Decay constants and masses of light tensor mesons ($J^P = 2^+$)

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Abstract

The masses and decay constants of the light tensor mesons were calculated with quantum numbers $J^P = 2^+$ in the framework of the QCD sum rules in the standard model. The non-perturbative contributions up to dimension-5 are considered as important terms of the operator product expansion.

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I. INTRODUCTION

In the flavor $SU(3)$ symmetry, the light p-wave tensor mesons with the angular momentum $L = 1$ and total spin $S = 1$ form an 1^3P_2 nonet. In other words, iso-vector mesons $a_2(1320)$, iso-doublet states $K_2^*(1430)$, and two iso-singlet mesons $f_2(1270)$ and $f_2'(1525)$, build the ground state nonet which has been experimentally known [1, 2]. The quark content, $q\bar{q}$ for the iso-vector and iso-doublet tensor resonances is obvious. The iso-scalar tensor states, $f_2(1270)$ and $f_2'(1525)$ have a mixing wave functions where mixing angle should be small [3, 4]. Therefore, $f_2(1270)$ is primarily a $(u\bar{u} + d\bar{d})/\sqrt{2}$ state, while $f_2'(1525)$ is dominantly $s\bar{s}$ [5].

Studying the light tensor mesons properties can be useful for understanding the QCD in low energy. In this work, an attempt was made to consider masses and decay constants of the light tensor mesons via the QCD sum rules (SR). The SR has been successfully applied to a wide variety of problems in hadron physics (for details of this method, see [6, 7]). In this method, calculation starts with correlation function investigated in two phenomenological and theoretical sides. The computation of the theoretical part of the correlation function via the operator product expansion (OPE) consists of two perturbative and non-perturbative parts, the last part of which is called condensate contributions. The condensate term of dimension-3 is related to the contribution of the quark-quark condensate and dimension-4 and 5 are connected to the gluon-gluon and gluon-quark condensate, respectively. After the two sides of correlation function, the phenomenological and theoretical, are equated, and the Borel transformation is applied to suppress the contribution of the higher states and continuum, the physical quantities are estimated.

The masses and decay constants of the light tensor mesons have been calculated in the framework of the SR using different approaches [5, 8–10]. In addition, several studies have derived decay constant of f_2 from the measurement of $\Gamma(f_2 \to \pi\pi)$ [11, 12]. In the present study a new approach is used in that an attempt was made to calculate the masses and decay constants of the light tensor mesons by extracting the Wilson coefficients $C^{(0)}$, and $C^{(3)}$ related to the bare loop and quark-quark correction, respectively. Our results for the masses and decay constants of the light tensor mesons are in consistent agreement with the mass experimental values and decay constant predictions made using other methods. The obtained results for the masses and decay constants can be used to calculate the magnetic dipole moments of the light tensor mesons [13].

This paper includes three sections. In section II, the method of the SR for the calculation of the masses and decay constants of the light tensor mesons are presented. Section III is devoted to the numerical analysis of the masses and decay constants as well as their comparison with the experimental data and predicted values by other methods.

II. THE METHOD

The computation of the decay constants and masses of the tensor mesons using the two-point QCD sum rules, starts with the correlation function as

$$\Pi_{\mu\nu\alpha\beta} = i\int d^4x e^{iq(x-y)} \langle 0|\, T[j_{\mu\nu}(x) j^{\dagger}_{\alpha\beta}(y)]\, |0\rangle, \tag{1}$$

the current $j_{\mu\nu}$, responsible for the production of the tensor meson from the QCD vacuum, is:

$$j_{\mu\nu}(x) = \frac{i}{2}\left[\bar{q}_1(x)\gamma_\mu \overleftrightarrow{D}_\nu (x) q_2(x) + \bar{q}_1(x)\gamma_\nu \overleftrightarrow{D}_\mu (x) q_2(x)\right], \tag{2}$$

where q_1 and q_2 are wave functions related to two quarks composing the tensor meson. Also

$$\begin{aligned}
\overleftrightarrow{D}_\mu (x) &= \frac{1}{2}\left[\overrightarrow{D}_\mu (x) - \overleftarrow{D}_\mu (x)\right], \\
\overrightarrow{D}_\mu (x) &= \overrightarrow{\partial}_\mu (x) - i\frac{g}{2}\lambda^a \boldsymbol{A}^a_\mu(x), \\
\overleftarrow{D}_\mu (x) &= \overleftarrow{\partial}_\mu (x) + i\frac{g}{2}\lambda^a \boldsymbol{A}^a_\mu(x),
\end{aligned}$$

