Analysis of rare semileptonic $B_c \to D^*_{s0}(2317) l^+ l^-$ decays

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We analyze the rare semileptonic $B_c \to D^*_{s0} l^+ l^-$, $(l = e,\ \mu,\ \tau)$ and $B_c \to D^*_{s0} \nu\bar{\nu}$ decays in the framework of the three-point QCD sum rules. D^*_{s0} is a scalar meson with total spin 0 and even parity. Taking into account the gluon condensate contributions as the important correction of the nonperturbative parts of the correlation function and the large effects of Coulombic corrections, the relevant form factors of these transitions have been calculated. The branching fractions and longitudinal lepton polarization asymmetry are also investigated.

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I. INTRODUCTION

The $\bar{b}c$ is the only system, composed of two heavy quarks b and c with different flavors. Therefore among the hidden charm $c\bar{c}$ and beauty $b\bar{b}$ families, the family of B_c mesons with the open charm and beauty have a particular place. Both experimental and theoretical study on the B_c meson could be useful for physicists, since in contrast to the charmonium and bottomonium, the $\bar{b}c$ system levels have no strong and electromagnetic annihilation channels of decays. In fact the pseudoscalar meson $B_c(0^-)$ is the long-living particle that decays only via the weak interaction. So the study of the B_c meson decays is the rich field of the heavy quark physics, where one may extract the important information about the weak theory, leptonic decay constants, as well as the origin of the CP violation and also search for new physics beyond the standard model (SM) (for details of the physics of the B_c meson, see [1]).

A scalar meson is a meson with total spin 0 and even parity. In comparison with pseudoscalar meson, these mesons are most often produced in proton-antiproton annihilation, radiative decays of vector mesons, and meson-meson scattering. The discovery of the very narrow $D^*_{s0}(2317)$ [2] charm-strange meson with mass 2317 MeV and spin-parity structure of 0^+ has triggered hot discussion on its nature. The reason is its surprisingly low mass, some 170 MeV below the predictions of standard relativistic constituent quark models for the ground-state scalar $c\bar{s}$ meson [3]. Also its measured mass is smaller than the expected value in the potential model [4]. Many theoretical papers have been dedicated to the understanding of the nature of this state and its quark content [5–18]. Many physicists presume that the quark content of this meson is $c\bar{s}$ [16–23] and others believe that it might be a $\bar{c}qs\bar{q}$ four-quark state [5–8] and so on. The mass of the $c\bar{s}$ state is much smaller than that of the four-quark state within quark model [24]. Furthermore, there are two 0^+ states in the four-quark system. But only one 0^+ state is in the $c\bar{s}$ system, which is consistent with the experimental search by *BABAR* [25]. Analysis of the radiative decay $D_{s0}(2317) \to D^*_s \gamma$ also concludes that this state is the conventional $c\bar{s}$ meson [23].

The rare semileptonic $B_c \to D^*_{s0} l^+ l^- / \nu\bar{\nu}$ decays occur at loop level by electroweak penguin and weak box diagrams in the SM via the flavor changing neutral current (FCNC) transition of $b \to s$. The FCNC decays of B_c meson are sensitive to new physics (NP) contributions to penguin operators. So to test the SM and look for NP, we need to determine the SM predictions for FCNC decays and compare these results to the corresponding experimental values.

The QCD sum rules have been successfully applied to a wide variety of problems in hadron physics [26]. Some possible semileptonic decays of B_c involving charmed mesons such as $B_c \to D^0 l\nu$ [27], $B_c \to D^{*0} l\nu$ [27,28], $B_c \to D^*_{(s,d)} l^+ l^-$ [29], $B_c \to D_{(s,d)} l^+ l^- / \nu\bar{\nu}$ [30], $B_c \to X\nu\bar{\nu}$, $(X = D^*_{(s,d)}, D_{s1}(2460))$ [31], and $B_c \to D_{s1} l^+ l^-$ [32] have been studied in the framework of the three-point QCD sum rules (3PSR).

In the present work, the $B_c \to D^*_{s0} l^+ l^- / \nu\bar{\nu}$ transitions are investigated within the 3PSR approach. For analysis of the above-mentioned decays, we calculate the transition form factors of the semileptonic $B_c \to D^*_{s0}$ decay. Also, we consider the branching ratio values and longitudinal lepton polarization asymmetry of these semileptonic decays. Any future experimental measurement on these form factors as well as decay rates and branching fractions and their comparison with the obtained results in the present work can give considerable information about the structure of D^*_{s0} meson.

This paper is organized as follow. In Sec. II, we calculate the form factors for the $B_c \to D^*_{s0}$ transition in the 3PSR. In Sec. III, the two-gluon condensate contributions as the nonperturbative corrections are calculated. Finally, Sec. IV is devoted to the numeric results and discussions.

II. QCD SUM RULES FOR TRANSITION FORM FACTORS

The $B_c \to D^*_{s0} l^+ l^- / \nu\bar{\nu}$ decays are described via the loop $b \to s l^+ l^- / \nu\bar{\nu}$ transitions at quark level. The $b \to s l^+ l^-$

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transition receives contributions from electromagnetic and Z-penguin and box diagrams, but the $b \to s\nu\bar{\nu}$ transition takes place only via Z-penguin and box diagrams (see Fig. 1). In these loop transitions u, c, t are the intermediate quarks in which the main contribution comes from the intermediate top quark.

The effective Hamiltonian responsible for the $b \to sl^+l^-$ transition in the standard model can be written in the following form:

$$H_{\rm eff} = \frac{G_F\alpha}{2\pi\sqrt{2}}V_{tb}V_{ts}^*\bigg[C_9^{\rm eff}\bar{s}\gamma_\mu(1-\gamma_5)b\bar{l}\gamma_\mu l + C_{10}\bar{s}\gamma_\mu(1-\gamma_5)b\bar{l}\gamma_\mu\gamma_5 l - 2C_7^{\rm eff}\frac{m_b}{q^2}\bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\bar{l}\gamma_\mu l\bigg], \quad (1)$$

where $C_7^{\rm eff}$, $C_9^{\rm eff}$, and C_{10} are the Wilson coefficients, G_F is the Fermi constant, α is the fine structure constant at Z mass scale, and V_{ij} are elements of the Cabibbo, Kabayashi, Maskawa (CKM) matrix. For the $\nu\bar{\nu}$ case, we consider only the term containing C_{10}. To derive the matrix elements for $B_c \to D_{s0}^* l^+l^-/\nu\bar{\nu}$ decays, we need to sandwich Eq. (1) between initial and final meson states. So the amplitude of these decays is given by

