

Vertices of the vector mesons with the strange charmed mesons in QCD

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We investigate the strong form factors and coupling constants of vertices containing the strange charmed mesons D_{s0}^* , D_s , D_s^* , and D_{s1} with the vector mesons ϕ and J/ψ in the framework of the three point QCD sum rules. Taking into account the nonperturbative part contributions of the correlation functions, the condensate terms of dimension 3, 4 and 5 related to the contributions of the quark-quark, gluon-gluon, and quark-gluon condensate, respectively, are evaluated. The present work can give considerable information about the hadronic processes involving the strange charmed mesons.

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I. INTRODUCTION

The strong form factors and coupling constants associated with vertices involving mesons are important for the explanation of hadronic processes in the strong interaction. They have received wide attention for the new research of the nature of the charmed pseudoscalar and axial vector mesons. The strong coupling constants among the charmed meson such as $g_{D^*D^*P}$, g_{D^*DP} , g_{DDV} , and $g_{D^*D^*V}$, where P and V stand for pseudoscalar and vector mesons, respectively, play an important role in understanding the final state interactions in QCD [1].

The QCD sum rules have been successfully applied to a wide variety of problems in hadron physics [2] (for details about this method, see Refs. [3,4]). Some possible vertices involving charmed mesons such as $D^*D\pi$ [5,6], $DD\rho$ [7], $D^*D^*\rho$ [8], DDJ/ψ [9], D^*DJ/ψ [10], D^*D_sK , D_s^*DK , D_0D_sK , $D_{s0}DK$ [11], D^*D^*P , D^*DV , DDV [12], $D^*D^*\pi$ [13], D^*D^*J/ψ [14], D_sD^*K , D_s^*DK [15], and $DD\omega$ [16] have been studied in the framework of the QCD sum rules. It is very important to know the precise functional form of the form factors in these vertices and even to know how this form changes when one or the other (or both) mesons are off shell [8].

In this work, we consider vertices of vector mesons with the strange charmed mesons via the three point QCD sum rules (3PSR); i.e., we calculate the strong form factors and coupling constants associated with the $VD_{s0}^*D_{s0}^*$, VD_sD_s , $VD_s^*D_s^*$, and $VD_{s1}D_{s1}$ vertices, where V can be ϕ or J/ψ . In the 3PSR theory, the calculation begins with a three point correlation function. The correlation function is investigated in two phenomenological and theoretical sides. The strong form factors are estimated by equating two sides and applying the double Borel transformations with respect to the momentum of the initial and final states to suppress the contribution of the higher states and continuum. Calculating the theoretical part of the correlation

function consists of two perturbative and nonperturbative contributions. The nonperturbative part of the correlation function is called the condensate contribution. The condensate term of dimension 3 is related to the contribution of the quark-quark condensate, and dimension 4 and 5 are connected to the gluon-gluon and gluon-quark condensate, respectively. The main points in the present work are the calculation of the quark-quark, gluon-gluon, and gluon-quark corrections; these are the most important corrections of the nonperturbative part of the correlation function in the 3PSR method.

This paper includes four sections. The calculation of the sum rules for the strong form factors of the $\phi D_{s0}^*D_{s0}^*$, ϕD_sD_s , $\phi D_s^*D_s^*$, and $\phi D_{s1}D_{s1}$ vertices are presented in Sec. II. With the necessary changes in the expressions derived for the strong form factors of the vertices involving the ϕ meson, such as variations in the type of quarks, we can easily apply the same calculations for the $J/\psi D_{s0}^*D_{s0}^*$, $J/\psi D_sD_s$, $J/\psi D_s^*D_s^*$, and $J/\psi D_{s1}D_{s1}$ vertices. In Sec. III, the calculations of the quark-quark, gluon-quark, and gluon-gluon condensate contributions in the Borel transform scheme are presented. In this section the strong form factors are derived. Section IV presents our numerical analysis of the strong form factors as well as the coupling constants.

II. THE THREE POINT QCD SUM RULES METHOD

We start with the correlation function in 3PSR to calculate the strong form factors associated with the $\phi D_{s0}^*D_{s0}^*$, ϕD_sD_s , $\phi D_s^*D_s^*$, and $\phi D_{s1}D_{s1}$ meson vertices when both strange charmed and ϕ mesons can be off shell, in 3PSR we start with the correlation function. For off-shell charmed mesons, these correlation functions are given by

$$\begin{aligned} \Pi_\nu^{D'}(p, p') \\ = i^2 \int d^4x d^4y e^{i(p'x - py)} \langle 0 | \mathcal{T} \{ j^{D'}(x) j^{D'\dagger}(0) j_\nu^\phi(y) \} | 0 \rangle, \end{aligned} \quad (1)$$

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$$\begin{aligned} \Pi_{\nu\alpha\mu}^{D''}(p, p') &= i^2 \int d^4x d^4y e^{i(p'x - py)} \langle 0 | \mathcal{T}\{j_\nu^{D''}(x) j_\alpha^{D''\dagger}(0) j_\mu^\phi(y)\} | 0 \rangle, \\ (2) \end{aligned}$$

where D' stands for the scalar or pseudoscalar charmed mesons (D_{s0}^* , D_s), and D'' stands for the vector or axial vector charmed mesons (D_s^* , D_{s1}). For the off-shell ϕ meson, these quantities are

$$\begin{aligned} \Pi_\nu^\phi(p, p') &= i^2 \int d^4x d^4y e^{i(p'x - py)} \langle 0 | \mathcal{T}\{j_\nu^{D'}(x) j_\nu^\phi(0) j^{D'\dagger}(y)\} | 0 \rangle, \\ (3) \end{aligned}$$

$$\begin{aligned} \Pi_{\nu\mu\alpha}^\phi(p, p') &= i^2 \int d^4x d^4y e^{i(p'x - py)} \langle 0 | \mathcal{T}\{j_\nu^{D''}(x) j_\mu^\phi(0) j_\alpha^{D''\dagger}(y)\} | 0 \rangle, \\ (4) \end{aligned}$$

where $j_{s0}^{D^*} = \bar{s}Uc$, $j_s^{D_s} = \bar{s}\gamma_5 c$, $j_s^{D_s^*} = \bar{s}\gamma_\nu c$, $j_{s1}^{D_{s1}} = \bar{s}\gamma_\nu \gamma_5 c$, and $j_\nu^\phi = \bar{s}\gamma_\nu s$ are interpolating currents of D_{s0}^* , D_s , D_s^* , D_{s1} , and ϕ mesons, respectively, and have the same quantum numbers of the associative mesons. Also, \mathcal{T} is the time ordering product, p and p' are the

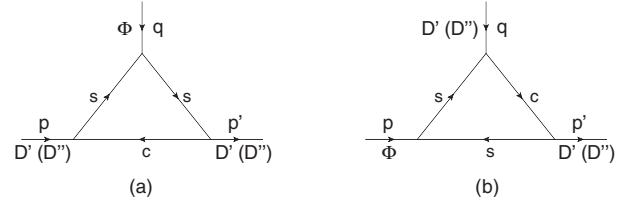


FIG. 1. Perturbative diagrams for the off-shell ϕ (a) and the off-shell $D'(D'')$ (b).

four-momentum of the initial and final mesons, respectively (see Fig. 1).

Equations (1)–(4) can be calculated in two different ways: In the physical or phenomenological part, the representation is in terms of hadronic degrees of freedom which is responsible for the introduction of the form factors, decay constants, and masses. In QCD or theoretical representation, we evaluate the correlation function in quark-gluon language and in terms of QCD degrees of freedom like quark-quark condensate, gluon-gluon condensate, etc., with the help of the Wilson operator product expansion (OPE).

In order to calculate the phenomenological parts of the correlation functions in Eqs. (1)–(4), three complete sets of intermediate states with the same quantum number should be inserted in these equations. For Π_ν^ϕ and $\Pi_{\nu\mu\alpha}^\phi$, we have

$$\Pi_\nu^\phi = \frac{\langle 0 | j_\nu^{D'} | D'(p) \rangle \langle 0 | j_\nu^{D'} | D'(p') \rangle \langle D'(p) D'(p') | \phi(q, \epsilon'') \rangle \langle \phi(q, \epsilon'') | j_\nu^\phi | 0 \rangle}{(p^2 - m_{D'}^2)(p'^2 - m_{D'}^2)(q^2 - m_\phi^2)} + \text{higher and continuum states}, \quad (5)$$

$$\Pi_{\nu\mu\alpha}^\phi = \frac{\langle 0 | j_\nu^{D''} | D''(p, \epsilon) \rangle \langle 0 | j_\mu^{D''} | D''(p', \epsilon') \rangle \langle D''(p, \epsilon) D''(p', \epsilon') | \phi(q, \epsilon'') \rangle \langle \phi(q, \epsilon'') | j_\alpha^\phi | 0 \rangle}{(p^2 - m_{D''}^2)(p'^2 - m_{D''}^2)(q^2 - m_\phi^2)} + \text{higher and continuum states}. \quad (6)$$

The same process can be easily used for $\Pi_\nu^{D'}$ and $\Pi_{\nu\mu\alpha}^{D''}$. The following matrix elements are defined in the standard way in terms of strong form factors as well as leptonic decay constants of the charmed and ϕ mesons as

$$\begin{aligned} \langle D'(p) D'(p') | \phi(q, \epsilon'') \rangle &= -g_{\phi D'D'}^\phi(q^2)(p_\nu + p'_\nu)\epsilon''^\nu(q), \\ \langle D''(p, \epsilon) D''(p', \epsilon') | \phi(q, \epsilon'') \rangle &= -ig_{\phi D''D''}^\phi(q^2)[(p' - p)_\mu g_{\nu\alpha} - (q + p')_\nu g_{\alpha\mu} + (q + p)_\alpha g_{\mu\nu}] \epsilon''^\mu(q) \epsilon_{D''}^{\nu\alpha}(p') \epsilon_{D''}^\nu(p), \\ \langle 0 | j_\nu^{D'} | D'(p) \rangle &= C(D') m_{D'} f_{D'}, \\ \langle 0 | j_\nu^{D''} | D''(p, \epsilon) \rangle &= m_{D''} f_{D''} \epsilon_\nu(p), \\ \langle 0 | j_\mu^\phi | \phi(q, \epsilon'') \rangle &= m_\phi f_\phi \epsilon_\mu''(q), \end{aligned} \quad (7)$$

where $q = p' - p$, $g_{\phi D'D'}^\phi(q^2)$, and $g_{\phi D''D''}^\phi(q^2)$ are the strong form factors, m_i and f_i , $i = D', D''$ and ϕ are the masses and decay constants of mesons, ϵ , ϵ' and ϵ'' are the polarization vector of the vector mesons. Also, $C(D_{s0}^*) = 1$ and $C(D_s) = \frac{m_{D_s}}{(m_s + m_c)}$.

Using Eq. (7) in Eqs. (5) and (6), and after some calculations, we obtain

$$\Pi_\nu^\phi = -g_{\phi D'D'}^\phi(q^2) \frac{[C(D')]^2 m_\phi m_{D'}^2 f_\phi f_{D'}^2}{(p^2 - m_\phi^2)(p'^2 - m_{D'}^2)(q^2 - m_\phi^2)} (p_\nu + \text{other structures}) + \text{higher and continuum states}, \quad (8)$$

$$\Pi_{\mu\alpha\nu}^\phi = -g_{\phi D''D''}^\phi(q^2) \frac{m_\phi f_\phi f_{D''}^2 (4m_{D''}^2 - q^2)}{2(p^2 - m_\phi^2)(p'^2 - m_{D''}^2)(q^2 - m_\phi^2)} (g_{\mu\alpha} q_\nu + \text{other structures}) + \text{higher and continuum states}, \quad (9)$$

$$\Pi_\nu^{D'} = -g_{\phi D'D'}^{D'}(q^2) \frac{[C(D')]^2 m_{D'}^2 f_\phi f_{D'}^2 (m_{D'}^2 + m_\phi^2 - q^2)}{m_\phi(p^2 - m_\phi^2)(p'^2 - m_{D'}^2)(q^2 - m_{D'}^2)} (p_\nu + \text{other structures}) + \text{higher and continuum states}, \quad (10)$$

$$\Pi_{\mu\alpha\nu}^{D''} = -g_{\phi D''D''}^{D''}(q^2) \frac{m_\phi f_\phi f_{D''}^2 (3m_{D''}^2 + m_\phi^2 - q^2)}{2(p^2 - m_\phi^2)(p'^2 - m_{D''}^2)(q^2 - m_{D''}^2)} (g_{\mu\alpha} q_\nu + \text{other structures}) + \text{higher and continuum states}. \quad (11)$$

With the help of the OPE in the Euclidean region, where $p^2, p'^2 \rightarrow -\infty$, we calculate the QCD side of the correlation functions containing perturbative and nonperturbative parts. For this aim, the correlation functions for the $D'D'\phi$ and the $D''D''\phi$ vertices are written, respectively, as follows:

$$\Pi_\nu^{D'(\phi)}(p^2, p'^2, q^2) = (\Pi_{\text{per}}^{D'(\phi)} + \Pi_{\text{nonper}}^{D'(\phi)}) p_\nu + \dots, \quad \Pi_{\mu\alpha\nu}^{D''(\phi)}(p^2, p'^2, q^2) = (\Pi_{\text{per}}^{D''(\phi)} + \Pi_{\text{nonper}}^{D''(\phi)}) g_{\mu\alpha} q_\nu + \dots, \quad (12)$$

where \dots denotes other structures and higher states.