$\lambda^a (a = 1, ..., 8)$ are the Gell-man matrixes and $\boldsymbol{A}^a_\mu(x)$ are the external (vacuum) gluon fields. In Fock-Schwinger gauge, $x^\mu \boldsymbol{A}^a_\mu(x) = 0$.

As noted, the correlation function is a complex function of which the imaginary part comprises the computations of the phenomenology and real part comprises the computations of the theoretical part (QCD). By linking these two parts via the dispersion relation as

$$\Pi(q^2) = \frac{1}{\pi}\int \frac{\mathrm{Im}\Pi(s)}{s-q^2} ds, \tag{3}$$

the physical quantities such as the decay constants and masses of the tensor mesons are calculated. A complete set of the quantum states of mesons is inserted in the correlation function (Eq.(1)) to compute the phenomenology part of the correlation function. After performing integral over x and separating the contribution of the higher states and continuum and opting $y = 0$, we obtain:

$$\Pi_{\mu\nu\alpha\beta} = \frac{\langle 0|\, j_{\mu\nu}\, |T(q)\rangle \, \langle T(q)|\, j_{\alpha\beta}\, |0\rangle}{m_T^2 - q^2} + \text{higher states and continuum}, \qquad (4)$$

where m_T is the mass of the tensor meson T. The matrix elements of Eq. (4) can be defined as follows [8, 14]:

$$\langle 0|j_{\mu\nu}|T(q)\rangle = f_T\ m_T^3\ \varepsilon_{\mu\nu}, \qquad (5)$$

where f_T and $\varepsilon_{\mu\nu}$ are the decay constant and polarization of the tensor meson, respectively. Inserting Eq. (5) into Eq. (4) and using the relation

$$\varepsilon_{\mu\nu}\varepsilon_{\alpha\beta} = \frac{1}{2}T_{\mu\alpha}T_{\nu\beta} + \frac{1}{2}T_{\mu\beta}T_{\nu\alpha} - \frac{1}{3}T_{\mu\nu}T_{\alpha\beta},$$

where $T_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_T^2}$, and choosing a suitable independent tensor structure, we obtain:

$$\Pi_{\mu\nu\alpha\beta} = \left\{ \frac{1}{2}(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha})\frac{f_T^2\ m_T^6}{m_T^2 - q^2} + \text{other structures} \right\} + \ \text{higher states}. \qquad (6)$$

In QCD, the correlation function can be evaluated by the operator product expansion (OPE), in the deep Euclidean region, as

$$\Pi^{QCD}_{\mu\nu\alpha\beta} = C^{(0)}_{\mu\nu\alpha\beta} + \langle 0|\bar{q}q|0\rangle C^{(3)}_{\mu\nu\alpha\beta} + \langle 0|G^a_{\varphi\lambda}G^{\varphi\lambda}_a|0\rangle C^{(4)}_{\mu\nu\alpha\beta} + \langle 0|\bar{q}\ \sigma_{\varphi\lambda}T^a G^{\varphi\lambda}_a\ q|0\rangle C^{(5)}_{\mu\nu\alpha\beta} + ...,$$

where $C^{(i)}_{\mu\nu\alpha\beta}$ are the Wilson coefficients, $\bar{q}$ is the local fermion filed operator of the light quark and $G^a_{\varphi\lambda}$ is the gluon strength tensor. The Wilson coefficients are determined by the contribution of the bare-loop, and power corrections coming from the quark-gluon condensates of dimension-3, 4 and higher dimensions. The diagrams corresponding to the perturbative (bare loop), and non-perturbative part contributions up to dimension-5 are depicted in Fig. 1.

