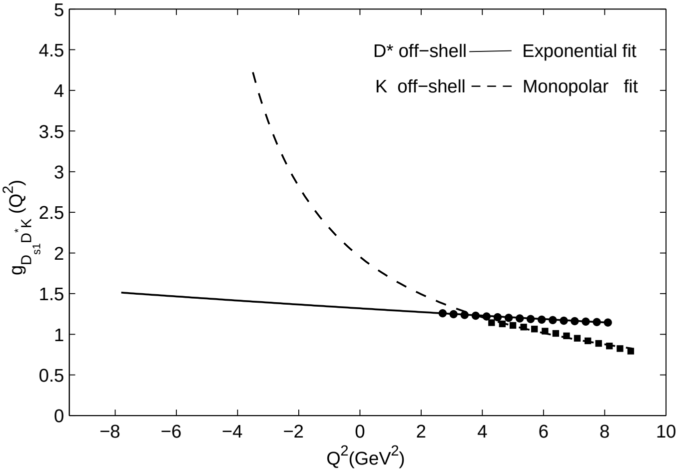

Fig. 5. The strong form factors $g^{D^*}_{D_{s1}D^*K}(Q^2)$ and $g^{K}_{D_{s1}D^*K}(Q^2)$ as a function of Q^2.

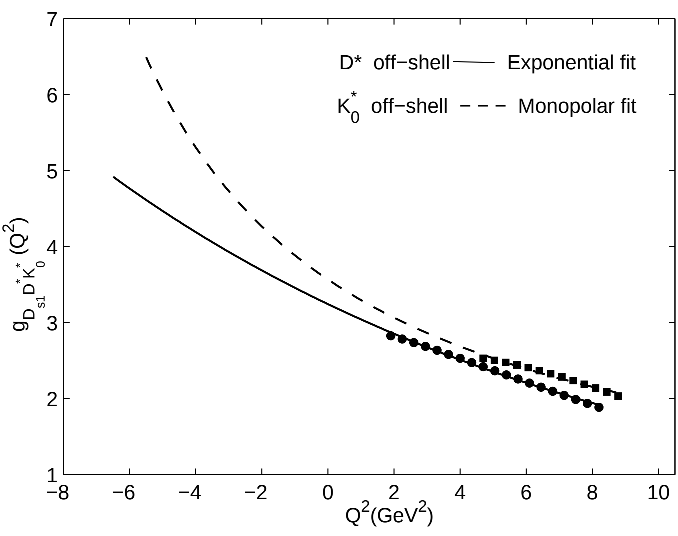

Fig. 6. The strong form factors $g^{D^*}_{D_{s1}D^*K_0^*}(Q^2)$ and $g^{K_0^*}_{D_{s1}D^*K_0^*}(Q^2)$ as a function of Q^2.

Considering the uncertainties results with the continuum threshold, we vary $r_{1,2}$ between 0.4 GeV $\leq r_1 \leq$ 0.6 GeV and 0.5 GeV $\leq r_2 \leq$ 0.7 GeV in the case of D^* off-shell and 0.4 GeV $\leq r_{1,2} \leq$ 0.6 GeV for $K(K_0^*)$ off-shell, the leptonic decay constants $f_{D_{s1}}$, f_{D^*}, f_K and $f_{K_0^*}$ and uncertainties in the values of the other input parameters, we obtain the values of the strong coupling constants shown in Table 3. We can see that the two cases considered here, off shell D^* or K meson, give compatible results for the coupling constant.

Table 3. The strong coupling constants $g_{D_{s1}D^*K}$ and $g_{D_{s1}D^*K_0^*}$ in different approaches: 3PSR (our), light cone sum rules (LCSR),[4] heavy chiral unitary,[30] in GeV^{-1}.

Strong coupling constant	K off-shell	D^* off-shell	Average (our)	Ref. 4	Ref. 30
$g_{D_{s1}D^*K}$	2.03 ± 0.23	1.41 ± 0.18	1.72 ± 0.21	1.67 ± 0.57	1.78
$g_{D_{s1}D^*K_0^*}$	4.28 ± 0.38	4.20 ± 0.42	4.24 ± 0.42	—	—

In summary, we estimate the strong form factors and the coupling constants for $D_{s1}D^*K$ and $D_{s1}D^*K_0^*$ vertices within QCD sum rules. We compare our result for $g_{D_{s1}D^*K}$ with the results of the other approaches such as light cone QCD sum rules and heavy chiral unitary. Detection of these strong form factors and the coupling constants and their comparison with the phenomenological models like QCD sum rules could give useful information about the structure of the $D_{s1}(2460)$ axial vector meson.