$$M = \frac{G_F\alpha}{2\sqrt{2}\pi}V_{tb}V_{ts}^*\bigg[C_9^{\rm eff}\langle D_{s0}^*(p')|\bar{s}\gamma_\mu(1-\gamma_5)b|B_c(p)\rangle\bar{l}\gamma_\mu l + C_{10}\langle D_{s0}^*(p')|\bar{s}\gamma_\mu(1-\gamma_5)b|B_c(p)\rangle\bar{l}\gamma_\mu\gamma_5 l - 2C_7^{\rm eff}\frac{m_b}{q^2}\langle D_{s0}^*(p')|\bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B_c(p)\rangle\bar{l}\gamma_\mu l\bigg], \quad (2)$$

where p and p' are the momentum of initial and final meson states, respectively. Because of the parity conservation, only the axial vector parts of $s\gamma_\mu(1-\gamma_5)b$ and $si\sigma_{\mu\nu}q^\nu(1+\gamma_5)b$ contribute to the above matrix elements, i.e.,

$$\langle D_{s0}^*(p')|\bar{s}\gamma_\mu b|B_c(p)\rangle = 0,$$
$$\langle D_{s0}^*(p')|\bar{s}i\sigma_{\mu\nu}q^\nu b|B_c(p)\rangle = 0. \quad (3)$$

Considering Lorentz invariance and parity conservation the matrix elements $\langle D_{s0}^*(p')|\bar{s}\gamma_\mu\gamma_5 b|B_c(p)\rangle$ and $\langle D_{s0}^*(p')|\bar{s}i\sigma_{\mu\nu}q^\nu\gamma_5 b|B_c(p)\rangle$ are parametrized in terms of the form factors as

$$\langle D_{s0}^*(p')|\bar{s}\gamma_\mu\gamma_5 b|B_c(p)\rangle = -i(P_\mu f_1(q^2) + q_\mu f_2(q^2)),$$
$$\langle D_{s0}^*(p')|\bar{s}i\sigma_{\mu\nu}q^\nu\gamma_5 b|B_c(p)\rangle = \frac{f_T(q^2)}{m_{B_c}+m_{D_{s0}^*}}[P_\mu q^2 - q_\mu(m_{B_c}^2 - m_{D_{s0}^*}^2)], \quad (4)$$

where $f_1(q^2)$, $f_2(q^2)$, and $f_T(q^2)$ are the transition form factors, $P_\mu = (p+p')_\mu$ and $q_\mu = (p-p')_\mu$. Since $f_T(q^2)$ is related to the photon vertex $(\sigma_{\mu\nu}q^\nu\gamma_5)$, it is not considered for the $b \to s\nu\bar{\nu}$ transition. To calculate the form factors $f_1(q^2)$, $f_2(q^2)$, and $f_T(q^2)$ in the framework of the QCD sum rules method, we start with the correlation function, as follows:

$$\Pi_\mu^{A,T} = i^2\int d^4x d^4y e^{-ipx}e^{ip'y}\langle 0|\tau\{J_{D_{s0}^*}(y)J_\mu^{A,T}(0)J_{B_c}^\dagger(x)\}|0\rangle, \quad (5)$$

where $J_{D_{s0}^*}(y) = \bar{c}s$, $J_{B_c}(x) = \bar{c}\gamma_5 b$ are the interpolating currents of the D_{s0}^* and B_c mesons and $J_\mu^A = \bar{s}\gamma_\mu\gamma_5 b$, $J_\mu^T = \bar{s}\sigma_{\mu\nu}q^\nu\gamma_5 b$ are the weak flavor changing quark currents. From the general philosophy of the QCD sum rules, the correlation function can be calculated in two phenomenological and QCD or theoretical sides. Equating the coefficient of the selected structures from the phenomenological and the expressions, which is calculated by the help of the operator product expansion (OPE) in the QCD or theoretical side, the QCD sum rules for our form factors can be obtained. We first calculate the phenomenological part of the correlation function given in Eq. (5). For this aim, we need to insert the complete set of the intermediate states with the same quantum numbers as the currents $J_{D_{s0}^*}$ and J_{B_c}. After standard calculations

$$\Pi_\mu^{A,T}(p^2, p'^2, q^2) = \frac{\langle 0|J_{D_{s0}^*}|D_{s0}^*(p')\rangle\langle D_{s0}^*(p')|J_\mu^{A,T}|B_c(p)\rangle\langle B_c(p)|J_{B_c}^\dagger|0\rangle}{(m_{D_{s0}^*}^2 - p'^2)(m_{B_c}^2 - p^2)} + \text{higher resonances and continuum states} \quad (6)$$

is obtained. The decay constants $f_{D_{s0}^*}$ and f_{B_c} are defined as

$$\langle 0|J_{D_{s0}^*}|D_{s0}^*\rangle = f_{D_{s0}^*}m_{D_{s0}^*}, \qquad \langle 0|J_{B_c}|B_c\rangle = -i\frac{f_{B_c}m_{B_c}^2}{m_b+m_c}. \quad (7)$$

Using Eqs. (4) and (7) in Eq. (6), we obtain

$$\Pi_\mu^A(p^2, p'^2, q^2) = \frac{f_{B_c}m_{B_c}^2}{(m_b+m_c)}\frac{f_{D_{s0}^*}m_{D_{s0}^*}}{(m_{D_{s0}^*}^2 - p'^2)(m_{B_c}^2 - p^2)} \times[f_1 P_\mu + f_2 q_\mu] + \text{excited states}, \quad (8)$$

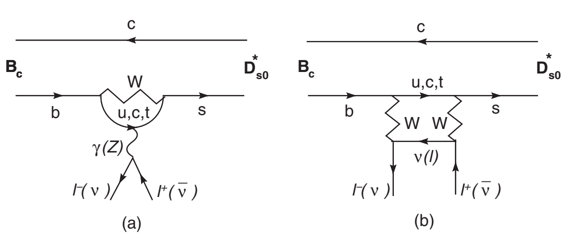

FIG. 1. Semileptonic decay of B_c involving D_{s0}^*. The electroweak penguin and box diagrams are shown in parts (a) and (b), respectively.

$$\Pi^T_\mu(p^2,p'^2,q^2) = \frac{f_{B_c} m^2_{B_c}}{(m_b+m_c)} \frac{f_{D^*_{s0}} m_{D^*_{s0}}}{(m^2_{D^*_{s0}}-p'^2)(m^2_{B_c}-p^2)} \times \left[\frac{f_T}{m_{B_c}+m_{D^*_{s0}}} \times [q^2 P_\mu - (m^2_{B_c}-m^2_{D^*_{s0}})q_\mu]\right] + \text{excited states.} \quad (9)$$

To calculate the form factors $f_1(q^2)$, $f_2(q^2)$, and $f_T(q^2)$, we will choose the structures P_μ, q_μ from Π^A_μ and q_μ from Π^T_μ, respectively.