The portion of the perturbative part (Fig 1-(a)), is computed by using the Feynman rules for the bare loop:

$$C^{(0)}_{\mu\nu\alpha\beta} \ = \ P_{\mu\nu\alpha\beta} + P_{\mu\nu\beta\alpha} + P_{\nu\mu\alpha\beta} + P_{\nu\mu\beta\alpha}, \qquad (7)$$

where

$$P_{\mu\nu\alpha\beta} \ = \ -\frac{i}{4}\int d^4x\ e^{iqx}\ \mathrm{Tr}\left[S_{q_1}(y-x)\gamma_\mu \overleftrightarrow{D}_\nu (x) S_{q_2}(x-y)\gamma_\alpha \overleftrightarrow{D}_\beta (y)\right]_{|y=0}. \qquad (8)$$

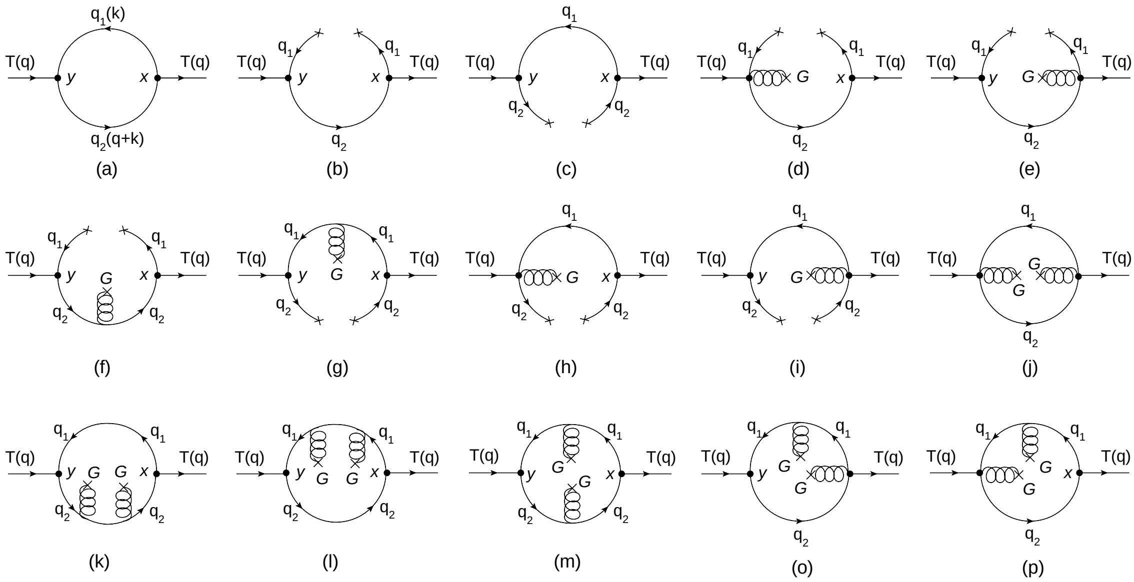

FIG. 1: The Fynnmans graphs corresponding to the perturbative (a), and non-perturbative part contributions (b,...,p), up to dimension-5.

Taking the partial derivative with respect to x and y of the light quark free propagators, and performing the Fourier transformation and using the Cutkosky rules, i.e., $\frac{1}{p^2-m^2} \to -2i\pi\delta(p^2-m^2)$, imaginary part of the $P_{\mu\nu\alpha\beta}$ is calculated as

$$\begin{aligned} \mathrm{Im}(P_{\mu\nu\alpha\beta}) &= -\frac{1}{(8\pi)^2}\int d^4k\ \delta(k^2-m_1^2)\delta((q+k)^2-m_2^2)\ (q_\nu q_\beta + 2q_\nu k_\beta + 2k_\nu q_\beta + 4k_\nu k_\beta) \\ &\times\ \mathrm{Tr}\left[(\not k + m_1)\gamma_\mu(\not q + \not k + m_2)\gamma_\alpha\right], \end{aligned} \tag{9}$$