First, we calculate the perturbative part whose diagrams are shown in Fig. 1.

Using the double dispersion relation for each coefficient of the Lorentz structures p_ν and $g_{\mu\alpha} q_\nu$ appearing in the correlation functions [Eq. (12)], we get

$$\Pi_{\text{per}}^M(p^2, p'^2, q^2) = -\frac{1}{4\pi^2} \int ds \int ds' \frac{\rho^M(s, s', q^2)}{(s - p^2)(s' - p'^2)} + \text{subtraction terms}, \quad (13)$$

where $\rho^M(s, s', q^2)$ is the spectral density, and M stands for off-shell mesons D' , D'' , and ϕ . We calculate the spectral densities in terms of the usual Feynman integrals with the help of the Cutkosky rules, where the quark propagators are replaced by Dirac delta functions $\frac{1}{p^2 - m^2} \rightarrow (-2\pi i)\delta(p^2 - m^2)$.

(i) For the p_ν structure related to the $D'D'\phi$ vertex:

$$\begin{aligned} \rho_{\phi D'D'}^{D'} &= N_c [-2\kappa\Delta'I_0 - 2I_0 m_s(m_s + \kappa m_c) + 2B_1(4m_s m_c + \kappa(4m_s^2 + u - 2\Delta'))], \\ \rho_{\phi D'D'}^\phi &= N_c [-2\kappa\Delta'I_0 + 4I_0 m_c(m_s + \kappa m_c) + 2B_1(4m_s m_c + 2\kappa(m_c^2 + m_s^2 - u))], \end{aligned}$$

where $\kappa = 1$ for $D' = D_{s0}^*$ and $\kappa = -1$ for $D' = D_s$

(ii) For the $g_{\mu\alpha} q_\nu$ structure related to the $D''D''\phi$ vertex:

$$\begin{aligned} \rho_{\phi D''D''}^{D''} &= -N_c [4A - 8(C_1 - C_2) - 2(B_1 - B_2)\Delta + I_0(\Delta + \Delta') - 2m_s^2(B_1 - B_2) \\ &\quad + 2\kappa I_0 m_s m_c + 2(B_1 - B_2 - I_0)m_s^2 + u(B_1 - B_2)], \\ \rho_{\phi D''D''}^\phi &= -N_c [4A - 8(C_1 - C_2) - 2(B_1 - B_2)\Delta + I_0(\Delta + \Delta') - 2m_s^2(B_1 - B_2) \\ &\quad + 4\kappa I_0 m_s m_c + 2(B_1 - B_2 - 2I_0)m_c^2 + u(B_1 - B_2)], \end{aligned}$$

where $\kappa = 1$ for $D'' = D_s^*$ and $\kappa = -1$ for $D'' = D_{s1}$. The explicit expressions of the coefficients in the spectral densities entering the sum rules are given as

$$\begin{aligned}
I_0(s, s', q^2) &= \frac{1}{4\lambda^{\frac{1}{2}}(s, s', q^2)}, & \lambda(a, b, c) &= a^2 + b^2 + c^2 - 2ac - 2bc - 2ac, & \Delta &= (s + m_3^2 - m_1^2), \\
\Delta' &= (s' + m_3^2 - m_2^2), & u &= s + s' - q^2, \\
B_1 &= \frac{I_0}{\lambda(s, s', q^2)} [2s'\Delta - \Delta'u], & B_2 &= \frac{I_0}{\lambda(s, s', q^2)} [2s\Delta' - \Delta u], \\
A &= -\frac{I_0}{2\lambda(s, s', q^2)} [4ss'm_3^2 - s\Delta'^2 - s'\Delta^2 - u^2m_3^2 + u\Delta\Delta'], \\
C_1 &= \frac{I_0}{2\lambda^2(s, s', q^2)} [8s'^2m_3^2\Delta s - 2s'm_3^2\Delta u^2 - 4um_3^2\Delta'ss' + u^3m_3^2\Delta' - 2s'^2\Delta^3 + 3s'u\Delta^2\Delta' \\
&\quad - 2\Delta'^2\Delta ss' - \Delta'^2\Delta u^2 + us\Delta^3], \\
C_2 &= \frac{I_0}{2\lambda^2(s, s', q^2)} [8s^2m_3^2\Delta's' - 2s^2\Delta'^3 - 4um_3^2\Delta ss' - 2\Delta^2\Delta'ss' + 3us\Delta'^2\Delta - 2sm_3^2\Delta u^2 \\
&\quad + s'u\Delta^3 + u^3m_3^2\Delta - \Delta^2\Delta'u^2],
\end{aligned}$$

where $N_c = 3$ is the color factor. It should be noted that in these coefficients, $m_1 = m_2 = m_s$, $m_3 = m_c$ for $\rho_{\phi D'D'}^\phi$, $\rho_{\phi D''D''}^\phi$, and $m_1 = m_3 = m_s$, $m_2 = m_c$ for $\rho_{\phi D'D'}^{D''}$, $\rho_{\phi D''D''}^{D''}$ (see Fig. 1).

III. CONDENSATE CONTRIBUTIONS

In this section, the nonperturbative part contributions of the correlation function are discussed [Eq. (12)]. In QCD, the three point correlation function can be evaluated by the OPE in the deep Euclidean region. For this aim, we expand the time ordered products of currents contained in the three point correlation function in terms of a series of local operators with increasing dimension. Taking into account the vacuum expectation value of OPE, the expansion of the correlation function in terms of local operators is written as follows:

$$\begin{aligned}
\Pi_{\text{per}}(p^2, p'^2, q^2) &= C_0, \\
\Pi_{\text{nonper}}(p^2, p'^2, q^2) &= C_3\langle\bar{q}q\rangle + C_4\langle G_{\alpha\beta}^a G^{a\alpha\beta}\rangle \\
&\quad + C_5\langle\bar{q}\sigma_{\alpha\beta} T^a G^{a\alpha\beta} q\rangle \\
&\quad + C_6\langle\bar{q}\Gamma q\bar{q}\Gamma' q\rangle + \dots,
\end{aligned} \tag{14}$$

where C_i are the Wilson coefficients, $G_{\alpha\beta}^a$ is the gluon field strength tensor, and Γ and Γ' are the matrices appearing in the calculations. In Eq. (14), C_0 is related to the contribution of the perturbative part of the correlation function, and the rest of the terms are related to the nonperturbative contributions of it. The perturbative part contribution of the correlation function was discussed before. For the calculation of the nonperturbative contributions (condensate terms), we consider these points:

- (a) The condensate terms of dimension 3, 4, and 5 are related to the contributions of the quark-quark, gluon-gluon, and quark-gluon condensate, respectively, and are more important than the other terms in OPE.

- (b) In 3PSR, when the light quark is a spectator, the gluon-gluon condensate contributions can be easily ignored (see Fig. 2) [17].
- (c) When the heavy quark is a spectator, the quark-quark condensate contributions are suppressed by the inverse of the heavy quark mass and can be safely omitted (see Fig. 3) [17].
- (d) The quark condensate contributions of the light quark, which is a nonspectator, are zero after applying the double Borel transformation with respect to both variables p^2 and p'^2 , because only one variable appears in the denominator.

Therefore, only three important diagrams of dimension 3 and 5 remain from the nonperturbative part contributions when the charmed mesons are off shell. These diagrams named quark-quark and quark-gluon condensate are

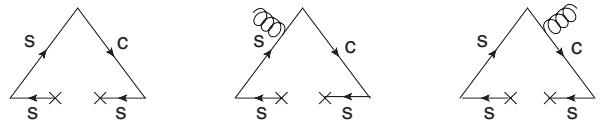


FIG. 2. Nonperturbative diagrams for the off-shell charmed mesons in the $\phi D_s^* D_{s0}^*$, $\phi D_s D_s$, $\phi D_s^* D_s^*$, and $\phi D_{s1} D_{s1}$ vertices.

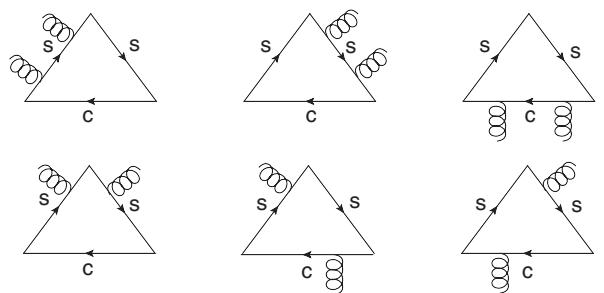


FIG. 3. Nonperturbative diagrams for the off-shell ϕ meson in considering vertices.

shown in Fig. 2. When ϕ is an off-shell meson, the important diagrams are the six diagrams of dimension 3. Figure 3 shows these diagrams related to gluon-gluon condensate.

After some straightforward calculations and applying the double Borel transformations with respect to the $p^2(p^2 \rightarrow M_1^2)$ and $p'^2(p'^2 \rightarrow M_1^2)$ as

$$\begin{aligned} B_{p^2}(M_1^2) \left(\frac{1}{p^2 - m_s^2} \right)^m &= \frac{(-1)^m}{\Gamma(m)} \frac{e^{-\frac{m_s^2}{M_1^2}}}{(M_1^2)^m}, \\ B_{p'^2}(M_2^2) \left(\frac{1}{p'^2 - m_c^2} \right)^n &= \frac{(-1)^n}{\Gamma(n)} \frac{e^{-\frac{m_c^2}{M_2^2}}}{(M_2^2)^n}, \end{aligned} \quad (15)$$

where M_1^2 and M_2^2 are the Borel parameters, respectively, the following results are obtained for the quark-quark and quark-gluon contributions (Fig. 2):

$$\Pi_{\text{nonper}}^{D'(D'')} = \frac{\langle s\bar{s} \rangle}{(M_1^2 M_2^2)^2} \frac{C^{D'(D'')}}{12}. \quad (16)$$

The explicit expressions for $C_{\phi D'D'}^{D'}$ and $C_{\phi D''D''}^{D''}$ associated with the $\phi D'D'$ and $\phi D''D''$ vertices are given in Appendix A.

To obtain the gluon condensate contributions (Fig. 3), we will follow the same procedure as stated in Ref. [18]. The calculations of the gluon condensate diagrams are performed in the Fock-Schwinger fixed-point gauge $x^\mu A_\mu^a = 0$, where A_μ^a is the gluon field. In the calculation of these diagrams, the following type of integrals appears:

$$\begin{aligned} I_{\mu_1 \mu_2 \dots \mu_n}(a, b, c) &= \int \frac{d^4 k}{(2\pi)^4} \frac{k_{\mu_1 \mu_2 \dots \mu_n}}{[k^2 - m_c^2]^a [(p+k)^2 - m_s^2]^b [(p'+k)^2 - m_s^2]^c}, \end{aligned} \quad (17)$$

where k is the momentum of the spectator quark c . These integrals can be calculated using the Schwinger representation for the Euclidean propagator, i.e.,

$$\frac{1}{[p^2 + m^2]} = \frac{1}{\Gamma(\alpha)} \int_0^\infty d\alpha \alpha^{n-1} e^{-\alpha(p^2 + m^2)}. \quad (18)$$