References

1. Z. G. Wang, *Nucl. Phys. A* **796**, 61 (2007).
2. M. E. Bracco, M. Chiapparini, F. S. Navarra and M. Nielson, *Phys. Lett. B* **605**, 326 (2005).
3. P. Colangelo and F. De Fazio, *Phys. Lett. B* **559**, 49 (2003).
4. Z. G. Wang, *J. Phys. G* **34**, 753 (2007).
5. Z. G. Wang, *Phys. Rev. D* **77**, 054024 (2008).
6. E. Kolomeitsev and M. Lutz, *Phys. Lett. B* **582**, 39 (2004).
7. T. Barnes, F. E. Close and H. J. Lipkin, *Phys. Rev. D* **68**, 054006 (2003).
8. A. P. Szczepaniak, *Phys. Lett. B* **567**, 23 (2003).
9. H. Y. Cheng and W. S. Hou, *Phys. Lett. B* **566**, 193 (2003).
10. K. Terasaki, *Phys. Rev. D* **68**, 011501 (2003).
11. T. E. Browder, S. Pakvasa and A. A. Petrov, *Phys. Lett. B* **578**, 365 (2004).
12. U. Dmitrasinovic, *Phys. Rev. D* **70**, 096011 (2004).
13. U. Dmitrasinovic, *Phys. Rev. Lett.* **94**, 162002 (2005).
14. M. E. Bracco, A. Lozea, R. D. Matheus, F. S. Navarra and M. Nielsen, *Phys. Lett. B* **624**, 217 (2005).
15. H. Kim and Y. Oh, *Phys. Rev. D* **72**, 074012 (2005).
16. S. Godfrey, *Phys. Lett. B* **568**, 254 (2003).
17. W. A. Bardeen, E. J. Eichten and C. T. Hill, *Phys. Rev. D* **68**, 054024 (2003).
18. M. A. Nowak, M. Rho and I. Zahed, *Acta Phys. Pol. B* **35**, 2377 (2004).
19. A. Deandrea, G. Nardulli and A. D. Polosa, *Phys. Rev. D* **68**, 097501 (2003).
20. R. N. Cahn and J. D. Jackson, *Phys. Rev. D* **68**, 037502 (2003).
21. Y. B. Dai, C. S. Huang, C. Liu and S. L. Zhu, *Phys. Rev. D* **68**, 114011 (2003).
22. W. Lucha and F. Schobert, *Mod. Phys. Lett. A* **18**, 2837 (2003).
23. J. Hofmann and M. F. M. Lutz, *Nucl. Phys. A* **733**, 142 (2004).
24. M. Sadzikowski, *Phys. Lett. B* **579**, 39 (2004).
25. D. Becirevic *et al.*, *Phys. Lett. B* **599**, 59 (2004).
26. T. Lee, I. W. Lee, D. P. Min and B. Y. Park, *Eur. Phys. J. C* **49**, 737 (2007).
27. P. Colangelo, F. De Fazio and A. Ozpineci, *Phys. Rev. D* **72**, 074004 (2005).
28. M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, *Nucl. Phys. B* **147**, 448 (1979).

29. H. Sundu, J. Y. Sungu, S. Sahin, N. Yinelek and K. Azizi, *Phys. Rev. D* **83**, 114009 (2011).
30. F. K. Guo, P. N. Shen and H. C. Chiang, *Phys. Lett. B* **647**, 133 (2007).
31. H. G. Dosch, M. Jamin and S. Narison, *Phys. Lett. B* **220**, 251 (1989).
32. V. M. Belyaev and B. L. Ioffe, *Sov. Phys. JETP* **57**, 716 (1982).
33. B. L. Ioffe, *Prog. Part. Nucl. Phys.* **56**, 232 (2006).
34. B. L. Loffe and A. V. Smilga, *Nucl. Phys. B* **216**, 373 (1983).
35. Particle Data Group (K. Nakamura *et al.*), *J. Phys. G* **37**, 075021 (2010).
36. C. E. Thomas, *Phys. Rev. D* **73**, 054016 (2006).
37. P. Colangelo, F. De Fazio and A. Ozpineci, *Phys. Rev. D* **72**, 074004 (2005).
38. H. Y. Cheng, C. K. Chua and K. C. Yang, *Phys. Rev. D* **73**, 014017 (2006).
39. F. S. Navarra, M. Nielsen, M. E. Bracco, M. Chiapparini and C. L. Schat, *Phys. Lett. B* **489**, 319 (2000).
40. F. S. Navarra, M. Nielsen and M. E. Bracco, *Phys. Rev. D* **65**, 037502 (2002).
41. M. E. Bracco, M. Chiapparini, F. S. Navarra and M. Nielsen, *Phys. Lett. B* **659**, 559 (2008).