where q is the four-momentum of the tensor meson, m_1 and m_2 are the masses of the two quarks q_1, and q_2, respectively. To solve the integral in Eq. (9), we will have to deal with the integrals as $I_{\mu\nu...} = \int d^4k\ [k_\mu k_\nu ...]\delta(k^2-m_1^2)\delta((q+k)^2-m_2^2)$. Each integral can be taken as an appropriate tensor structure of the q_μ, q_ν and $g_{\mu\nu}$ as

$$\begin{aligned} I_0 &= \frac{\pi}{2s}\sqrt{\lambda(s,m_1^2,m_2^2)}, \\ I_\mu &= A(q_\mu), \\ I_{\mu\nu} &= B_1(g_{\mu\nu}) + B_2(q_\mu q_\nu), \\ I_{\mu\nu\alpha} &= C_1(q_\mu q_\nu q_\alpha) + C_2(q_\mu g_{\nu\alpha} + q_\nu g_{\mu\alpha} + q_\alpha g_{\mu\nu}), \\ I_{\mu\nu\alpha\beta} &= E_1(q_\mu q_\nu q_\alpha q_\beta) + E_2(g_{\mu\nu}g_{\alpha\beta} + g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) + E_3(q_\mu q_\nu g_{\alpha\beta} + q_\mu q_\alpha g_{\nu\beta} + q_\mu q_\beta g_{\nu\alpha} \\ &+\ q_\alpha q_\beta g_{\mu\nu} + q_\alpha q_\nu g_{\mu\beta} + q_\nu q_\beta g_{\alpha\mu}), \end{aligned} \tag{10}$$

where $s=q^2$. The quantities of the coefficient $\lambda(s,m_1^2,m_2^2)$, A, B_i, C_i, $i=1,2$, and E_j, $j=1,...,3$, are stated in the appendix. By computing the trace realized in Eq. (9) and using the relations in Eq. (10) and dispersion relation (to calculate the real part from the imaginary), the perturbative part contribution of the correlation function, for the suitable structure corresponding to Eq. (6), can be shown as follows:

$$\begin{aligned} C^{(0)}_{\mu\nu\alpha\beta} &= \frac{1}{16\pi^3}(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha})\int_0^\infty \frac{\psi(s)}{s-q^2}ds + \text{other structures}, \\ \psi(s) &= B_1(m_1-m_2)^2 - B_1 s - 8E_2. \end{aligned} \tag{11}$$

Now, the condensate terms of dimension 3, 4 and 5 are considered. The non-perturbative part contains the quark and gluon condensate diagrams. The calculations show that the important contribution comes from dimension-3 related to Fig 1-(b) and (c). The remaining contributions are either zero such as (d) to (i), or so small in comparison with the contributions of dimension-3 that, could be easily ignored such as (j) to (p). It should be reminded that in the SR method, when the light quark is a spectator, the gluon-gluon condensate contributions are very small [15]. The computation of the QCD part of the correlation function is continued by extracting Wilson coefficient $C^{(3)}$ corresponding to the Feynman graphs (b) and (c). For Fig. 1-(b), there is:

$$C^{(3),b}_{\mu\nu\alpha\beta} = N_{\mu\nu\alpha\beta}+N_{\mu\nu\beta\alpha}+N_{\nu\mu\alpha\beta}+N_{\nu\mu\beta\alpha}, \tag{12}$$

where

$$N_{\mu\nu\alpha\beta} = -\frac{i}{4}\int d^4x\, e^{iqx}\langle 0|\bar{q}_{1\rho}(x)\Big[\gamma_\mu \overleftrightarrow{D}_\nu (x) S_{q_2}(x-y)\gamma_\alpha \overleftrightarrow{D}_\beta (y)\Big]_{\rho\sigma} q_{1\sigma}(y)|0\rangle_{|_{y=0}}. \tag{13}$$

To extract the $N_{\mu\nu\alpha\beta}$, we can expand $q_1(x)$ around the origin as follows:

$$q_1(x) = q_1(0) + x^\xi \overrightarrow{D}_\xi\, q_1(0) + \frac{1}{2}x^\xi x^{\xi'} \overrightarrow{D}_\xi \overrightarrow{D}_{\xi'}\, q_1(0) + \ldots.$$