On the QCD side, using the operator product expansion (OPE), we can obtain the correlation function in quark-gluon language in the deep Euclidean region where $p^2 \ll (m_b+m_c)^2$, $p'^2 \ll (m_c^2+m_s^2)$. For this aim, the correlators are written as

$$\Pi^A_\mu(p^2,p'^2,q^2) = \Pi^A_1(p^2,p'^2,q^2)P_\mu + \Pi^A_2(p^2,p'^2,q^2)q_\mu, \quad (10)$$
$$\Pi^T_\mu(p^2,p'^2,q^2) = \Pi^T_T(p^2,p'^2,q^2)q_\mu,$$

where each Π_i function is defined in terms of the perturbative and nonperturbative parts as

$$\Pi_i(p^2,p'^2,q^2) = \Pi^{\text{per}}_i(p^2,p'^2,q^2) + \Pi^{\text{nonper}}_i(p^2,p'^2,q^2), \quad (11)$$

where i stands for 1, 2, and T. To obtain the perturbative part of the correlation function, we should study the bare loop diagrams in Fig. 1. In calculating the bare loop contributions, we first write the double dispersion representation for the coefficients of the corresponding Lorentz structures appearing in each correlation function as

$$\Pi^{\text{per}}_i = -\frac{1}{(2\pi)^2}\int ds'\int ds \frac{\rho_i(s,s',q^2)}{(s-p^2)(s'-p'^2)} + \text{subtraction terms}, \quad (12)$$

where $\rho^{\text{per}}_i(s,s',q^2)$ are the spectral densities. Using the usual Feynman integral, the spectral densities can be calculated with the help of the Cutkosky rules, i.e., by replacing the propagators with Dirac-delta functions:

$$\frac{1}{p^2-m^2} \to -2i\pi\delta(p^2-m^2),$$

which implies that all quarks are real. After some straightforward calculations for the corresponding spectral densities, we obtain

$$\rho^A_1(s,s',q^2) = -I_0N_c\{\Delta+\Delta'-2m_c[(2+E_1+E_2)m_c+(1+E_1+E_2)m_s] + 2m_b[(1+E_1+E_2)m_c+(E_1+E_2)m_s]+(E_1+E_2)u\},$$
$$\rho^A_2(s,s',q^2) = I_0N_c\{\Delta-\Delta'+2m_c[(E_1-E_2+1)m_s+(E_1-E_2)m_c] - 2m_b[(E_1-E_2-1)m_c+(E_1-E_2)m_s]+(E_2-E_1)u\},$$
$$\rho^T_T(s,s',q^2) = I_0N_c\{\Delta(m_b-2m_c-m_s)+\Delta'(2m_c-m_b+m_s)+2[m_c(E_2-E_1+1)+m_s(E_2-E_1)]s + 2[m_b(E_1-E_2)+m_c(E_2-E_1-1)]s'+(E_1-E_2)(2m_c-m_b+m_s)u\}, \kappa \quad (13)$$

where

$$I_0(s,s',q^2) = \frac{1}{4\lambda^{1/2}(s,s',q^2)},$$
$$\lambda(s,s',q^2) = s^2+s'^2+q^4-2sq^2-2s'q^2-2ss',$$
$$E_1 = \frac{1}{\lambda(s,s',q^2)}[2s'\Delta-\Delta'u],$$
$$E_2 = \frac{1}{\lambda(s,s',q^2)}[2s\Delta'-\Delta u], \qquad u = s+s'-q^2,$$
$$\Delta = s+m_c^2-m_b^2, \qquad \Delta' = s'+m_c^2-m_s^2, \quad (14)$$

and $N_c = 3$ is the color factor.

The integration region in Eq. (12) is obtained requiring that the argument of three delta vanish, simultaneously. The physical region in the s and s' plane is described by the following inequalities:

$$-1 \le \frac{2ss'+(s+s'-q^2)(m_b^2-s-m_c^2)+(m_c^2-m_s^2)2s}{\lambda^{1/2}(m_b^2,s,m_c^2)\lambda^{1/2}(s,s',q^2)} \le +1. \quad (15)$$

III. GLUON CONDENSATE CONTRIBUTIONS

With the help of the operator product expansion (OPE), in the deep Euclidean region where $p^2 \ll (m_b+m_c)^2$ and $p'^2 \ll m_c^2$, the vacuum expectation value of the expansion of the correlation function in terms of the local operators, is written as follows:

$$\Pi_\mu(p^2,p'^2,q^2) = (C_0)_\mu + (C_3)_\mu\langle\bar{q}q\rangle + (C_4)_\mu\langle G^a_{\alpha\beta}G^{a\alpha\beta}\rangle + (C_5)_\mu\langle\bar{q}\sigma_{\alpha\beta}T^aG^{a\alpha\beta}q\rangle + (C_6)_\mu\langle\bar{q}\Gamma q\bar{q}\Gamma' q\rangle + \cdots, \quad (16)$$

where $(C_i)_\mu$ are the Wilson coefficients, $G^a_{\alpha\beta}$ is the gluon field strength tensor, Γ and Γ' are the matrices appearing in the calculations. The nonperturbative part contains the quark and gluon condensate diagrams. We consider the condensate terms of dimension 3, 4, and 5. It is found that the heavy quark condensate contributions are suppressed by inverse of the heavy quark mass and can be safely omitted. So there are three diagrams with mass dimension 3 and 5. These diagrams are shown in Fig. 2. Let us consider the condensate contribution of quark-quark. After some calculations for diagram (a), we have:

$$(C_3)_\mu = -\frac{1}{4}\,\mathrm{Tr}[F(p,k)] + \frac{m_u}{16} \times \mathrm{Tr}\Big[\Big(\frac{\partial}{\partial p^\alpha}+\frac{\partial}{\partial k^\alpha}\Big)F(p,k)\gamma_\alpha\Big] + \frac{1}{32}\Big(m_u^2-\frac{m_0^2}{2}\Big) \times \mathrm{Tr}\Big[\Big(\frac{\partial^2}{\partial p^\alpha\partial k^\alpha}+\frac{\partial^2}{(\partial p^\alpha)^2}+\frac{\partial^2}{(\partial k^\alpha)^2}\Big)F(p,k)\Big], \quad (17)$$

where $m_0^2 = 0.8 \pm 0.2\ \mathrm{GeV}^2$ and $F(p,k)$ is

$$F(p,k) = \Big(\gamma_\mu(1-\gamma_5)\frac{i}{(\not p+\not k)-m_b}\gamma_5\frac{i}{\not k - m_c}\Big), \quad (18)$$

for the $\gamma_\mu(1-\gamma_5)$ vertex, and is

$$F(p,k) = \Big(\sigma_{\mu\nu}q^\nu(1+\gamma_5)\frac{i}{(\not p+\not k)-m_b}\gamma_5\frac{i}{\not k - m_c}\Big), \quad (19)$$

for the $\sigma_{\mu\nu}q^\nu(1+\gamma_5)$ vertex. In Eqs. (18) and (19), k is the momentum of the c quark. The contribution of this diagram is zero after applying the double Borel transformation with respect to both variables p^2 and p'^2, because only one variable appears in the denominator. In a similar way, it can be shown that the condensate contributions of other diagrams are zero after applying the double Borel transformation.