After the Borel transformation using

$$B_{p^2}(M^2) e^{-\alpha p^2} = \delta(1/M^2 - \alpha), \quad (19)$$

we obtain

$$\begin{aligned} \hat{I}_0(a, b, c) &= \frac{(-1)^{a+b+c}}{16\pi^2 \Gamma(a) \Gamma(b) \Gamma(c)} (M_1^2)^{2-a-b} (M_2^2)^{2-a-c} U_0(a+b+c-4, 1-c-b), \\ \hat{I}_\mu(a, b, c) &= \hat{I}_1(a, b, c) p_\mu + \hat{I}_2(a, b, c) p'_\mu, \\ \hat{I}_{\mu\nu}(a, b, c) &= \hat{I}_3(a, b, c) p_\mu p_\nu + \hat{I}_4(a, b, c) (p_\mu p'_\nu + p'_\mu p_\nu) + \hat{I}_5(a, b, c) p'_\mu p'_\nu + \hat{I}_6(a, b, c) g_{\mu\nu}, \\ \hat{I}_{\mu\nu\alpha}(p, p') &= \hat{I}_7(a, b, c) (g_{\mu\nu} p_\alpha + g_{\mu\alpha} p_\nu + g_{\nu\alpha} p_\mu) + \hat{I}_8(a, b, c) (g_{\mu\nu} p'_\alpha + g_{\mu\alpha} p'_\nu + g_{\nu\alpha} p'_\mu) + \dots, \end{aligned} \quad (20)$$

where $\hat{I}_{\mu_1 \mu_2 \dots \mu_n}$ in the above equations represents the double Borel transformed form of integrals as

$$B_{p^2}(M_1^2) B_{p'^2}(M_2^2) [p^2]^m [p'^2]^n I_{\mu_1 \mu_2 \dots \mu_n} = [M_1^2]^m [M_2^2]^n \frac{d^m}{d(M_1^2)^m} \frac{d^n}{d(M_2^2)^n} [M_1^2]^m [M_2^2]^n \hat{I}_{\mu_1 \mu_2 \dots \mu_n},$$

also \hat{I}_l ($l = 1, \dots, 8$) are defined as

$$\begin{aligned} \hat{I}_k(a, b, c) &= i \frac{(-1)^{a+b+c+1}}{16\pi^2 \Gamma(a) \Gamma(b) \Gamma(c)} (M_1^2)^{1-a-b+k} (M_2^2)^{4-a-c-k} U_0(a+b+c-5, 1-c-b), \\ \hat{I}_m(a, b, c) &= i \frac{(-1)^{a+b+c+1}}{16\pi^2 \Gamma(a) \Gamma(b) \Gamma(c)} (M_1^2)^{-a-b-1+m} (M_2^2)^{7-a-c-m} U_0(a+b+c-5, 1-c-b), \\ \hat{I}_6(a, b, c) &= i \frac{(-1)^{a+b+c+1}}{32\pi^2 \Gamma(a) \Gamma(b) \Gamma(c)} (M_1^2)^{3-a-b} (M_2^2)^{3-a-c} U_0(a+b+c-6, 2-c-b), \\ \hat{I}_n(a, b, c) &= i \frac{(-1)^{a+b+c}}{32\pi^2 \Gamma(a) \Gamma(b) \Gamma(c)} (M_1^2)^{-4-a-b+n} (M_2^2)^{11-a-c-n} U_0(a+b+c-7, 2-c-b), \end{aligned}$$

where $k = 1, 2$, $m = 3, 4, 5$ and $n = 7, 8$. We can define the function $U_0(\alpha, \beta)$ as

$$U_0(a, b) = \int_0^\infty dy (y + M_1^2 + M_2^2)^a y^b \exp\left[-\frac{B_{-1}}{y} - B_0 - B_1 y\right], \quad (21)$$

where

$$\begin{aligned} B_{-1} &= \frac{1}{M_2^2 M_1^2} (m_s^2 (M_1^2 + M_2^2)^2 - M_2^2 M_1^2 Q^2), \\ B_0 &= \frac{1}{M_1^2 M_2^2} (m_s^2 + m_c^2)(M_1^2 + M_2^2), \quad B_1 = \frac{m_c^2}{M_1^2 M_2^2}. \end{aligned} \quad (22)$$

After straightforward but lengthy calculations, we get the following results for the gluon condensate contributions:

$$\Pi_{\text{nonper}}^\phi = i \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{C_\phi^\phi}{6}, \quad (23)$$

where the explicit expressions for $C_{\phi D'D'}^\phi$ and $C_{\phi D''D''}^\phi$ are given in Appendix B.

The QCD sum rules for the strong form factors are obtained by equating two representations of the correlation function and applying the Borel transformations with respect to the $p^2(p^2 \rightarrow M_1^2)$ and $p'^2(p'^2 \rightarrow M_2^2)$ on the phenomenological as well as the perturbative and non-perturbative parts of the correlation function in order to suppress the contributions of the higher states and continuum. We obtain the equations for the strong form factors as follows:

$$\begin{aligned} g_{\phi D'D'}^{D'}(q^2) &= - \frac{m_\phi(q^2 - m_{D'}^2)}{C(D') m_{D'}^2 f_\phi f_{D'}^2 (m_{D'}^2 + m_\phi^2 - q^2)} e^{\frac{m_\phi^2}{M_1^2}} e^{\frac{m_{D'}^2}{M_2^2}} \\ &\times \left\{ -\frac{1}{4\pi^2} \int_{(m_c + m_s)^2}^{s_0^{D'}} ds' \int_{2m_s^2}^{s_0^\phi} ds \rho_{\phi D'D'}^{D'}(s, s', q^2) e^{-\frac{s}{M_1^2}} e^{-\frac{s'}{M_2^2}} + \frac{\langle s\bar{s} \rangle}{M_1^2 M_2^2} \frac{C_{\phi D'D'}^{D'}}{12} \right\}, \end{aligned} \quad (24)$$

$$\begin{aligned} g_{\phi D''D''}^{D''}(q^2) &= - \frac{2(q^2 - m_{D''}^2)}{m_\phi f_\phi f_{D''}^2 (3m_{D''}^2 + m_\phi^2 - q^2)} e^{\frac{m_\phi^2}{M_1^2}} e^{\frac{m_{D''}^2}{M_2^2}} \\ &\times \left\{ -\frac{1}{4\pi^2} \int_{(m_c + m_s)^2}^{s_0^{D''}} ds' \int_{2m_s^2}^{s_0^\phi} ds \rho_{\phi D''D''}^{D''}(s, s', q^2) e^{-\frac{s}{M_1^2}} e^{-\frac{s'}{M_2^2}} + \frac{\langle s\bar{s} \rangle}{M_1^2 M_2^2} \frac{C_{\phi D''D''}^{D''}}{12} \right\}, \end{aligned} \quad (25)$$

$$\begin{aligned} g_{\phi D'D'}^\phi(q^2) &= - \frac{(q^2 - m_\phi^2)}{C(D') m_\phi m_{D'}^2 f_\phi f_{D'}^2} e^{\frac{m_{D'}^2}{M_1^2}} e^{\frac{m_{D'}^2}{M_2^2}} \\ &\times \left\{ -\frac{1}{4\pi^2} \int_{(m_c + m_s)^2}^{s_0^{D'}} ds' \int_{(m_c + m_s)^2}^{s_0^{D'}} ds \rho_{\phi D'D'}^\phi(s, s', q^2) e^{-\frac{s}{M_1^2}} e^{-\frac{s'}{M_2^2}} - i M_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{C_{\phi D'D'}^\phi}{6} \right\}, \end{aligned} \quad (26)$$

$$\begin{aligned} g_{\phi D''D''}^\phi(q^2) &= - \frac{2(q^2 - m_\phi^2)}{m_\phi f_\phi f_{D''}^2 (4m_{D''}^2 - q^2)} e^{\frac{m_{D''}^2}{M_1^2}} e^{\frac{m_{D''}^2}{M_2^2}} \\ &\times \left\{ -\frac{1}{4\pi^2} \int_{(m_c + m_s)^2}^{s_0^{D''}} ds' \int_{(m_c + m_s)^2}^{s_0^{D''}} ds \rho_{\phi D''D''}^\phi(s, s', q^2) e^{-\frac{s}{M_1^2}} e^{-\frac{s'}{M_2^2}} - i M_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{C_{\phi D''D''}^\phi}{6} \right\}, \end{aligned} \quad (27)$$

where s_0^ϕ and $s_0^{D'(D'')}$ are the continuum thresholds in the ϕ and $D'(D'')$ mesons, respectively.

IV. NUMERICAL ANALYSIS

In this section, we analyze the strong form factors and coupling constants for the $VD_{s0}^* D_{s0}^*$, $VD_s D_s$, $VD_s^* D_s^*$, and $VD_{s1} D_{s1}$, ($V = \phi, J/\psi$) vertices. We choose the values of meson masses as $m_\phi = 1.680$ GeV, $m_{J/\psi} = 3.097$ GeV, $m_{D_{s0}^*} = 2.317$ GeV, $m_{D_s} = 1.968$ GeV, $m_{D_s^*} = 2.112$ GeV, $m_{D_{s1}} = 2.459$ GeV [19], $\langle s\bar{s} \rangle = (0.8 \pm 0.2)\langle u\bar{u} \rangle$, and

$\langle u\bar{u} \rangle = \langle d\bar{d} \rangle = -(0.240 \pm 0.010 \text{ GeV})^3$ for which we choose the value of the condensates at a fixed renormalization scale of about 1 GeV [20]. Also, the leptonic decay constants used in the calculation of the QCD sum rule for these vertices are presented in Table I. For a comprehensive analysis of the strong form factors and coupling constants, we use the following values of the quark masses m_c and m_s in three sets presented in Table II.

The expressions for the strong form factors in Eqs. (24)–(27) contain also four auxiliary parameters: Borel mass parameters M_1 and M_2 , and continuum

TABLE I. The leptonic decay constants in MeV.

$f_{J/\psi}$	f_ϕ	$f_{D_{s0}^*}$	f_{D_s}	$f_{D_s^*}$	$f_{D_{s1}}$
405 ± 10 [14]	234 ± 10 [21]	225 ± 25 [22]	274 ± 13 [23]	266 ± 32 [22]	240 ± 25 [24]

TABLE II. Different values of the quark masses in GeV in three sets.

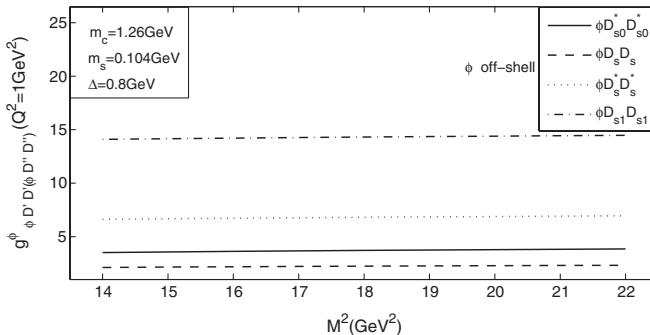
	Set I	Set II	Set III
m_c	1.30 <i>should</i> [14]	1.26 [25]	1.47 [25]
m_s	$0.142(\mu = 1 \text{ GeV})$ [26]	0.104 [19]	0.104 [19]

thresholds s_0^ϕ and $s_0^{D'(D'')}$. These are mathematical objects, so the physical quantities, i.e., strong form factors and coupling constants, should be independent of them. The values of the continuum thresholds are taken to be $s_0^\phi = (m_\phi + \Delta)^2$ and $s_0^{D'(D'')} = (m_{D'(D'')} + \Delta)^2$. We use $0.4 \text{ GeV} \leq \Delta \leq 1.0 \text{ GeV}$, where $Q^2 = -q^2$ [25]. The working regions for M_1^2 and M_2^2 are determined by requiring that the contributions of the higher states and continuum be effectively suppressed, and therefore it guarantees that the contributions of higher dimensional operators are small. In this work, we use the following relation between the Borel masses M_1 and M_2 [8,9]:

$$\frac{M_1^2}{M_2^2} = \frac{m_i^2}{m_o^2}, \quad (28)$$

where m_i and m_o are the masses of the incoming and outgoing meson, respectively. According to this relation between the M_1 and M_2 [Eq. (28)], we will have only one independent Borel mass parameter M . We found good stability of the sum rule in the interval $14 \text{ GeV}^2 \leq M^2 \leq 22 \text{ GeV}^2$ for all vertices. The dependence of the strong form factors $g_{\phi D_{s0}^* D_{s0}^*}$, $g_{\phi D_s D_s}$, $g_{\phi D_s^* D_s^*}$, and $g_{\phi D_{s1} D_{s1}}$ on Borel mass parameters M^2 in $Q^2 = 1 \text{ GeV}^2$ considering set II and $\Delta = 0.8 \text{ GeV}$ is shown in Fig. 4.

We calculated the Q^2 dependence of the strong coupling form factors in the region where the sum rule is valid. So to extend our results to the full region, we look for



parametrization of the form factors in such a way that in the validity region of 3PSR, this parametrization coincides with the sum rules prediction. For off-shell charmed mesons, our numerical calculations show that the sufficient parametrization of the form factors with respect to Q^2 is (monopole fit function)

$$g(Q^2) = \frac{A}{Q^2 + B}, \quad (29)$$

and for the off-shell ϕ meson the strong form factors can be fitted by the exponential fit function as given (Gaussian fit function)

$$g(Q^2) = Ae^{-Q^2/B}. \quad (30)$$

For different values of the three sets (Table II) and Δ ($0.4 \text{ GeV} \leq \Delta \leq 1.0 \text{ GeV}$), we analyze the parameters A and B for the $\phi D_{s0}^* D_{s0}^*$, $\phi D_s D_s$, $\phi D_s^* D_s^*$, and $\phi D_{s1} D_{s1}$ vertices. The values of the parameters A and B are given in Table III. As Table III shows, the values of the parameters A and B in set II are increased by increasing the Δ value. Therefore, the dependence of the strong form factors on Q^2 are changed. For example, Fig. 5 shows the variation of the strong form factors $g_{\phi D_{s0}^* D_{s0}^*}$ for different amounts of Δ . Our calculations show that the left and right diagrams in Fig. 5 nearly cross each other at one point for different values of Δ , which means that the strong coupling constant can be defined. In the next figure, we clearly show the intersection point of two diagrams of the strong form factors in $\Delta = 0.8$. With regard to the values of set II and set III, the dependence of the strong form factors on Q^2 for the $\phi D_{s0}^* D_{s0}^*$, $\phi D_s D_s$, $\phi D_s^* D_s^*$, and $\phi D_{s1} D_{s1}$ vertices is shown in Fig. 6. In this figure, the small circles and boxes correspond to the form factors via the 3PSR calculations. As it is seen, the form factors and their fit functions coincide well together.