It should be noted that the Wilson coefficients are evaluated in the deep Euclidean region $(x-y\ll 1)$, and y is chosen as the origin in our calculations, therefore $x\ll 1$. Hence, we include only the first term of this expansion in Eq. (13). Additionally, using the definition for the following matrix elements as

$$\begin{aligned} \langle 0|\bar{q}_{1\rho}(0) q_{1\sigma}(0)|0\rangle &= \frac{1}{4}\delta_{\rho\sigma}\langle 0|\bar{q}_1 q_1|0\rangle, \\ \langle 0|\bar{q}_{1\rho}(0) \overrightarrow{D}_\mu\, q_{1\sigma}(0)|0\rangle &= -i\frac{m_1}{16}(\gamma_\mu)_{\rho\sigma}\langle 0|\bar{q}_1 q_1|0\rangle, \\ \langle 0|\bar{q}_{1\rho}(0) \overleftarrow{D}_\mu \overrightarrow{D}_\nu\, q_{1\sigma}(0)|0\rangle &= \frac{1}{16}(\frac{m_0^2}{2}-m_1^2)(g_{\mu\nu})_{\rho\sigma}\langle 0|\bar{q}_1 q_1|0\rangle, \end{aligned}$$

and after some calculations, we obtain:

$$C^{(3),b}_{\mu\nu\alpha\beta} = \frac{1}{16}(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha})\frac{\kappa\langle 0|\bar{q}_1 q_1|0\rangle}{q^2 - m_2^2} + \text{other structures},$$

where $\kappa = m_2(m_1^2 - \frac{m_0^2}{2})$ and $m_0^2(1GeV) = (0.8 \pm 0.2)GeV^2$ [16]. After similar calculations for Fig 1-(c), the final result for the non-perturbative contributions, $C^{(3)}_{\mu\nu\alpha\beta}$ is obtained as follows:

$$C^{(3)}_{\mu\nu\alpha\beta} = \frac{1}{16}(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha})\left(\frac{\kappa\langle 0|\bar{q}_1 q_1|0\rangle}{q^2 - m_2^2} + \frac{\kappa'\langle 0|\bar{q}_2 q_2|0\rangle}{q^2 - m_1^2}\right) + \text{other structures}, \quad (14)$$

where $\kappa' = m_1(m_2^2 - \frac{m_0^2}{2})$.

Now, equating the two phenomenology part, Eq .(6), and QCD part, Eqs .(11) and (14), of the correlation function as well as applying the Borel transformation

$$\hat{B}_{M^2}(q^2)\frac{1}{m^2 - q^2} = \frac{1}{M^2}e^{-\frac{m^2}{M^2}},$$

to both sides, the decay constant of the tensor meson is computed as

$$f_T^2 = \frac{e^{m_T^2/M^2}}{8\ m_T^6}\left\{\frac{3}{\pi^3}\int_0^{s_T}\psi(s)e^{-s/M^2}ds - \kappa\langle 0|\bar{q}_1 q_1|0\rangle e^{-m_2^2/M^2} - \kappa'\langle 0|\bar{q}_2 q_2|0\rangle e^{-m_1^2/M^2}\right\}, \quad (15)$$

where s_T is the continuum threshold of the tensor meson. In the above equation, the quark-hadron duality assumption is also used to subtract the contributions of the higher states and the continuum. In other words, it is assumed that [15]:

$$\text{higher states} \simeq \frac{1}{\pi}\int_{s_T}^{\infty}\frac{\rho^{\text{OPE}}}{s - q^2}ds, \quad (16)$$

where $\rho^{\text{OPE}} = \frac{3}{8\pi^2}\psi(s)$. In fact ρ^{OPE} is related to the Wilson coefficient $C^{(0)}_{\mu\nu\alpha\beta}$.