In the QCD sum rule method, OPE is truncated at some finite order such that Borel transformations play an important role in this cutting. Usually, the proper regions of the Borel parameters are adopted by demanding that in the truncated OPE, the condensate term with the highest dimension remains a small fraction of the sum of all terms. These regions keep the convergence of the condensate expansion under control and guarantee that one does not introduce a large error neglecting the higher-dimensional terms. In the numerical analysis section, we explain how these proper regions are obtained. So we will not consider the condensates with $d \geq 6$ that play a minor role in our calculations.

Therefore in this case, we consider the two-gluon condensate diagrams with mass dimension 4 as an important term of the nonperturbative corrections. The diagrams for contribution of the gluon condensates are depicted in Fig. 3.

To obtain the gluon condensate contributions, we will follow the same procedure as stated in [33]. The calculation of the gluon condensate diagrams are performed in the Fock-Schwinger fixed-point gauge, $x^\mu A_\mu^a = 0$, where A_μ^a is the gluon field. In calculation of these diagrams, the following type of integrals appear:

$$I_0(a,b,c) = \int \frac{d^4k}{(2\pi)^4} \times \frac{1}{[k^2-m_c^2]^a[(p+k)^2-m_b^2]^b[(p'+k)^2-m_s^2]^c},$$
$$I_\mu(a,b,c) = \int \frac{d^4k}{(2\pi)^4} \times \frac{k_\mu}{[k^2-m_c^2]^a[(p+k)^2-m_b^2]^b[(p'+k)^2-m_s^2]^c}, \quad (20)$$

where k is the momentum of the spectator quark c. These integrals can be calculated using the Schwinger representation for the Euclidean propagator, i.e.,

$$\frac{1}{[p^2+m^2]} = \frac{1}{\Gamma(\alpha)}\int_0^\infty d\alpha\,\alpha^{n-1}e^{-\alpha(p^2+m^2)}. \quad (21)$$

After Borel transformation using

$$B_{\hat p^2}(M^2)e^{-\alpha p^2} = \delta(1/M^2-\alpha), \quad (22)$$

we obtain

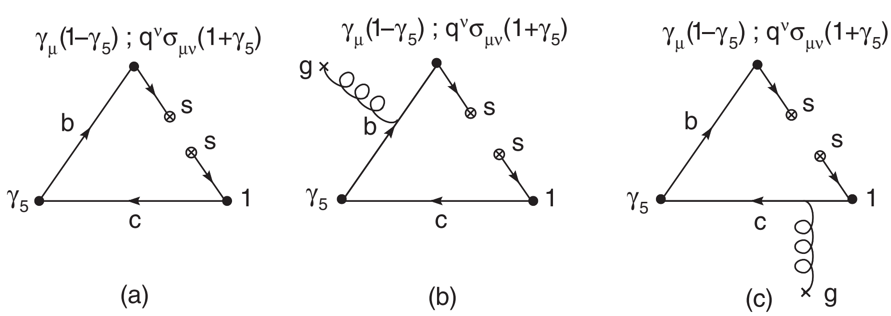

FIG. 2. Contribution of condensates with mass dimension 3 and 5. The quark-quark condensate and quark-gluon condensate are shown in parts (a) and (b, c) for $B_c \to D_{s0}^*$ transition, respectively.

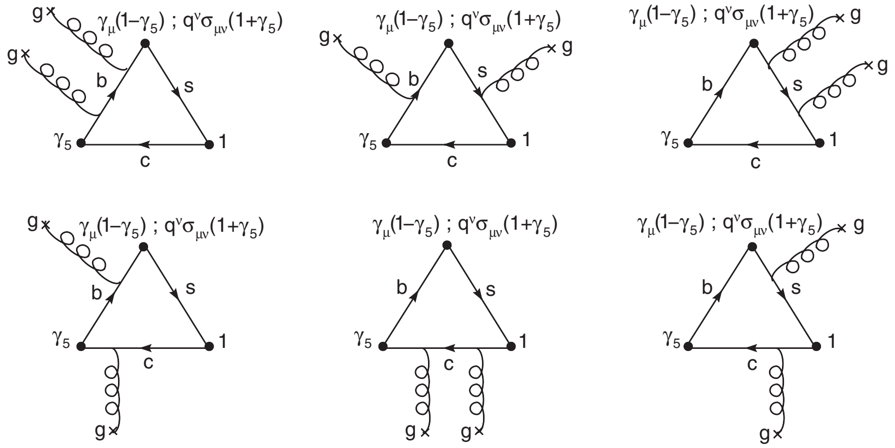

FIG. 3. Contribution of two-gluon condensates for the $B_c \to D_{s0}^*$ transition.

$$\hat{I}_0(a,b,c) = \frac{(-1)^{a+b+c}}{\Gamma(a)\Gamma(b)\Gamma(c)16\pi^2}(M_1^2)^{2-a-c}(M_2^2)^{2-a-b} \times U_0(a+b+c-4, 1-c-b),$$
$$\hat{I}_\mu(a,b,c) = \frac{1}{2}[\hat{I}_1(a,b,c) + \hat{I}_2(a,b,c)]P_\mu + \frac{1}{2}[\hat{I}_1(a,b,c) - \hat{I}_2(a,b,c)]q_\mu, \tag{23}$$

where

$$\hat{I}_1(a,b,c) = \frac{i(-1)^{a+b+c+1}}{\Gamma(a)\Gamma(b)\Gamma(c)16\pi^2}(M_1^2)^{2-a-b} \times (M_2^2)^{3-a-c}U_0(a+b+c-5, 1-c-b),$$
$$\hat{I}_2(a,b,c) = \frac{i(-1)^{a+b+c+1}}{\Gamma(a)\Gamma(b)\Gamma(c)16\pi^2}(M_1^2)^{3-a-b} \times (M_2^2)^{2-a-c}U_0(a+b+c-5, 1-c-b). \tag{24}$$