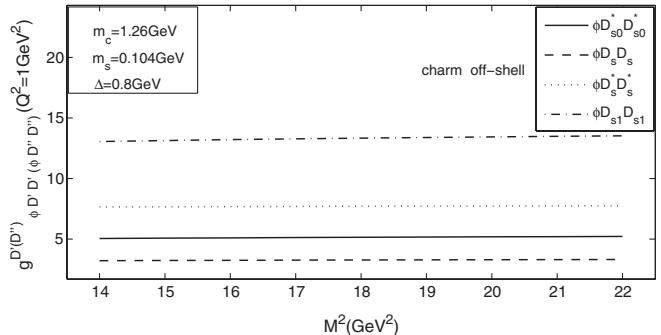
FIG. 4. The strong form factors $g_{\phi D'D'(\phi D''D'')}$ as functions of the Borel mass parameter M^2 with the values of set II for the ϕ off shell meson (left) and the charmed off shell mesons (right).

TABLE III. Parameters appearing in the fit functions for the $\phi D_{s0}^* D_{s0}^*$, $\phi D_s D_s$, $\phi D_s^* D_s^*$, and $\phi D_{s1} D_{s1}$ vertices for various m_c , m_s , and Δ , where $\Delta_1 = 0.5$ GeV, $\Delta_2 = 0.6$ GeV, $\Delta_3 = 0.8$ GeV, and $\Delta_4 = 1.0$ GeV.

	Set I		Set II				Set III			
Form factor	$A(\Delta_1)$	$B(\Delta_1)$	$A(\Delta_2)$	$B(\Delta_2)$	$A(\Delta_3)$	$B(\Delta_3)$	$A(\Delta_4)$	$B(\Delta_4)$	$A(\Delta_3)$	$B(\Delta_3)$
$g_{\phi D_{s0}^* D_{s0}^*}$	3.11	2.98	4.74	4.93	5.89	6.03	7.13	6.92	4.26	5.23
$g_{\phi D_{s0}^* D_{s0}^*}$	129.77	21.53	147.73	22.59	202.12	25.98	265.42	29.78	213.26	35.38
$g_{\phi D_s D_s}$	1.47	2.72	2.76	4.82	3.14	5.28	3.87	6.68	2.83	5.02
$g_{\phi D_s D_s}$	67.11	19.69	87.48	21.67	117.56	24.7	152.17	28.13	166.62	35.94
$g_{\phi D_s^* D_s^*}$	4.89	125.45	6.02	127.39	7.38	171.69	8.66	225.45	7.54	234.78
$g_{\phi D_s^* D_s^*}$	90.20	21.39	132.178	23.23	202.75	31.39	290.73	36.72	162.15	25.45
$g_{\phi D_{s1} D_{s1}}$	12.83	315.78	13.08	317.13	13.82	478.09	14.94	641.45	14.31	868.31
$g_{\phi D_{s1} D_{s1}}$	237.70	24.35	295.22	28.12	896.13	69.03	741.61	51.19	522.89	42.01

We define the coupling constant as the value of the strong coupling form factor at $Q^2 = -m_m^2$ in Eqs. (24)–(27), where m_m is the mass of the off-shell meson. Considering the uncertainties in the values of the other input parameters, we obtain the values of the strong coupling constants in different values of the three sets shown in Table IV.

Now, we will provide the same results for the $J/\psi D_{s0}^* D_{s0}^*$, $J/\psi D_s D_s$, $J/\psi D_s^* D_s^*$, and $J/\psi D_{s1} D_{s1}$ vertices. With little change in the expressions presented in Secs. II and III, such as the change in the quark permutations, we can easily find similar results in Eqs. (24)–(27) for strong form factors of the new vertices as

$$g_{J/\psi D'D'}^{D'}(q^2) = \frac{m_{J/\psi}(q^2 - m_{D'}^2)}{C(D')m_{D'}^2 f_{J/\psi} f_{D'}^2(m_{D'}^2 + m_{J/\psi}^2 - q^2)} e^{\frac{m_{J/\psi}^2}{M_1^2}} e^{\frac{m_{D'}^2}{M_2^2}} \left\{ -\frac{1}{4\pi^2} \int_{(m_c + m_s)^2}^{s_0^{D'}} ds' \right. \\ \times \int_{2m_c^2}^{s_0^{J/\psi}} ds \rho_{J/\psi D'D'}^{D'}(s, s', q^2) e^{-\frac{s}{M_1^2} - \frac{s'}{M_2^2}} - iM_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{C_{J/\psi D'D'}^{J/\psi}}{6} \left. \right\}, \quad (31)$$

where $\rho_{J/\psi D'D'}^{D'} = \rho_{\phi D'D'}^{D'}|_{s \leftrightarrow c}$ and $C_{J/\psi D'D'}^{D'} = C_{\phi D'D'}^{\phi}|_{s \leftrightarrow c}$. $s_0^{J/\psi}$ is the continuum threshold in the J/ψ meson, and its value is $(m_{J/\psi} + \Delta)^2$ where 0.4 GeV $\leq \Delta \leq 1.0$ GeV,

$$g_{J/\psi D''D''}^{D''}(q^2) = \frac{2(q^2 - m_{D''}^2)}{m_{J/\psi} f_{J/\psi} f_{D''}^2(3m_{D''}^2 + m_{J/\psi}^2 - q^2)} e^{\frac{m_{J/\psi}^2}{M_1^2}} e^{\frac{m_{D''}^2}{M_2^2}} \left\{ -\frac{1}{4\pi^2} \int_{(m_c + m_s)^2}^{s_0^{D''}} ds' \right. \\ \times \int_{2m_c^2}^{s_0^{J/\psi}} ds \rho_{J/\psi D''D''}^{D''}(s, s', q^2) e^{-\frac{s}{M_1^2} - \frac{s'}{M_2^2}} - iM_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{C_{J/\psi D''D''}^{J/\psi}}{6} \left. \right\}, \quad (32)$$

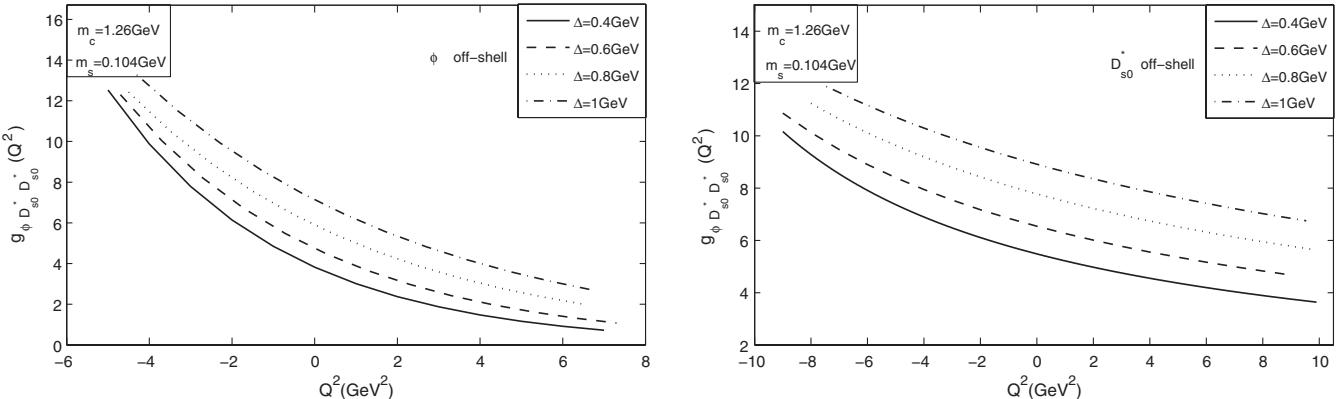


FIG. 5. The strong form factor $g_{\phi D_{s0}^* D_{s0}^*}$ on Q^2 for different values of Δ for the ϕ off shell (left) and the D_{s0}^* off shell (right).

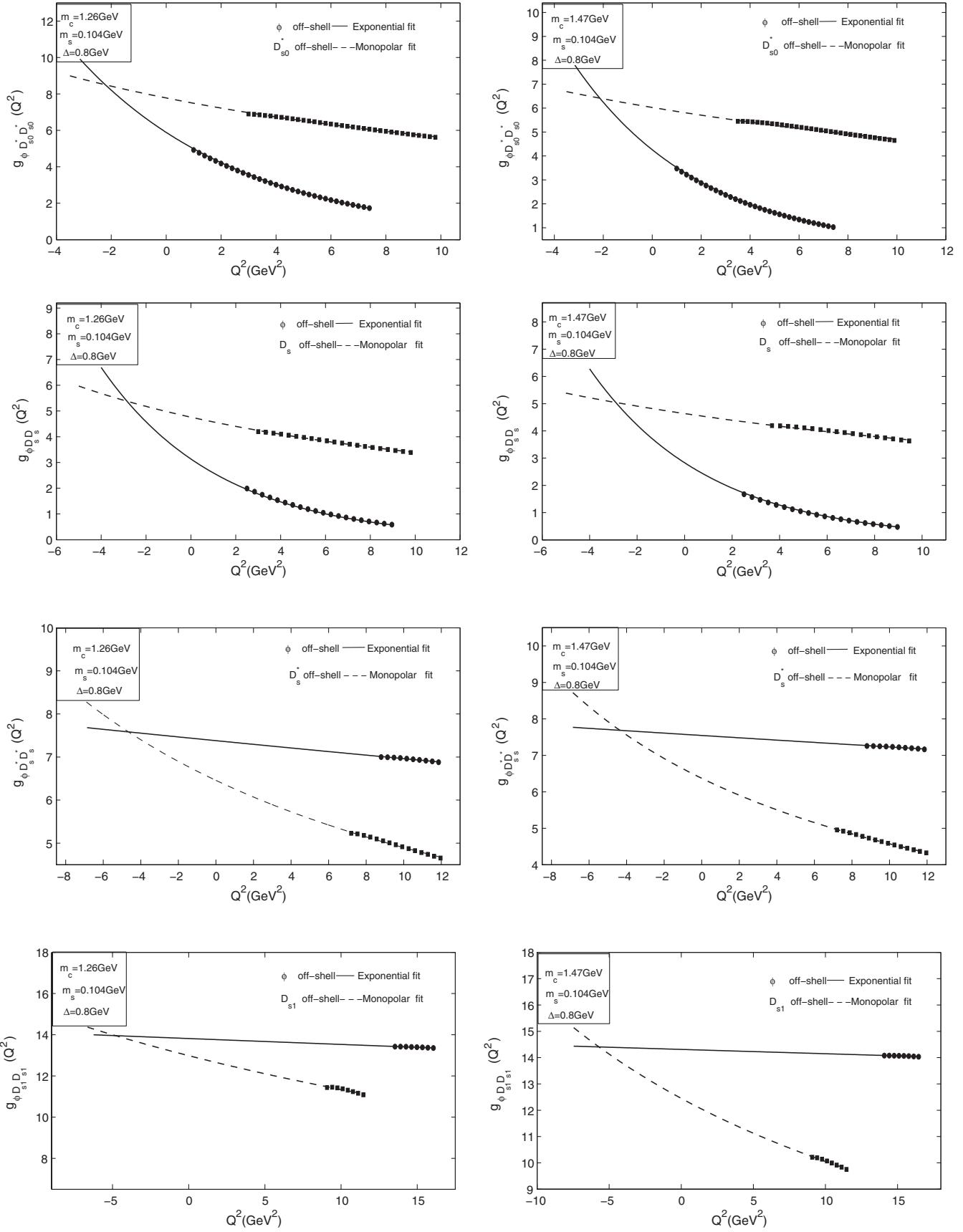


FIG. 6. The strong form factors $g_{\phi D'D'}$ and $g_{\phi D''D''}$ on Q^2 for different values of the m_c and $\Delta = 0.8$ for the ϕ off shell and the charmed off shell mesons. The small circles and boxes correspond to the form factors via the 3PSR calculations.

TABLE IV. The strong coupling constants $g_{\phi D'D'}$ and $g_{\phi D''D''}$ in GeV^{-1} for various m_c and m_s .