Furthermore, by applying derivation to both sides of Eq. (15) in term of M^2, the mass of the tensor meson is obtained as

$$m_T^2 = \frac{\frac{3}{\pi^3}\int_0^{s_T} s\ \psi(s)e^{-s/M^2}ds - \kappa\langle 0|\bar{q}_1 q_1|0\rangle\ m_2^2 e^{-m_2^2/M^2} - \kappa'\langle 0|\bar{q}_2 q_2|0\rangle\ m_1^2 e^{-m_1^2/M^2}}{\frac{3}{\pi^3}\int_0^{s_T}\psi(s)e^{-s/M^2}ds - \kappa\langle 0|\bar{q}_1 q_1|0\rangle e^{-m_2^2/M^2} - \kappa'\langle 0|\bar{q}_2 q_2|0\rangle e^{-m_1^2/M^2}}. \quad (17)$$

III. NUMERICAL ANALYSIS AND CONCLUSION

In this section, Eqs. (15) and (17) are used to compute the masses and decay constants for the four light tensor mesons $K_2^*(1430)$, $a_2(1320)$, $f_2(1270)$, and $f_2'(1525)$. To this aim,

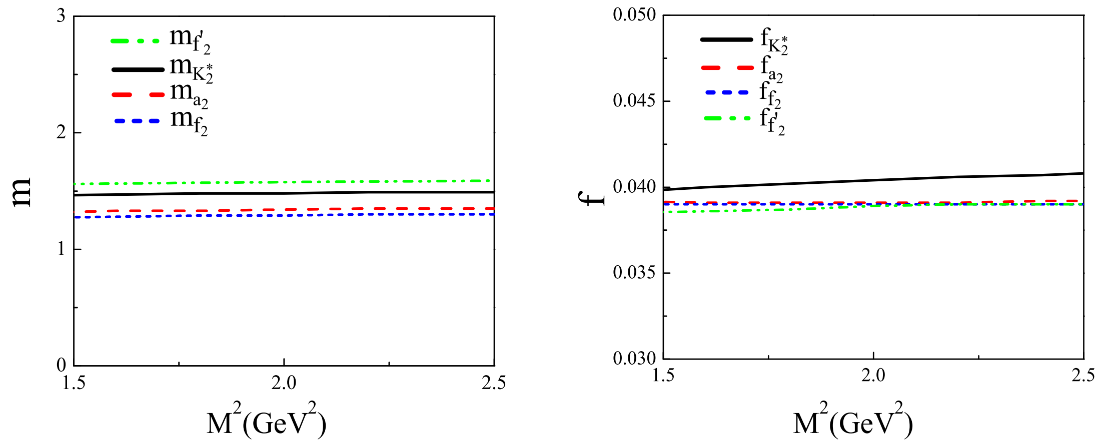

FIG. 2: The dependence of the tensor meson masses on the Borel parameter M^2 (left). The same as it but for the decay constants (right).

we need to insert the parameters s_T and the light quark masses in these equations. The masses of u and d quarks can be numerically neglected. The mass of the s quark, at the scale $1\ GeV$, is: $m_s = 142\ MeV$ [17]. The continuum threshold s_T is correlated with the energy of the first excited state of the tensor meson under consideration. In this study, the value of the continuum threshold is considered to be $s_T = (m_T+\Delta)^2\ GeV^2$, where $\Delta = (0.20-0.30)$. Also $\langle 0|\bar{s}s|0\rangle = (0.8 \pm 0.2)\langle 0|\bar{u}u|0\rangle$, $\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle = -(0.240 \pm 0.010\ GeV)^3$ in which the value of the condensates are chosen at a fixed renormalization scale of about $1\ GeV$ [18].

The expressions for the mass and decay constant in Eqs .(15) and (17) contain also the Borel mass square M^2 that is not physical quantity. The physical quantities, mass and decay constant, should be independent of the parameter M^2. The dependence of the masses and decay constants of the tensor mesons on M^2 is shown in Fig. 2. As can be seen from the following graphs, in our analysis, the dependence of the masses and the decay constants on the Broel parameter is insignificant in the region $1.5\ GeV^2 \leq M^2 \leq 2.5\ GeV^2$.

The results of our analysis for the masses of the tensor mesons for different values of Δ and $M^2 = 2$, are given in Table I. This table contains also the experimental quantities of the mass of the light tensor mesons. As shown, the values for $\Delta_2 = 0.25\ GeV$ are in very good agreement with the experimental values.