$\hat{I}$ in the above equations represents the double Borel transformed form of integrals. M_1^2 and M_2^2 are the Borel parameters in the s and s' channels, respectively. We can define the function $U_0(\alpha, \beta)$ as

$$U_0(a,b) = \int_0^\infty dy(y + M_1^2 + M_2^2)^a y^b \times \exp\left[-\frac{B_{-1}}{y} - B_0 - B_1 y\right], \tag{25}$$

where

$$B_{-1} = \frac{1}{M_1^2 M_2^2}(m_s^2 M_1^4 + m_b^2 M_2^4 + M_2^2 M_1^2(m_b^2 + m_s^2 - q^2)),$$
$$B_0 = \frac{1}{M_1^2 M_2^2}(M_1^2(m_s^2 + m_c^2) + M_2^2(m_b^2 + m_c^2)),$$
$$B_1 = \frac{m_c^2}{M_1^2 M_2^2}. \tag{26}$$

After straightforward but lengthy calculations, we get the following results for the gluon condensate contributions:

$$\Pi_i^{\langle G^2\rangle} = i\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{C_i^4}{6}, \tag{27}$$

where the explicit expressions for C_i^4 are given in the Appendix.

The QCD sum rules for the form factors f_1, f_2, and f_T are obtained by equating two representations of the correlation function and applying the Borel transformations with respect to the $p^2(p^2 \to M_1^2)$ and $p'^2(p'^2 \to M_1^2)$ on the phenomenological as well as the perturbative and nonperturbative parts of the correlation function in order to suppress the contributions of the higher states and continuum:

$$f_i'(q^2) = \frac{(m_b + m_c)}{f_{B_c} m_{B_c}^2 f_{D_{s0}^*} m_{D_{s0}^*}} e^{m_{B_c}^2/M_1^2} e^{m_{D_{s0}^*}^2/M_2^2}\left\{-\frac{1}{4\pi^2}\int_{(m_c+m_s)^2}^{s_0'} \times ds' \int_{s_L}^{s_0} ds\rho_i^{A,T}(s,s',q^2)e^{-s/M_1^2}e^{-s'/M_2^2} + iM_1^2M_2^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{C_i^4}{6}\right\}, \tag{28}$$

where $f_1(q^2) = f_1'(q^2)$, $f_2(q^2) = f_2'(q^2)$, and $f_T(q^2) = f_T'(q^2)/(m_{B_c} - m_{D_{s0}^*})$. s_0 and s_0' are the continuum thresholds in B_c and D_{s0}^* channels, respectively. From the inequality (15), to use in the lower limit of the integration over s, it is easy to express s in terms of s', i.e., s_L is

$$s_L = \frac{(m_c^2 + q^2 - m_b^2 - s')(m_b^2 s' - q^2 m_c^2)}{(m_b^2 - q^2)(m_c^2 - s')}. \tag{29}$$

We use the quark-hadron duality assumption to subtract the contributions of the higher states and the continuum as

$$\rho^{\text{higher states}}(s,s') = \rho^{\text{OPE}}(s,s')\theta(s - s_0)\theta(s' - s_0'). \tag{30}$$

Using Eq. (2), we get the following expressions for the differential decay widths in terms of form factors:

$$\frac{d\Gamma}{dq^2}(B_c \to D_{s0}^* \nu \bar{\nu}) = \frac{G_F^2 |V_{ts} V_{tb}^*|^2 m_{B_s}^3 \alpha^2}{2^8 \pi^5} \times |C_{10}|^2 \phi^{3/2}(1, \hat{r}, \hat{s}) |f_1(q^2)|^2, \quad (31)$$

the function $\phi(1, \hat{r}, \hat{s})$ is given as

$$\phi(1, \hat{r}, \hat{s}) = 1 + \hat{r}^2 + \hat{s}^2 - 2\hat{r} - 2\hat{s} - 2\hat{r}\,\hat{s},$$

where

$$\hat{r} = \frac{m_{D_{s0}^*}^2}{m_{B_c}^2}, \qquad \hat{s} = \frac{q^2}{m_{B_c}^2},$$

and

$$\frac{d\Gamma}{dq^2}(B_c \to D_{s0}^* l^+ l^-) = \frac{G_F^2 |V_{ts} V_{tb}^*|^2 m_{B_c}^3 \alpha^2}{3 \cdot 2^9 \pi^5} \upsilon \phi^{1/2}(1, \hat{r}, \hat{s}) \times \left[\left(1 + \frac{2\hat{l}}{\hat{s}}\right)\right] \phi(1, \hat{r}, \hat{s}) \alpha_1 + 12 \hat{l} \beta_1 \Big], \quad (32)$$

where $\hat{l} = m_l^2/m_{B_c}^2$ and the expressions of α_1 and β_1 and υ are given as

$$\upsilon = \sqrt{1 - \frac{4m_l^2}{q^2}},$$

$$\alpha_1 = \left| C_9^{\mathrm{eff}} f_1(q^2) + \frac{2\hat{m}_b C_7^{\mathrm{eff}} f_T(q^2)}{1 + \sqrt{\hat{r}}} \right|^2 + |C_{10} f_1(q^2)|^2,$$

$$\beta_1 = |C_{10}|^2 \left[\left(1 + \hat{r} - \frac{\hat{s}}{2}\right) |f_1(q^2)|^2 + (1 - \hat{r}) \mathrm{Re}(f_1(q^2) f_2^*(q^2)) + \frac{1}{2} \hat{s} |f_2(q^2)|^2 \right],$$

where $\hat{m}_b = m_b/m_{B_c}$.

IV. NUMERICAL ANALYSIS

This section encompasses our numerical analysis of the form factors $f_1(q^2)$, $f_2(q^2)$, and $f_T(q^2)$, branching fractions, longitudinal lepton polarization asymmetries, and discussion. The sum rules expressions of the form factors depict that the main input parameters entering the expressions are Wilson coefficients C_7^{eff}, C_9^{eff}, and C_{10}; elements of the CKM matrix V_{tb} and V_{ts}; leptonic decay constants f_{B_c} and $f_{D_{s0}^*}$; Borel parameters M_1^2 and M_2^2, as well as the continuum thresholds s_0 and s_0'. We choose the values of the gluon condensate, leptonic decay constants, Wilson coefficients C_7^{eff}, C_9^{eff}, and C_{10}, CKM matrix elements, and quark and meson masses as $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012\ \mathrm{GeV}^4$ [34], $f_{D_{s0}^*} = (225 \pm 25)$ MeV [23], $f_{B_c} = (350 \pm 25)$ MeV [35], $C_7^{\mathrm{eff}} = -0.313$, $C_9^{\mathrm{eff}} = 4.344$, $C_{10} = -4.669$ [36], $|V_{tb}| = (0.77 \pm 0.18)$, $|V_{ts}| = (40.6 \pm 2.7) \times 10^{-3}$, $m_s = (104^{+26}_{-34})$ MeV, $m_c = (1270^{+70}_{-110})$ MeV, $m_b = (4200^{+170}_{-70})$ MeV, $m_{D_{s0}^*} = (2318.8 \pm 0.6)$ MeV, and $m_{B_c} = (6276 \pm 4)$ MeV [37].