Coupling constant	Set I		Set II		Set III	
	ch-off-sh	ϕ -off-sh	ch-off-sh	ϕ -off-sh	ch-off-sh	ϕ -off-sh
$g_{\phi D_{s0}^* D_{s0}^*}$	8.03 ± 1.30	8.10 ± 1.20	9.20 ± 1.67	9.00 ± 1.72	7.11 ± 1.29	7.31 ± 1.40
$g_{\phi D_s D_s}$	4.25 ± 0.92	4.15 ± 0.88	5.17 ± 1.34	5.06 ± 1.83	5.20 ± 1.35	4.97 ± 1.80
$g_{\phi D_s^* D_s^*}$	5.33 ± 0.21	4.89 ± 0.74	7.28 ± 1.74	6.78 ± 1.63	7.72 ± 1.86	7.64 ± 1.84
$g_{\phi D_{s1} D_{s1}}$	12.99 ± 0.64	12.95 ± 0.81	14.18 ± 2.25	13.41 ± 1.88	14.54 ± 2.31	14.35 ± 2.01

where $\rho_{J/\psi D''D''}^{D''} = \rho_{\phi D''D''}^{D''}|_{s \leftrightarrow c}$ and $C_{J/\psi D''D''}^{D''} = C_{\phi D''D''}^{\phi}|_{s \leftrightarrow c}$,

$$g_{J/\psi D'D'}^{J/\psi}(q^2) = \frac{(q^2 - m_{J/\psi}^2)}{C(D')m_{J/\psi}m_{D'}^2 f_{J/\psi} f_{D'}} e^{\frac{m_{D'}^2}{M_1^2}} e^{\frac{m_{D'}^2}{M_2^2}} \left\{ -\frac{1}{4\pi^2} \int_{(m_c+m_s)^2}^{s_0^{D'}} ds' \right. \\ \times \int_{(m_c+m_s)^2}^{s_0^{D'}} ds \rho_{J/\psi D'D'}^{J/\psi}(s, s', q^2) e^{-\frac{s}{M_1^2} - \frac{s'}{M_2^2}} + \frac{\langle s\bar{s} \rangle}{M_1^2 M_2^2} \frac{C_{J/\psi D'D'}^{J/\psi}}{12} \left. \right\}, \quad (33)$$

where $\rho_{J/\psi D'D'}^{J/\psi} = \rho_{\phi D'D'}^{\phi}|_{s \leftrightarrow c}$ and $C_{J/\psi D'D'}^{J/\psi} = C_{\phi D'D'}^{\phi}|_{s \leftrightarrow c}$,

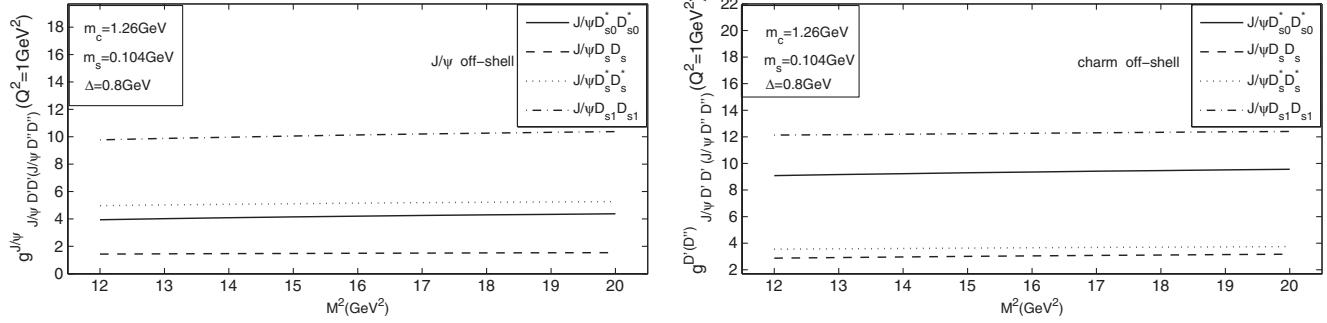


FIG. 7. The strong form factors $g_{J/\psi D'D'}(J/\psi D''D'')$ as functions of the Borel mass parameter M^2 with the values of set II for the J/ψ off shell (left) and the charmed off shell mesons (right).

TABLE V. Parameters appearing in the fit functions for the $g_{J/\psi D'D'(J/\psi D''D'')}$ form factors for various m_c , m_s , and Δ , where $\Delta_1 = 0.5$ GeV, $\Delta_2 = 0.6$ GeV, $\Delta_3 = 0.8$ GeV, and $\Delta_4 = 1.0$ GeV.

Form factor	Set I		Set II				Set III			
	$A(\Delta_1)$	$B(\Delta_1)$	$A(\Delta_2)$	$B(\Delta_2)$	$A(\Delta_3)$	$B(\Delta_3)$	$A(\Delta_4)$	$B(\Delta_4)$	$A(\Delta_3)$	$B(\Delta_3)$
$g_{J/\psi D_{s0}^* D_{s0}^*}^{J/\psi}$	3.73	14.31	5.62	31.07	7.20	48.83	8.02	46.03	4.64	46.73
$g_{J/\psi D_{s0}^* D_{s0}^*}^{D_{s0}^*}$	174.95	29.88	179.92	30.10	221.17	30.94	254.72	32.16	129.89	29.18
$g_{J/\psi D_s D_s}^{J/\psi}$	1.73	253.20	1.96	187.02	2.34	242.44	2.96	251.02	1.86	194.32
$g_{J/\psi D_s D_s}^{D_s}$	160.21	89.05	176.22	92.22	211.61	89.91	284.62	89.32	186.54	101.63
$g_{J/\psi D_s^* D_s^*}^{J/\psi}$	2.14	23.59	2.79	25.54	3.67	27.76	4.48	31.72	2.87	26.91
$g_{J/\psi D_s^* D_s^*}^{D_s^*}$	499.86	166.02	847.23	199.97	1452.91	282.26	2341.12	403.88	868.24	254.32
$g_{J/\psi D_{s1} D_{s1}}^{J/\psi}$	5.98	29.48	7.46	32.88	8.66	46.79	9.58	60.37	6.93	38.69
$g_{J/\psi D_{s1} D_{s1}}^{D_{s1}}$	538.94	70.62	724.31	83.10	1033.08	107.17	1489.78	138.33	658.84	86.47

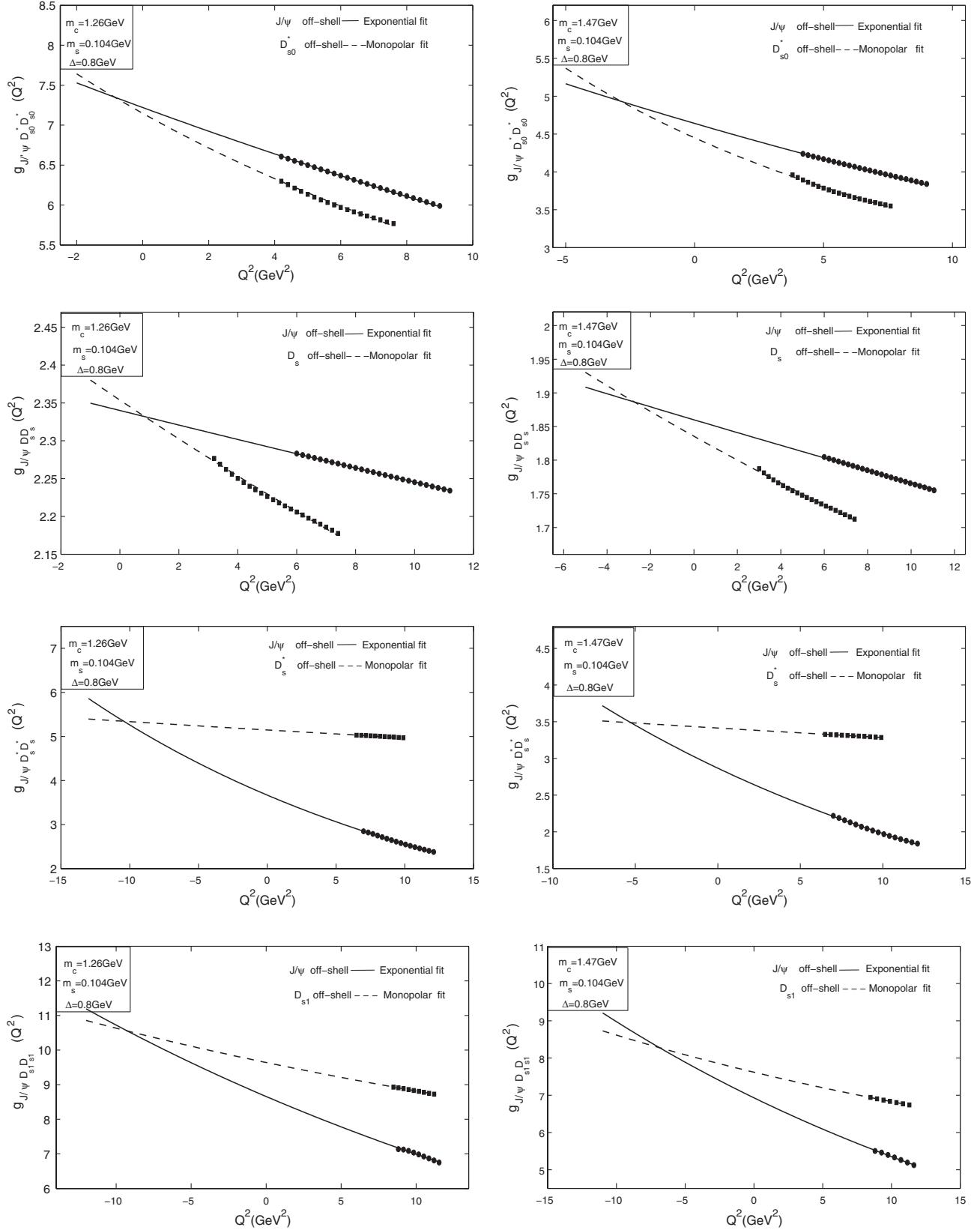


FIG. 8. The strong form factors $g_{J/\psi D'D''}$ and $g_{J/\psi D''D'''}$ on Q^2 for different values of the m_c and $\Delta = 0.8$ for the J/ψ off shell and the charmed off shell mesons. The small circles and boxes correspond to the form factors via the 3PSR calculations.

TABLE VI. The strong coupling constants $g_{J/\psi D'D'}$ and $g_{J/\psi D''D''}$ in GeV^{-1} for various m_c and m_s .

Coupling constant	Set I		Set II		Set III	
	ch-off-sh	J/ψ -off-sh	ch-off-sh	J/ψ -off-sh	ch-off-sh	J/ψ -off-sh
$g_{J/\psi D_{s0}^* D_{s0}^*}$	7.14 ± 1.50	7.32 ± 1.20	7.82 ± 1.97	8.13 ± 1.93	5.45 ± 1.37	5.70 ± 1.35
$g_{J/\psi D_s D_s}$	1.88 ± 0.73	1.80 ± 0.65	2.33 ± 1.01	2.26 ± 0.81	1.91 ± 0.83	1.95 ± 0.70
$g_{J/\psi D_s^* D_s^*}$	3.09 ± 0.52	3.21 ± 0.61	4.69 ± 1.34	4.59 ± 1.58	3.47 ± 0.99	4.09 ± 1.41
$g_{J/\psi D_{s1} D_{s1}}$	8.35 ± 0.91	8.29 ± 0.97	9.91 ± 1.35	10.19 ± 1.30	8.19 ± 1.12	8.89 ± 1.14

$$g_{J/\psi D''D''}^{J/\psi}(q^2) = \frac{2(q^2 - m_{J/\psi}^2)}{m_{J/\psi} f_{J/\psi} f_{D''}^2 (4m_{D''}^2 - q^2)} e^{\frac{m_{D''}^2}{M_1^2}} e^{\frac{m_{D''}^2}{M_2^2}} \left\{ -\frac{1}{4\pi^2} \int_{(m_c + m_s)^2}^{s_0^{D''}} ds' \right. \\ \times \int_{(m_c + m_s)^2}^{s_0^{D''}} ds \rho_{J/\psi D''D''}^{J/\psi}(s, s', q^2) e^{-\frac{s}{M_1^2} - \frac{s'}{M_2^2}} + \frac{\langle s\bar{s} \rangle}{M_1^2 M_2^2} \frac{C_{J/\psi D''D''}^{J/\psi}}{12} \left. \right\}, \quad (34)$$

where $\rho_{J/\psi D''D''}^{J/\psi} = \rho_{\phi D''D''}^\phi|_{s \leftrightarrow c}$ and $C_{J/\psi D''D''}^{J/\psi} = C_{\phi D''D''}^\phi|_{s \leftrightarrow c}$.

The dependence of the strong form factors $g_{J/\psi D_{s0}^* D_{s0}^*}$, $g_{J/\psi D_s D_s}$, $g_{J/\psi D_s^* D_s^*}$, and $g_{J/\psi D_{s1} D_{s1}}$ on the Borel mass parameters M^2 for the values of set II, $\Delta = 0.8$ GeV and $Q^2 = 1$ GeV 2 are shown in Fig. 7.

For all these vertices, good stability occurs in the interval $12 \text{ GeV}^2 \leq M^2 \leq 20 \text{ GeV}^2$.

TABLE VII. Parameters appearing in the fit functions for the $\phi D'D'(\phi D''D'')$ and $J/\psi D'D'(J/\psi D''D'')$ form factors in $SU_f(3)$ symmetry with $m_c = 1.26$ GeV and $\Delta = 0.8$ GeV.