In Table II, obtained results for the decay constants of the tensor mesons for different

TABLE I: Comparison of the light tensor meson masses in this work for various Δ, where $\Delta_1 = 0.20\ GeV, \Delta_2 = 0.25\ GeV, \Delta_3 = 0.30\ GeV$, with the experimental values in GeV.

Mass	$m_T(\Delta_1)$	$m_T(\Delta_2)$	$m_T(\Delta_3)$	EXP [19]
$m_{K_2^*}$	1.39 ± 0.22	1.42 ± 0.31	1.46 ± 0.42	1.43
m_{a_2}	1.28 ± 0.21	1.31 ± 0.30	1.35 ± 0.41	1.32
m_{f_2}	1.25 ± 0.21	1.28 ± 0.30	1.35 ± 0.41	1.28
$m_{f_2'}$	1.49 ± 0.23	1.52 ± 0.33	1.55 ± 0.45	1.53

values of Δ and $M^2 = 2$, as well as the obtained results via other ways in the framework of the SR are presented. It should be noted that the decay constant f_T defined in [5, 10] differs from those obtained in this study by a factor of $1/(2m_T)$. Therefore, their values have been rescaled and then presented in Table II. The results derived in this study, especially for

TABLE II: Comparison of the decay constant values of the tensor mesons in this work for various Δ, where $\Delta_1 = 0.20\ GeV, \Delta_2 = 0.25\ GeV, \Delta_3 = 0.30\ GeV$, with the values obtained by others (in units of 10^{-3}).

Decay Constant	$f_T(\Delta_1)$	$f_T(\Delta_2)$	$f_T(\Delta_3)$	[5]	[9]	[8]	[10]
$f_{K_2^*}$	34 ± 3	36 ± 4	39 ± 5	41	50	—	—
f_{a_2}	34 ± 3	37 ± 4	40 ± 5	41	—	—	—
f_{f_2}	35 ± 3	38 ± 4	41 ± 6	40	—	40	$52 - 72$
$f_{f_2'}$	33 ± 2	35 ± 3	37 ± 4	42	—	—	$37 - 50$

$\Delta_3 = 0.30\ GeV$, are in consistent agreement with other values.

The errors are estimated by the variation of the Borel parameter M^2, the variation of the continuum threshold s_T, and uncertainties in the values of the other input parameters. The main uncertainty comes from the continuum thresholds of the central value, while the other uncertainties are small, constituting a few percent.

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Appendix

In this appendix, the explicit expressions of the coefficients $\lambda(s,m_1^2,m_2^2)$, $A(s,m_1^2,m_2^2)$, $B_i(s,m_1^2,m_2^2)$, $C_i(s,m_1^2,m_2^2)$, $i=1,2$, and $E_j(s,m_1^2,m_2^2)$, $j=1,...,3$ are given.

$$\begin{aligned}
\lambda(s,m_1^2,m_2^2) &= \left(s-m_1^2-m_2^2\right)^2-4m_1^2m_2^2, & \Delta &= s+m_1^2-m_2^2,\\
A &= -\frac{\Delta}{2s}I_0, & B_1 &= \frac{I_0}{3s}\left(m_1^2 s-\frac{\Delta^2}{4}\right),\\
B_2 &= \frac{I_0}{s}\left[m_1^2+\frac{4}{3s}\left(\frac{\Delta^2}{4}-m_1^2 s\right)\right], & C_1 &= \frac{\Delta I_0}{s^3}\left[\frac{1}{22}\left(\frac{\Delta^2}{4}+m_1^2 s\right)-\frac{\Delta^2}{8}\right],\\
C_2 &= \frac{\Delta I_0}{22s^2}\left(\frac{\Delta^2}{4}-m_1^2 s\right), & E_1 &= I_0\left(\frac{\Delta^4}{12s^4}-\frac{\Delta^2 m_1^2}{8s^3}\right),\\
E_2 &= \frac{m_1^2 I_0}{36}\left(m_1^2-\frac{\Delta^2}{4s}\right), & E_3 &= -\frac{I_0}{72}\left(\frac{\Delta^4}{4s^3}+\frac{m_1^4}{s}+\frac{5}{4}\frac{\Delta^2 m_1^2}{s^2}\right).
\end{aligned}$$

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