Note that the values of the decay constants of the B_c and D_{s0}^* mesons have been obtained from the two-point QCD sum rules method.

The expressions for the form factors contain also four auxiliary parameters: Borel mass squares M_1^2 and M_2^2 and continuum threshold s_0 and s_0'. These are not physical quantities, so the physical quantities, form factors, should be independent of them. The parameters s_0 and s_0', which are the continuum thresholds of the B_c and D_{s0}^* mesons, respectively, are determined from the conditions that guarantee the sum rules to have the best stability in the allowed M_1^2 and M_2^2 region. The values of continuum thresholds calculated from the two-point QCD sum rules are taken to be $s_0 = 35\ \mathrm{GeV}^2$ [35,38] and $s_0' = (2.5\ \mathrm{GeV})^2$ [23]. The working regions for M_1^2 and M_2^2 are determined by requiring that not only the contributions of the higher states and continuum are effectively suppressed, but guaranteeing that the contributions of higher dimensional operators are small. Both conditions are satisfied in the regions $12\ \mathrm{GeV}^2 \le M_1^2 \le 25\ \mathrm{GeV}^2$ and $8\ \mathrm{GeV}^2 \le M_2^2 \le 14\ \mathrm{GeV}^2$.

The dependence of the form factors f_1, f_2, and f_T on M_1^2 and M_2^2 for $B_c \to D_{s0}^* l^+ l^- / \nu \bar{\nu}$ decays are shown in Fig. 4. This figure shows a good stability of the form factors with respect to the Borel mass parameters in the working regions. Our numerical analysis shows that the contribution of the nonperturbative part is about 9% of the total and the main contribution comes from the perturbative part of the form factors.

The values of the form factors at $q^2 = 0$ are given in Table I:

The sum rules for the form factors are truncated at about $4\ \mathrm{GeV}^2$ below the perturbative cut for the $B_c \to D_{s0}^*$; so to extend our results to the full physical region, we look for parametrization of the form factors in such a way that in the region $0 \le q^2 \le (m_{B_c} - m_{D_{s0}^*})^2$, this parametrization coincides with the sum rules prediction. Our numerical calculations show that the sufficient parametrization of the form factors with respect to q^2 is

$$f_i(q^2) = \frac{f_i(0)}{1 + \alpha \hat{q} + \beta \hat{q}^2}, \quad (33)$$

where $\hat{q} = q^2/m_{B_c}^2$. The values of the parameters $f_i(0)$, α, and β are given in the Table II. The dependence of the form factors $f_1(q^2)$, $f_2(q^2)$, and $f_T(q^2)$ on q^2 are given in Fig. 5.

Now we would like to present the values of the branching ratios for the considered decays. Integrating Eqs. (31) and (32) over q^2 in the whole physical region and using the total mean life time $\tau_{B_c} = (0.46 \pm 0.07)$ ps [37], the branching ratios of the $B_c \to D_{s0}^* \nu \bar{\nu}$ and $B_c \to D_{s0}^* l^+ l^-$ are obtained. The differential decay branching ratios for the $B_c \to D_{s0}^* l^+ l^- / \nu \bar{\nu}$ decays as functions of q^2 are shown in Figs. 6–8. The results for electron and muon are approximately the same, so we consider $l = \mu$, τ. Also the branching ratio values of these decays are obtained as

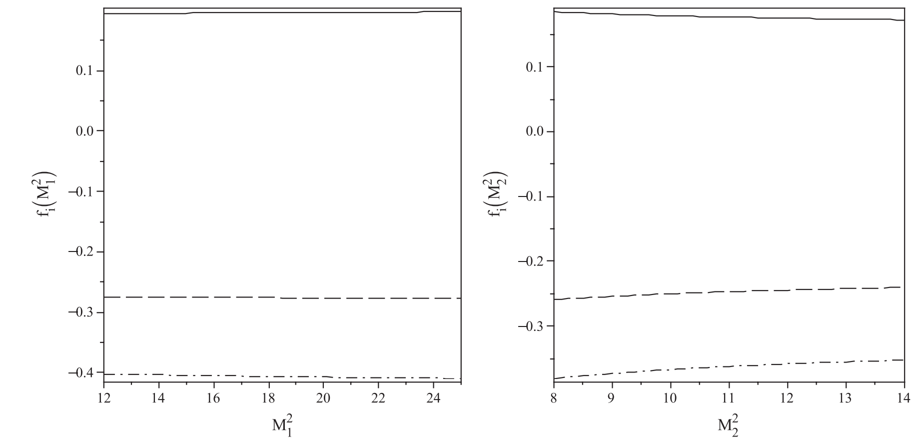

FIG. 4. The dependence of the form factors on M_1^2 and M_2^2 for $B_c \to D_{s0}^*$ decay. The solid line corresponds to the f_1, the dash line f_2, and the dash dot line f_T.

presented in Table III, when only the short distance (SD) effects are considered.

For the heavy quarkonium $b\bar{c}$, where the relative velocity of quark movement is small, an essential role is taken by the Coulomb-like α_s/v-corrections [27]. It leads to the finite renormalization for $\rho_i^{(A,T)}$, $(i = 1, 2, T)$ so that

TABLE I. The values of the form factors at $q^2 = 0$ for $M_1^2 = 18\ \text{GeV}^2$ and $M_2^2 = 10\ \text{GeV}^2$.

$f_i(0)$	Value
$f_1(0)$	0.20 ± 0.05
$f_2(0)$	-0.28 ± 0.07
$f_T(0)$	-0.41 ± 0.10

TABLE II. Parameters appearing in the form factors for $M_1^2 = 18\ \text{GeV}^2$, $M_2^2 = 10\ \text{GeV}^2$.