Form factor	A	B	Form factor	A	B
$g_{\phi D_{s0}^* D_{s0}^*}^\phi$	6.15	5.67	$g_{J/\psi D_{s0}^* D_{s0}^*}^{J/\psi}$	7.39	54.43
$g_{\phi D_{s0}^* D_{s0}^*}^{D_{s0}^*}$	214.64	26.40	$g_{J/\psi D_{s0}^* D_{s0}^*}^{D_{s0}^*}$	237.60	31.73
$g_{\phi D_s D_s}^\phi$	2.13	4.63	$g_{J/\psi D_s D_s}^{J/\psi}$	2.01	213.63
$g_{\phi D_s D_s}^{D_s}$	118.08	32.74	$g_{J/\psi D_s D_s}^{D_s}$	204.69	103.05
$g_{\phi D_s^* D_s^*}^\phi$	7.19	254.34	$g_{J/\psi D_s^* D_s^*}^{J/\psi}$	3.47	29.19
$g_{\phi D_s^* D_s^*}^{D_s^*}$	191.06	29.08	$g_{J/\psi D_s^* D_s^*}^{D_s^*}$	2570.27	522.58
$g_{\phi D_{s1} D_{s1}}^\phi$	15.14	370.05	$g_{J/\psi D_{s1} D_{s1}}^{J/\psi}$	9.13	43.75
$g_{\phi D_{s1} D_{s1}}^{D_{s1}}$	650.85	48.39	$g_{J/\psi D_{s1} D_{s1}}^{D_{s1}}$	1149.21	113.47

TABLE VIII. The strong coupling constants $g_{\phi D'D'}$, $g_{\phi D''D''}$, $g_{J/\psi D'D'}$, and $g_{J/\psi D''D''}$ in GeV^{-1} in $SU_f(3)$ symmetry with $m_c = 1.26$ GeV.

Coupling constant	ch-off-sh	ϕ -off-sh	Coupling constant	ch-off-sh	J/ψ -off-sh
$g_{\phi D_{s0}^* D_{s0}^*}$	10.21 ± 1.81	10.12 ± 1.79	$g_{J/\psi D_{s0}^* D_{s0}^*}$	9.02 ± 2.15	8.81 ± 2.01
$g_{\phi D_s D_s}$	4.09 ± 1.15	3.91 ± 1.03	$g_{J/\psi D_s D_s}$	2.07 ± 0.91	2.10 ± 0.92
$g_{\phi D_s^* D_s^*}$	7.76 ± 1.79	7.27 ± 1.61	$g_{J/\psi D_s^* D_s^*}$	4.96 ± 1.42	4.82 ± 1.38
$g_{\phi D_{s1} D_{s1}}$	15.37 ± 2.51	15.26 ± 2.47	$g_{J/\psi D_{s1} D_{s1}}$	10.70 ± 2.42	11.37 ± 2.55

TABLE IX. Values of the strong coupling constant using different approaches: 3PSR [14], QM [27], 3PSR [28], VMD [29], and CQM [30].

Coupling constant	Ours	Reference [14]	Reference [27]	Reference [28]	Reference [29]	Reference [30]
$g_{J/\psi D^* D^*}$	4.89 ± 1.40	6.2 ± 0.9	4.9	5.8 ± 0.8	7.6	8.0 ± 0.5

value, while the other uncertainties are small, constituting a few percent.

So far, the strong coupling constant values were investigated via the $SU_f(3)$ symmetry breaking, and the mass of the s quark was considered in the expressions of the condensate terms and spectral densities. Now, we want to analyze the strong coupling constants considering the $SU_f(3)$ symmetry. For the stated purpose, the mass of the s quark is ignored in all equations, i.e., $m_s \rightarrow m_u \simeq 0$ GeV and $\langle s\bar{s} \rangle \rightarrow \langle u\bar{u} \rangle$.

In view of the $SU_f(3)$ symmetry, the values of the parameters A and B for the $\phi D'D'(\phi D''D'')$ and $J/\psi D'D'(J/\psi D''D'')$ vertices in $m_c = 1.26$ GeV and $\Delta = 0.8$ GeV are given in Table VII. Also considering the $SU_f(3)$ symmetry, we obtain the values of the strong coupling constants in $m_c = 1.26$ GeV shown in Table VIII.

Finally, we would like to compare our results with the values predicted by other methods. It should be reminded that with the $SU_f(3)$ symmetry consideration, it is possible to compare the coupling constant value of $g_{J/\psi D_s^* D_s^*}$ with $g_{J/\psi D^* D^*}$. Taking an average of the two values of the strong form factor $g_{J/\psi D_s^* D_s^*}$ for the off-shell D_s^* and the off-shell J/ψ , we obtain

$$g_{J/\psi D_s^* D_s^*} = 4.89 \pm 1.40 \text{ GeV}^{-1}. \quad (35)$$

We compare our result for $g_{J/\psi D_s^* D_s^*}$ in Eq. (35) with those of other calculations for $g_{J/\psi D^* D^*}$ in Table IX. Our value is smaller than the values obtained using all the approaches such as 3PSR [14,28], the quark model (QM) [27], the vector meson dominance approach (VMD) [29], and the constituent quark-meson model (CQM) [30], but it is in good agreement with the QM calculation.

In summary, considering the contributions of the quark-quark, quark-gluon, and gluon-gluon condensate corrections, we estimate the strong form factors for the $\phi D'D'$, $\phi D''D''$, $J/\psi D'D'$, and $J/\psi D''D''$ ($D' = D_{s0}^*$, D_s and $D'' = D_s^*$, D_{s1}) vertices within 3PSR. The dependence of the strong form factors on the transferred momentum square Q^2 was plotted. We also evaluated the coupling constants of these vertices. Detection of these strong form factors and the coupling constants and their comparison with the phenomenological models like QCD sum rules could give useful information about strong interactions of the strange charmed mesons.

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APPENDIX A

In this Appendix, the explicit expressions of the coefficients of the quark-quark and quark-gluon condensate of the strong form factors for the vertices $\phi D'D'$ and $\phi D''D''$ with application of the double Borel transformations are given.

$$\begin{aligned} C_{\phi D_{s0}^* D_{s0}^*}^{D_{s0}^*} = & \left(-3m_s m_c^2 M_1^2 - 3m_s^2 m_c M_1^2 + 2m_0^2 m_s M_1^2 + 3m_s q^2 M_2^2 - 3m_s^2 m_c M_2^2 - 3m_s m_0^2 M_2^2 + m_0^2 m_c M_2^2 - 3m_s m_c^2 M_2^2 \right. \\ & - 3 \frac{m_s^3 m_c^2 M_1^2}{M_2^2} - \frac{3}{2} \frac{m_0^2 m_c^3 M_1^2}{M_2^2} + \frac{3}{2} \frac{m_0^2 m_s m_c^2 M_1^2}{M_2^2} + 3 \frac{m_c^3 m_s^2 M_1^2}{M_2^2} - 3 \frac{m_s^5 M_2^2}{M_1^2} - \frac{7}{4} \frac{m_0^2 m_s^2 m_c M_2^2}{M_1^2} + 3 \frac{m_s^4 m_c M_2^2}{M_1^2} \\ & + \frac{7}{4} \frac{m_0^2 m_s^3 M_2^2}{M_1^2} - 3m_s^5 - 3m_0^2 m_s^2 m_c - 3m_s^3 m_c^2 + 3m_s^3 q^2 + 3m_c^3 m_s^2 - m_0^2 m_s q^2 + 2m_0^2 m_c q^2 - 3m_c q^2 m_s^2 \\ & \left. + 3m_s^4 m_c + m_0^2 m_s^3 - 2m_0^2 m_c^3 \right) \times e^{-\frac{m_s^2}{M_1^2}} e^{-\frac{m_c^2}{M_2^2}}, \end{aligned}$$

$$\begin{aligned} C_{\phi D_s D_s}^{D_s} = & \left(-3m_0^2 m_c M_1^2 + 3m_s^3 M_1^2 - \frac{7}{2} m_0^2 m_s M_1^2 + 3m_s^2 m_c M_1^2 + 3m_s m_c^2 M_1^2 - 3m_s q^2 M_2^2 + 3m_s m_c^2 M_2^2 + 6m_s^3 M_2^2 - \frac{5}{2} m_0^2 m_c M_2^2 \right. \\ & - \frac{3}{2} \frac{m_0^2 m_s m_c^2 M_1^2}{M_2^2} + 3 \frac{m_s^3 m_c^2 M_1^2}{M_2^2} + 3 \frac{m_c^3 m_s^2 M_1^2}{M_2^2} - \frac{3}{2} \frac{m_0^2 m_c^3 M_1^2}{M_2^2} + 3 \frac{m_s^5 M_2^2}{M_1^2} + 3 \frac{m_s^4 m_c M_2^2}{M_1^2} - \frac{3}{2} \frac{m_0^2 m_s^2 m_c M_2^2}{M_1^2} - \frac{3}{2} \frac{m_0^2 m_s^3 M_2^2}{M_1^2} \\ & \left. + \frac{1}{2} m_0^2 m_c q^2 - \frac{3}{2} m_0^2 m_s^2 m_c + \frac{3}{2} m_0^2 m_s m_c^2 - \frac{1}{2} m_0^2 m_s q^2 + \frac{1}{2} m_0^2 m_s^3 - \frac{1}{2} m_0^2 m_c^3 \right) \times e^{-\frac{m_s^2}{M_1^2}} e^{-\frac{m_c^2}{M_2^2}}, \end{aligned}$$

$$\begin{aligned}
C_{\phi D_s^* D_s^*}^{D_s^*} = & \left(3m_s M_1^2 M_2^2 - 3m_s^3 M_1^2 + 9m_s^2 m_c M_1^2 - 6m_s m_c^2 M_1^2 - 2m_0^2 m_c M_1^2 + m_0^2 m_s M_1^2 + 3m_s q^2 M_1^2 + 3m_s^3 M_2^2 \right. \\
& - 3m_s m_0^2 M_2^2 - 3m_s^2 m_c M_2^2 - 3 \frac{m_s^3 m_c^2 M_1^2}{M_2^2} + \frac{3}{2} \frac{m_0^2 m_c^3 M_1^2}{M_2^2} + \frac{3}{2} \frac{m_0^2 m_s m_c^2 M_1^2}{M_2^2} - 3 \frac{m_c^3 m_s^2 M_1^2}{M_2^2} - 3 \frac{m_s^4 m_c M_2^2}{M_1^2} \\
& - 3 \frac{m_s^5 M_2^2}{M_1^2} + \frac{3}{2} \frac{m_s^2 m_c m_0^2 M_2^2}{M_1^2} + \frac{3}{2} \frac{m_0^2 m_s^3 M_2^2}{M_1^2} - 3m_s^5 + 2m_0^2 m_s^3 + m_0^2 m_c^3 + 3m_s^3 q^2 - 3m_c^3 m_s^2 - 3m_s^4 m_c - 3m_s^3 m_c^2 \\
& \left. - 2m_0^2 m_s q^2 - m_0^2 m_c q^2 + 3m_0^2 m_s m_c^2 + 3m_c q^2 m_s^2 \right) \times e^{-\frac{m_s^2}{M_1^2}} e^{-\frac{m_c^2}{M_2^2}}, \\
C_{\phi D_{s1} D_{s1}}^{D_{s1}} = & \left(3m_s M_1^2 M_2^2 - 3m_s^3 M_1^2 - 9m_s^2 m_c M_1^2 - 6m_s m_c^2 M_1^2 + 2m_0^2 m_c M_1^2 + m_0^2 m_s M_1^2 + 3m_s q^2 M_1^2 + 3m_s^3 M_2^2 \right. \\
& - 3m_s m_0^2 M_2^2 + 3m_s^2 m_c M_2^2 - 3 \frac{m_s^3 m_c^2 M_1^2}{M_2^2} - \frac{3}{2} \frac{m_0^2 m_c^3 M_1^2}{M_2^2} + \frac{3}{2} \frac{m_0^2 m_s m_c^2 M_1^2}{M_2^2} + 3 \frac{m_c^3 m_s^2 M_1^2}{M_2^2} + 3 \frac{m_s^4 m_c M_2^2}{M_1^2} \\
& - 3 \frac{m_s^5 M_2^2}{M_1^2} - \frac{3}{2} \frac{m_s^2 m_c m_0^2 M_2^2}{M_1^2} + \frac{3}{2} \frac{m_0^2 m_s^3 M_2^2}{M_1^2} - 3m_s^5 + 2m_0^2 m_s^3 - m_0^2 m_c^3 + 3m_s^3 q^2 + 3m_c^3 m_s^2 + 3m_s^4 m_c - 3m_s^3 m_c^2 \\
& \left. - 2m_0^2 m_s q^2 + m_0^2 m_c q^2 + 3m_0^2 m_s m_c^2 - 3m_c q^2 m_s^2 \right) \times e^{-\frac{m_s^2}{M_1^2}} e^{-\frac{m_c^2}{M_2^2}}.
\end{aligned}$$

APPENDIX B

In this Appendix, the coefficients of the gluon condensate contributions of the strong form factors for the $\phi D'D'$ and $\phi D''D''$ vertices in the Borel transform scheme are presented.

$$\begin{aligned}
C_{\phi D_{s0}^* D_{s0}^*}^\phi = & \hat{I}_2(3, 2, 2)m_c^6 - \hat{I}_1(3, 2, 2)m_c^6 + \hat{I}_1(3, 2, 2)m_c^4 m_s^2 - \hat{I}_2(3, 2, 2)m_c^4 m_s^2 - \hat{I}_2(3, 2, 2)m_c^3 m_s^3 \\
& + \hat{I}_1(3, 2, 2)m_c^3 m_s^3 + 3\hat{I}_0(3, 2, 1)m_c^4 - \hat{I}_1(3, 2, 1)m_c^4 + 3\hat{I}_0(2, 2, 2)m_c^4 + \hat{I}_0(3, 1, 2)m_c^4 \\
& + \hat{I}_2(3, 2, 1)m_c^4 + 2\hat{I}_1(2, 3, 1)m_c^3 m_s - 3\hat{I}_1(4, 1, 1)m_c^3 m_s - 4\hat{I}_0(2, 3, 1)m_c^3 m_s + 3\hat{I}_2(4, 1, 1)m_c^3 m_s \\
& - 2\hat{I}_1(2, 2, 2)m_c^3 m_s + 2\hat{I}_2(2, 2, 2)m_c^3 m_s + \hat{I}_2^{[1,0]}(3, 2, 2)m_c^3 m_s + \hat{I}_1^{[1,0]}(3, 2, 2)m_c^3 m_s \\
& - 2\hat{I}_2(2, 3, 1)m_c^3 m_s + \hat{I}_2(3, 2, 1)m_c^3 m_s - \hat{I}_1(3, 2, 1)m_c^3 m_s - \hat{I}_1^{[0,1]}(3, 2, 2)m_c^2 m_s^2 \\
& - \hat{I}_2^{[0,1]}(3, 2, 2)m_c^2 m_s^2 + 3\hat{I}_0(3, 2, 1)m_c^2 m_s^2 - 2\hat{I}_1(3, 2, 1)m_c m_s^3 + 2\hat{I}_2(3, 2, 1)m_c m_s^3 \\
& - 2\hat{I}_1(1, 2, 2)m_c^2 - 3/2\hat{I}_1(1, 3, 1)m_c^2 - 3\hat{I}_0^{[1,0]}(3, 2, 1)m_c^2 - 3\hat{I}_1^{[0,1]}(3, 1, 2)m_c^2 - 3\hat{I}_1^{[1,0]}(3, 2, 1)m_c^2 \\
& + \hat{I}_0^{[0,1]}(3, 2, 1)m_c^2 + 6\hat{I}_0(1, 3, 1)m_c^2 + 2\hat{I}_2(1, 2, 2)m_c^2 - 3\hat{I}_2^{[0,1]}(3, 1, 2)m_c^2 - \hat{I}_0^{[0,2]}(3, 2, 2)m_c^2 \\
& - 3\hat{I}_2^{[1,0]}(3, 2, 1)m_c^2 + 2\hat{I}_0(1, 2, 2)m_c^2 - 3\hat{I}_0^{[0,1]}(4, 1, 1)m_c^2 + 5\hat{I}_2^{[1,1]}(3, 2, 2)m_c^2 + 5\hat{I}_1^{[1,1]}(3, 2, 2)m_c^2 \\
& + 3/2\hat{I}_2(1, 3, 1)m_c^2 - 3\hat{I}_2^{[0,1]}(3, 1, 2)m_c m_s + \hat{I}_2^{[1,1]}(3, 2, 2)m_c m_s - 2\hat{I}_1^{[1,0]}(3, 2, 1)m_c m_s \\
& + 4\hat{I}_1(2, 2, 1)m_c m_s + \hat{I}_1^{[1,1]}(3, 2, 2)m_c m_s - 2\hat{I}_2^{[1,0]}(3, 2, 1)m_c m_s - 3\hat{I}_1^{[0,1]}(3, 1, 2)m_c m_s \\
& + 2\hat{I}_1(1, 2, 2)m_c m_s - 4\hat{I}_2(2, 2, 1)m_c m_s - 2\hat{I}_2(1, 2, 2)m_c m_s - 4\hat{I}_0(1, 3, 1)m_c m_s - 2\hat{I}_0^{[0,1]}(2, 2, 2)m_s^2 \\
& + \hat{I}_0(1, 2, 2)m_s^2 + \hat{I}_0^{[0,2]}(3, 2, 2)m_s^2 - 2\hat{I}_0^{[0,1]}(2, 2, 1) + 2\hat{I}_1(1, 2, 1) + 2\hat{I}_0^{[1,1]}(2, 2, 2) - 2\hat{I}_2(1, 2, 1) \\
& + 2\hat{I}_1(2, 1, 1) - 2\hat{I}_2(2, 1, 1) + \hat{I}_1^{[1,1]}(2, 2, 2) - \hat{I}_0^{[1,0]}(2, 2, 1) \\
& + 3\hat{I}_0(2, 1, 1) - \hat{I}_0^{[0,1]}(1, 2, 2) + \hat{I}_2^{[1,1]}(2, 2, 2) - \hat{I}_0^{[1,2]}(3, 2, 2) - 6\hat{I}_0^{[0,1]}(1, 3, 1) - 3\hat{I}_0^{[0,1]}(2, 1, 2),
\end{aligned}$$

$$\begin{aligned}
C_{\phi D_s D_s}^\phi = & -\hat{I}_1(3, 2, 2)m_c^5 m_s + \hat{I}_2(3, 2, 2)m_c^5 m_s + \hat{I}_1(3, 2, 2)m_c^3 m_s^3 - \hat{I}_2(3, 2, 2)m_c^3 m_s^3 - \hat{I}_1^{[0,1]}(3, 2, 2)m_c^4 - 3\hat{I}_0(3, 2, 1)m_c^4 \\
& + 3\hat{I}_1(4, 1, 1)m_c^4 + 3\hat{I}_1(2, 2, 2)m_c^4 - 3\hat{I}_2(4, 1, 1)m_c^4 - \hat{I}_2(3, 2, 1)m_c^4 - 3\hat{I}_2(2, 2, 2)m_c^4 - \hat{I}_2^{[0,1]}(3, 2, 2)m_c^4 \\
& - 3\hat{I}_0(2, 2, 2)m_c^4 + \hat{I}_1(3, 2, 1)m_c^4 - \hat{I}_0^{[1,0]}(3, 2, 2)m_c^4 + 2\hat{I}_2(2, 2, 2)m_c^3 m_s + 3\hat{I}_0(3, 2, 1)m_c^3 m_s + 3\hat{I}_2(4, 1, 1)m_c^3 m_s \\
& + \hat{I}_2(3, 2, 1)m_c^3 m_s - 3\hat{I}_1(4, 1, 1)m_c^3 m_s - 2\hat{I}_1(2, 2, 2)m_c^3 m_s - \hat{I}_1(3, 2, 1)m_c^3 m_s + 6\hat{I}_2(1, 4, 1)m_c^2 m_s^2 \\
& + 2\hat{I}_1(3, 2, 1)m_c^2 m_s^2 - 2\hat{I}_2(3, 2, 1)m_c^2 m_s^2 - 6\hat{I}_1(1, 4, 1)m_c^2 m_s^2 - 3\hat{I}_0(3, 2, 1)m_c^2 m_s^2 - 2\hat{I}_1(2, 3, 1)m_c m_s^3 \\
& - \hat{I}_1^{[0,1]}(3, 2, 2)m_c m_s^3 - \hat{I}_2^{[0,1]}(3, 2, 2)m_c m_s^3 + 2\hat{I}_2(2, 3, 1)m_c m_s^3 + \hat{I}_0(2, 1, 2)m_c^2 + \hat{I}_0^{[0,2]}(3, 2, 2)m_c^2 \\
& + 2\hat{I}_0^{[0,1]}(2, 2, 2)m_c^2 + 3\hat{I}_1^{[0,1]}(3, 1, 2)m_c^2 - 5\hat{I}_2^{[1,1]}(3, 2, 2)m_c^2 + 3\hat{I}_0^{[1,0]}(3, 2, 1)m_c^2 - 5\hat{I}_1^{[1,1]}(3, 2, 2)m_c^2 \\
& + 3\hat{I}_2^{[0,1]}(3, 1, 2)m_c^2 - \hat{I}_0(3, 1, 1)m_c^2 + \hat{I}_2^{[1,1]}(3, 2, 2)m_c m_s + 3\hat{I}_1(1, 3, 1)m_c m_s - 3\hat{I}_2(1, 3, 1)m_c m_s \\
& - 2\hat{I}_2^{[1,0]}(2, 3, 1)m_c m_s - 2\hat{I}_1^{[1,0]}(2, 3, 1)m_c m_s + \hat{I}_0^{[0,1]}(3, 2, 1)m_c m_s + 4\hat{I}_1(2, 2, 1)m_c m_s - 4\hat{I}_2(2, 2, 1)m_c m_s \\
& + \hat{I}_1^{[1,1]}(3, 2, 2)m_c m_s - \hat{I}_0(2, 2, 1)m_c m_s + \hat{I}_1^{[0,1]}(2, 2, 2)m_s^2 + \hat{I}_2^{[0,1]}(2, 2, 2)m_s^2 + 2\hat{I}_0^{[0,1]}(2, 2, 2)m_s^2 - \hat{I}_1(2, 2, 1)m_s^2 \\
& - \hat{I}_0(2, 2, 1)m_s^2 + \hat{I}_2(2, 2, 1)m_s^2 - 2\hat{I}_0^{[0,2]}(3, 1, 2) + \hat{I}_0^{[0,1]}(1, 2, 2) + \hat{I}_0(2, 1, 1) - 2\hat{I}_1(1, 2, 1) + 2\hat{I}_2(1, 1, 2) \\
& + \hat{I}_0^{[1,0]}(1, 2, 2) - 3\hat{I}_0^{[1,1]}(3, 2, 1) - 2\hat{I}_1(1, 1, 2) + \hat{I}_0^{[0,1]}(2, 1, 2) + 2\hat{I}_2(1, 2, 1) + \hat{I}_0^{[1,2]}(3, 2, 2) + 2\hat{I}_0^{[0,1]}(3, 1, 1) \\
& + \hat{I}_0(1, 2, 1),
\end{aligned}$$