$f_i(q^2)$	$f(0)$	α	β
$f_1(q^2)$	0.20	1.10	−1.84
$f_2(q^2)$	−0.28	−0.81	−2.98
$f_T(q^2)$	−0.41	−0.77	−2.47

$$\rho_i^c = \mathcal{C}\rho_i^{(A,T)}, \tag{34}$$

with

$$\mathcal{C}^2 = \frac{4\pi\alpha_s^{\mathcal{C}}}{3v}\frac{1}{1 - \exp(-\frac{4\pi\alpha_s^{\mathcal{C}}}{3v})}, \tag{35}$$

where $\alpha_s^{\mathcal{C}}$ is the coupling constant of effective Coulomb interactions. Also v is the relative velocity of quarks in the $b\bar{c}$-system,

$$v = \sqrt{1 - \frac{4m_b m_c}{p^2 - (m_b - m_c)^2}}. \tag{36}$$

The value of the $\alpha_s^{\mathcal{C}}$ for the B_c meson is [27]

$$\alpha_s^{\mathcal{C}}[b\bar{c}] = 0.45.$$

Considering the Coulomb corrections, the value of $f_i(0)$ is given in Table IV. The values of the branching ratios for $B_c \to D_{s0}^* l^+ l^- / \nu\bar{\nu}$ decays, by considering Coulomb corrections, are also presented in Table V.

Note that, the long-distance (LD) effects for the charged lepton modes are not included in the values of Table V. With the LD effects, we introduce some cuts close to $q^2 = 0$ and around the resonances of J/ψ and ψ' and study the

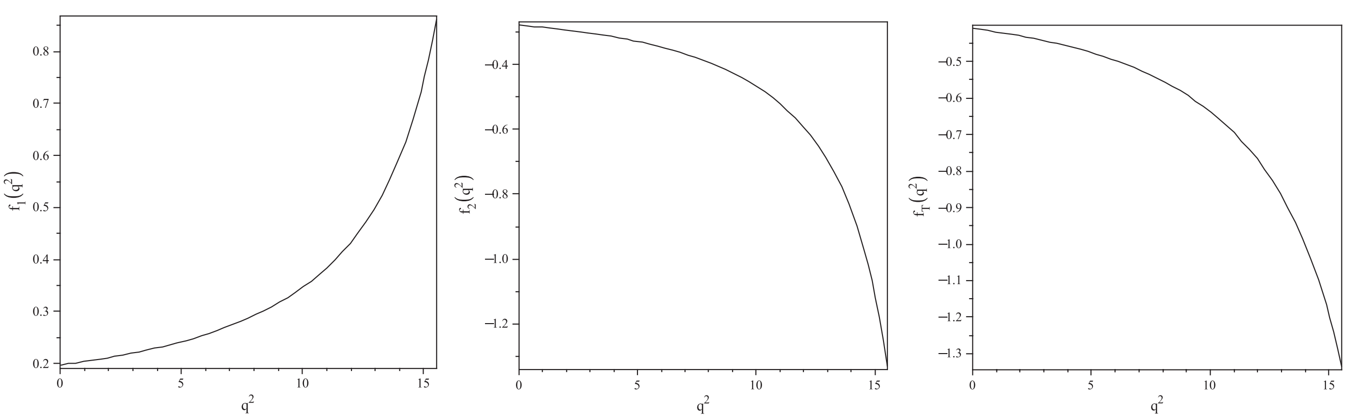

FIG. 5. The dependence of the form factors on q^2 at $M_1^2 = 18\ \text{GeV}^2$ and $M_2^2 = 10\ \text{GeV}^2$ for the $B_c \to D_{s0}^*$ transition.

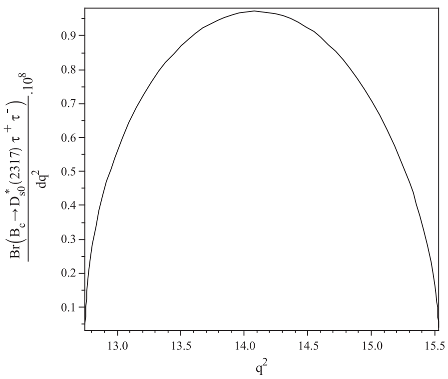

FIG. 6. The dependence of the differential decay branching fraction of the $B_c \to D_{s0}^{*}\tau^{+}\tau^{-}$ decay on q^2.

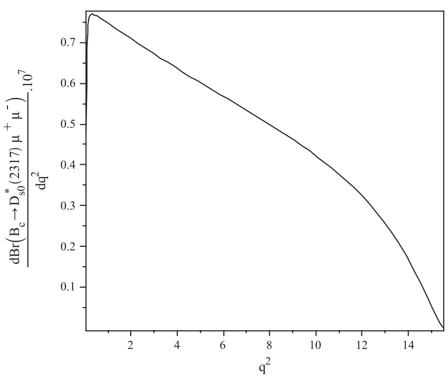

FIG. 7. The same as Fig. 6 but for the $B_c \to D_{s0}^{*}\mu^{+}\mu^{-}$.

three regions as follows:

$$I: \sqrt{q^2_{\min}} \leq \sqrt{q^2} \leq M_{J/\psi} - 0.20,$$

$$II: M_{J/\psi} + 0.04 \leq \sqrt{q^2} \leq M_{\psi'} - 0.10,$$

$$III: M_{\psi'} + 0.02 \leq \sqrt{q^2} \leq m_{B_c} - m_{D_{s1}}, \qquad (37)$$

where $\sqrt{q^2_{\min}} = 2m_l$. In Table VI, we present the branching ratios in terms of the regions shown in Eq. (37).

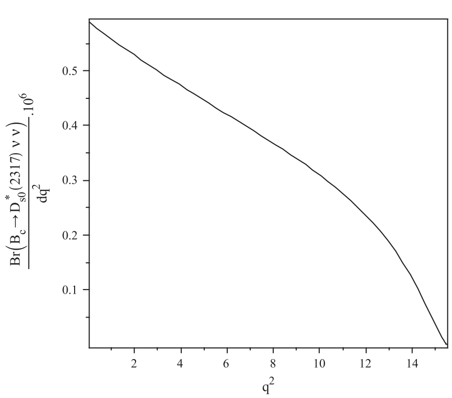

FIG. 8. The same as Fig. 6 but for the $B_c \to D_{s0}^{*}\nu\bar{\nu}$.

TABLE III. Our values for the branching fractions of the mentioned decays by considering $M_1^2 = 18\ \text{GeV}^2$ and $M_2^2 = 10\ \text{GeV}^2$.