$$\begin{aligned}
C_{\phi D_s^* D_s^*}^\phi = & \hat{I}_0(3, 2, 2)m_c^6 - \hat{I}_0(3, 2, 2)m_c^5 m_s + \hat{I}_2(3, 2, 2)m_c^4 m_s^2 - \hat{I}_1(3, 2, 2)m_c^4 m_s^2 + \hat{I}_0(3, 2, 2)m_c^3 m_s^3 + \hat{I}_1(3, 2, 2)m_c^2 m_s^4 \\
& - \hat{I}_2(3, 2, 2)m_c^2 m_s^4 + 3\hat{I}_0(4, 1, 1)m_c^4 + 3\hat{I}_0(2, 2, 2)m_c^4 + 2\hat{I}_6(3, 2, 2)m_c^4 + 2\hat{I}_1(2, 3, 1)m_c^3 m_s + \hat{I}_2^{[0,1]}(3, 2, 2)m_c^3 m_s \\
& - 2\hat{I}_0(2, 2, 2)m_c^3 m_s - \hat{I}_0^{[1,0]}(3, 2, 2)m_c^3 m_s - 3\hat{I}_0(4, 1, 1)m_c^3 m_s - 2\hat{I}_2(2, 2, 2)m_c^3 m_s - 2\hat{I}_2(2, 3, 1)m_c^3 m_s \\
& + 2\hat{I}_0(2, 3, 1)m_c^3 m_s + 2\hat{I}_1(2, 2, 2)m_c^3 m_s - \hat{I}_1^{[0,1]}(3, 2, 2)m_c^3 m_s - 2\hat{I}_1^{[1,0]}(3, 2, 2)m_c^2 m_s^2 - 2\hat{I}_6(3, 2, 2)m_c^2 m_s^2 \\
& + 2\hat{I}_2^{[1,0]}(3, 2, 2)m_c^2 m_s^2 + 2\hat{I}_2(2, 3, 1)m_c m_s^3 + \hat{I}_0^{[0,1]}(3, 2, 2)m_c m_s^3 + 6\hat{I}_0(1, 4, 1)m_c m_s^3 - \hat{I}_0(2, 2, 2)m_c m_s^3 \\
& - 2\hat{I}_1(2, 3, 1)m_c m_s^3 - \hat{I}_1(2, 2, 2)m_s^4 + \hat{I}_2^{[0,1]}(3, 2, 2)m_s^4 - \hat{I}_1^{[0,1]}(3, 2, 2)m_s^4 + \hat{I}_2(2, 2, 2)m_s^4 + \hat{I}_0(2, 1, 2)m_s^2 \\
& + 8\hat{I}_8(3, 1, 2)m_c^2 - \hat{I}_1(3, 1, 1)m_c^2 - \hat{I}_0^{[1,0]}(2, 2, 2)m_c^2 + 2\hat{I}_6^{[0,1]}(3, 2, 2)m_c^2 + 4\hat{I}_8^{[1,0]}(3, 2, 2)m_c^2 - 4\hat{I}_6(3, 2, 1)m_c^2 \\
& + 4\hat{I}_8(2, 2, 2)m_c^2 + 2\hat{I}_6^{[1,0]}(3, 2, 2)m_c^2 + \hat{I}_2(3, 1, 1)m_c^2 - 8\hat{I}_7(3, 1, 2)m_c^2 - 4\hat{I}_7(2, 2, 2)m_c^2 - 4\hat{I}_7^{[1,0]}(3, 2, 2)m_c^2 \\
& + 6\hat{I}_6(4, 1, 1)m_c^2 + 6\hat{I}_6(3, 1, 2)m_c^2 + 8\hat{I}_6(2, 3, 1)m_c m_s - 2\hat{I}_0(1, 2, 2)m_c m_s + \hat{I}_1^{[0,1]}(2, 2, 2)m_c m_s \\
& + 2\hat{I}_1(1, 2, 2)m_c m_s + \hat{I}_0^{[0,1]}(2, 2, 2)m_c m_s + \hat{I}_2^{[1,1]}(3, 2, 2)m_c m_s - \hat{I}_2(2, 2, 1)m_c m_s - 2\hat{I}_0^{[0,1]}(2, 3, 1)m_c m_s \\
& - 2\hat{I}_2(1, 2, 2)m_c m_s - \hat{I}_2^{[0,1]}(2, 2, 2)m_c m_s + \hat{I}_1(2, 2, 1)m_c m_s - \hat{I}_1^{[1,1]}(3, 2, 2)m_c m_s + 9/2\hat{I}_1(1, 3, 1)m_s^2 \\
& + \hat{I}_0^{[1,1]}(3, 2, 2)m_s^2 - 12\hat{I}_6(1, 4, 1)m_s^2 + 24\hat{I}_7(1, 4, 1)m_s^2 - 6\hat{I}_1^{[1,0]}(1, 4, 1)m_s^2 - 2\hat{I}_0^{[1,0]}(3, 2, 1)m_s^2 \\
& + 6\hat{I}_2^{[1,0]}(1, 4, 1)m_s^2 - \hat{I}_0^{[0,1]}(2, 2, 2)m_s^2 - 9/2\hat{I}_2(1, 3, 1)m_s^2 + 4\hat{I}_6(3, 2, 1)m_s^2 - 2\hat{I}_6^{[0,1]}(3, 2, 2)m_s^2 + 2\hat{I}_6(2, 2, 2)m_s^2 \\
& - 4\hat{I}_7(3, 2, 1)m_s^2 + 2\hat{I}_1^{[1,0]}(2, 2, 2)m_s^2 + \hat{I}_0(3, 1, 1)m_s^2 - 24\hat{I}_8(1, 4, 1)m_s^2 - 2\hat{I}_2^{[1,0]}(2, 2, 2)m_s^2 + 4\hat{I}_8(3, 2, 1)m_s^2 \\
& + \hat{I}_2^{[2,1]}(3, 2, 2) - 4\hat{I}_8^{[1,0]}(2, 2, 2) - 2\hat{I}_6^{[1,0]}(2, 2, 2) - \hat{I}_1^{[2,0]}(3, 2, 1) - 2\hat{I}_2^{[1,0]}(1, 2, 2) - 4\hat{I}_6^{[1,0]}(3, 2, 1) \\
& - \hat{I}_1^{[2,1]}(3, 2, 2) + 4\hat{I}_7^{[1,0]}(2, 2, 2) + 2\hat{I}_0^{[1,1]}(2, 2, 2) + 2\hat{I}_6^{[1,1]}(3, 2, 2) + 8\hat{I}_7(2, 2, 1) + 4\hat{I}_8^{[1,1]}(3, 2, 2) + \hat{I}_0^{[0,2]}(3, 1, 2) \\
& - 8\hat{I}_8(2, 2, 1) + 3\hat{I}_6(1, 3, 1) + \hat{I}_2^{[2,0]}(3, 2, 1) + 4\hat{I}_6(2, 2, 1) - 8\hat{I}_6(2, 1, 2) + 12\hat{I}_7(3, 1, 1) - 2\hat{I}_0^{[0,1]}(1, 2, 2) \\
& - 3\hat{I}_0^{[1,0]}(2, 2, 1) + \hat{I}_0^{[1,0]}(2, 1, 2) - 2\hat{I}_6^{[0,1]}(2, 2, 2) - 6\hat{I}_7(1, 3, 1) + 6\hat{I}_8(1, 3, 1) - 4\hat{I}_7^{[1,1]}(3, 2, 2) - 4\hat{I}_6(3, 1, 1) \\
& + 4\hat{I}_7^{[0,1]}(2, 2, 2) - 4\hat{I}_8^{[0,1]}(2, 2, 2) - 2\hat{I}_0^{[0,1]}(2, 2, 1) + 2\hat{I}_1^{[1,0]}(1, 2, 2) - 12\hat{I}_8(3, 1, 1) - 2\hat{I}_6^{[0,1]}(3, 1, 2),
\end{aligned}$$

$$\begin{aligned}
C_{\phi D_{s_1} D_{s_1}}^\phi &= \hat{I}_2(3, 2, 2)m_c^5 m_s - \hat{I}_1(3, 2, 2)m_c^5 m_s + \hat{I}_1^{[1,0]}(3, 2, 2)m_c^4 + 4\hat{I}_8(3, 2, 2)m_c^4 + 2\hat{I}_6(3, 2, 2)m_c^4 - 4\hat{I}_7(3, 2, 2)m_c^4 \\
&\quad + 3\hat{I}_0(4, 1, 1)m_c^4 - \hat{I}_2^{[1,0]}(3, 2, 2)m_c^4 - 2\hat{I}_0(3, 2, 1)m_c^3 m_s + \hat{I}_0^{[1,0]}(3, 2, 2)m_c^3 m_s + 3\hat{I}_0(4, 1, 1)m_c^3 m_s \\
&\quad + 2\hat{I}_0(2, 2, 2)m_c^3 m_s + 2\hat{I}_2(3, 2, 1)m_c^2 m_s^2 - 3\hat{I}_1(4, 1, 1)m_c^2 m_s^2 + \hat{I}_2(2, 2, 2)m_c^2 m_s^2 - \hat{I}_1(2, 2, 2)m_c^2 m_s^2 \\
&\quad - 2\hat{I}_1(3, 2, 1)m_c^2 m_s^2 - 2\hat{I}_6(3, 2, 2)m_c^2 m_s^2 + 3\hat{I}_2(4, 1, 1)m_c^2 m_s^2 + \hat{I}_2(3, 2, 1)m_c m_s^3 - \hat{I}_1(3, 2, 1)m_c m_s^3 \\
&\quad - \hat{I}_0^{[0,1]}(3, 2, 2)m_c m_s^3 - 6\hat{I}_0(1, 4, 1)m_c m_s^3 - \hat{I}_1^{[0,1]}(3, 2, 2)m_c m_s^3 + \hat{I}_2^{[0,1]}(3, 2, 2)m_c m_s^3 + \hat{I}_2^{[0,1]}(3, 2, 2)m_s^4 \\
&\quad + \hat{I}_0(3, 2, 1)m_s^4 - \hat{I}_1^{[0,1]}(3, 2, 2)m_s^4 - \hat{I}_1(3, 2, 1)m_s^4 - 3\hat{I}_0(1, 4, 1)m_s^4 + \hat{I}_2(3, 2, 1)m_s^4 + 8\hat{I}_8(3, 2, 1)m_s^2 \\
&\quad - \hat{I}_2^{[2,0]}(3, 2, 2)m_c^2 + 8\hat{I}_8(3, 1, 2)m_c^2 - \hat{I}_0^{[0,1]}(2, 2, 2)m_c^2 + 4\hat{I}_8^{[0,1]}(3, 2, 2)m_c^2 - 4\hat{I}_7^{[0,1]}(3, 2, 2)m_c^2 \\
&\quad + 2\hat{I}_6^{[1,0]}(3, 2, 2)m_c^2 - 4\hat{I}_6(3, 2, 1)m_c^2 + \hat{I}_1^{[2,0]}(3, 2, 2)m_c^2 - 8\hat{I}_7(3, 2, 1)m_c^2 + 6\hat{I}_6(4, 1, 1)m_c^2 + 2\hat{I}_6^{[0,1]}(3, 2, 2)m_c^2 \\
&\quad + 4\hat{I}_0(1, 2, 2)m_c^2 + 6\hat{I}_6(3, 1, 2)m_c^2 - 8\hat{I}_7(3, 1, 2)m_c^2 + \hat{I}_1(1, 3, 1)m_c m_s - \hat{I}_0(2, 1, 2)m_c m_s - \hat{I}_2(1, 3, 1)m_c m_s \\
&\quad + \hat{I}_1(2, 1, 2)m_c m_s - \hat{I}_2^{[1,0]}(2, 2, 2)m_c m_s + 2\hat{I}_2^{[1,0]}(2, 3, 1)m_c m_s - 2\hat{I}_0^{[0,1]}(3, 1, 2)m_c m_s - 2\hat{I}_1^{[1,0]}(2, 3, 1)m_c m_s \\
&\quad + \hat{I}_0(1, 3, 1)m_c m_s - 8\hat{I}_6(2, 3, 1)m_c m_s - \hat{I}_2^{[1,1]}(3, 2, 2)m_c m_s + \hat{I}_1^{[0,1]}(2, 2, 2)m_c m_s + \hat{I}_1^{[1,1]}(3, 2, 2)m_c m_s \\
&\quad - \hat{I}_2(2, 1, 2)m_c m_s - 12\hat{I}_6(1, 4, 1)m_s^2 + 6\hat{I}_2^{[1,0]}(1, 4, 1)m_s^2 + 4\hat{I}_8(3, 2, 1)m_s^2 + \hat{I}_1(3, 1, 1)m_s^2 - 6\hat{I}_1^{[0,1]}(1, 4, 1)m_s^2 \\
&\quad - 4\hat{I}_8^{[0,1]}(3, 2, 2)m_s^2 - \hat{I}_1(1, 2, 2)m_s^2 + \hat{I}_0^{[1,1]}(3, 2, 2)m_s^2 - 2\hat{I}_6^{[0,1]}(3, 2, 2)m_s^2 + \hat{I}_0(3, 1, 1)m_s^2 + \hat{I}_2(1, 2, 2)m_s^2 \\
&\quad + 3\hat{I}_0^{[0,1]}(1, 4, 1)m_s^2 + 4\hat{I}_6(3, 2, 1)m_s^2 - \hat{I}_0^{[0,1]}(2, 2, 2)m_s^2 + 4\hat{I}_7^{[0,1]}(3, 2, 2)m_s^2 - \hat{I}_2(3, 1, 1)m_s^2 + 2\hat{I}_6(2, 2, 2)m_s^2 \\
&\quad + 3\hat{I}_0^{[1,0]}(1, 4, 1)m_s^2 - 4\hat{I}_7(3, 2, 1)m_s^2 - 2\hat{I}_0^{[0,1]}(2, 2, 1) - 12\hat{I}_8(3, 1, 1) + 4\hat{I}_7^{[0,1]}(2, 2, 2) - 4\hat{I}_8^{[0,1]}(2, 2, 2) \\
&\quad - \hat{I}_0^{[1,0]}(3, 1, 1) - 2\hat{I}_6^{[0,1]}(3, 1, 2) - 4\hat{I}_7^{[1,1]}(3, 2, 2) + 4\hat{I}_8^{[1,1]}(3, 2, 2) - 2\hat{I}_6^{[0,1]}(2, 2, 2) + 8\hat{I}_7(2, 1, 2) - 8\hat{I}_6(2, 1, 2) \\
&\quad + 4\hat{I}_7^{[1,0]}(2, 2, 2) + 2\hat{I}_1(2, 1, 1) - \hat{I}_2^{[1,1]}(2, 2, 2) + \hat{I}_1^{[1,1]}(2, 2, 2) - 2\hat{I}_2(2, 1, 1) + 8\hat{I}_7(2, 2, 1) - 8\hat{I}_8(2, 2, 1) \\
&\quad - 4\hat{I}_6(3, 1, 1) - 4\hat{I}_8^{[1,0]}(2, 2, 2) + 4\hat{I}_6(2, 2, 1) - 3\hat{I}_0(2, 1, 1) - 4\hat{I}_6^{[1,0]}(3, 2, 1) + 12\hat{I}_7(3, 1, 1) - 2\hat{I}_6^{[1,0]}(2, 2, 2) \\
&\quad + 2\hat{I}_6^{[1,1]}(3, 2, 2) + 3\hat{I}_1^{[0,1]}(2, 1, 2) - 3\hat{I}_2^{[0,1]}(2, 1, 2) - 8\hat{I}_8(2, 1, 2) + 2\hat{I}_1^{[1,0]}(2, 2, 1) - 2\hat{I}_2^{[1,0]}(2, 2, 1) \\
&\quad - \hat{I}_0^{[0,1]}(3, 1, 1) + 3\hat{I}_6(1, 3, 1) - 2\hat{I}_0^{[0,1]}(1, 2, 2) + 2\hat{I}_0^{[1,1]}(2, 2, 2),
\end{aligned}$$

where

$$\hat{I}_l^{[m,n]}(a, b, c) = [M_1^2]^m [M_2^2]^n \frac{d^m}{d(M_1^2)^m} \frac{d^n}{d(M_2^2)^n} [M_1^2]^m [M_2^2]^n \hat{I}_l(a, b, c).$$

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