Modes	Br
$B_c \to D_{s0}^{*}\nu\bar{\nu}$	$(3.06 \pm 0.76) \times 10^{-7}$
$B_c \to D_{s0}^{*}e^{+}e^{-}$	$(3.79 \pm 0.94) \times 10^{-8}$
$B_c \to D_{s0}^{*}\mu^{+}\mu^{-}$	$(3.76 \pm 0.92) \times 10^{-8}$
$B_c \to D_{s0}^{*}\tau^{+}\tau^{-}$	$(1.28 \pm 0.32) \times 10^{-9}$

TABLE IV. The values of the form factors at $q^2 = 0$ by considering Coulomb corrections.

$f_i(q^2)$	Value
$f_1(0)$	0.46 ± 0.10
$f_2(0)$	-0.65 ± 0.16
$f_T(0)$	-0.94 ± 0.25

The errors are estimated by the variation of the Borel parameters M_1^2 and M_2^2, the variation of the continuum thresholds s_0 and s_0', and the variation of b and c quark masses and leptonic decay constants f_{B_c} and $f_{D_{s0}^{*}}$. The main uncertainty comes from the thresholds and the decay constants, which is about ~21% of the central value, while the other uncertainties are small, constituting a few percent.

Finally, we consider the longitudinal lepton polarization asymmetries for these decays. We have [39]

$$P_L = \frac{2v}{(1 + \frac{2\hat{l}}{\hat{s}})\phi(1, \hat{r}, \hat{s})\alpha_1 + 12\hat{l}\beta_1} \text{Re}\Big[\phi(1, \hat{r}, \hat{s}) \times \Big(C_9^{\text{eff}} f_1(q^2) - \frac{2C_7 f_T(q^2)}{1 + \sqrt{\hat{r}}}\Big)(C_{10} f_1(q^2))^{*}\Big], \qquad (38)$$

where v, $\hat{l}$, $\hat{r}$, $\hat{s}$, $\phi(1, \hat{r}, \hat{s})$, α_1, and β_1 were defined before. The dependence of the longitudinal lepton polarization asymmetries of the $B_c \to D_{s0}^{*}$ decays on the transferred momentum square q^2 are plotted in Fig. 9 in the region $0 \leq q^2 \leq (m_{B_c} - m_{D_{s0}^{*}})^2\ \text{GeV}^2$. Note that the result for the electron mode is similar to this for the muon mode.

Any experimental measurements on the branching fractions of these decays and those comparisons with the results of the phenomenological models like QCD sum rules could give valuable information about the nature of the $D_{s0}^{*}(2317)$ meson and strong interactions inside them.

TABLE V. The values for the branching fractions of $B_c \to D_{s0}^{*}l^{+}l^{-}/\nu\bar{\nu}$ decays, by considering Coulomb corrections.

Modes	Br
$B_c \to D_{s0}^{*}\nu\bar{\nu}$	$(1.62 \pm 0.46) \times 10^{-6}$
$B_c \to D_{s0}^{*}e^{+}e^{-}$	$(2.01 \pm 0.59) \times 10^{-7}$
$B_c \to D_{s0}^{*}\mu^{+}\mu^{-}$	$(1.98 \pm 0.57) \times 10^{-7}$
$B_c \to D_{s0}^{*}\tau^{+}\tau^{-}$	$(7.80 \pm 2.26) \times 10^{-9}$

TABLE VI. The branching ratios of the semileptonic $B_c \to D_{s0}^* l^+ l^- / \nu\bar{\nu}$ decays including LD effects.

Modes	I	II	III
$\mathrm{Br}(B_c \to D_{s0}^* e^+ e^-)$	$(1.43 \pm 0.39) \times 10^{-7}$	$(3.31 \pm 0.86) \times 10^{-8}$	$(1.89 \pm 0.51) \times 10^{-9}$
$\mathrm{Br}(B_c \to D_{s0}^* \mu^+ \mu^-)$	$(1.42 \pm 0.38) \times 10^{-7}$	$(3.30 \pm 0.85) \times 10^{-8}$	$(1.90 \pm 0.52) \times 10^{-9}$
$\mathrm{Br}(B_c \to D_{s0}^* \tau^+ \tau^-)$	undefined	$(1.54 \pm 0.42) \times 10^{-9}$	$(2.44 \pm 0.64) \times 10^{-9}$

In summary, the $B_c \to D_{s0}^* l^+ l^- / \nu\bar{\nu}$ decays were considered. We computed the relevant form factors considering the contributions of the gluon condensate corrections. Also the total decay widths and the branching fractions of these decays were evaluated. Finally, the dependence of the longitudinal lepton polarization asymmetries of the $B_c \to D_{s0}^*$ transition on the transferred momentum square q^2 were plotted. Detection of these channels and their comparison with the phenomenological models like QCD sum rules could give useful information about the structure of the D_{s0}^* scalar meson.

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APPENDIX

In this appendix, the explicit expressions of the coefficients of the gluon condensate entering the sum rules of the form factors $f_i(q^2)$, $(i = 1, 2, T)$ are given.

$$
\begin{aligned}
C_1^4 :=\; & -10\hat{I}_0^{[0,1]}(3,2,2)m_c{}^2m_b{}^3 + 10\hat{I}_0(2,3,1)m_c{}^2m_b{}^3 + 10\hat{I}_0(2,2,2)m_c{}^2m_b{}^3 + 10\hat{I}_0(3,2,1)m_b{}^5 + 10\hat{I}_0(2,3,1)m_b{}^5 \\
& - 30\hat{I}_0(1,4,1)m_b{}^5 - 10\hat{I}_0(3,1,1)m_c{}^3 + 10\hat{I}_0^{[0,1]}(3,2,1)m_c{}^2m_b - 10\hat{I}_0(2,2,2)m_c{}^2m_b{}^3 + 30\hat{I}_0(2,3,1)m_c{}^2m_b{}^3 \\
& + 10\hat{I}_0^{[0,1]}(3,2,2)m_c{}^2m_b{}^3 + 10\hat{I}_0(3,1,1)m_c{}^2m_b - 10\hat{I}_0^{[0,1]}(3,1,2)m_c{}^2m_b + 10\hat{I}_0(2,1,2)m_c{}^2m_b + 10\hat{I}_0(1,2,2)m_b{}^3 \\
& + 15\hat{I}_1(4,1,1)m_c{}^4 - 5\hat{I}_2(3,2,1)m_c{}^4 - 5\hat{I}_2(3,1,2)m_c{}^4 + 5\hat{I}_1(3,1,2)m_c{}^4 + 15\hat{I}_1(2,2,2)m_c{}^4 - 15\hat{I}_2(4,1,1)m_c{}^4 \\
& + 15\hat{I}_2^{[0,1]}(3,2,2)m_c{}^4 - 15\hat{I}_1^{[0,1]}(3,2,2)m_c{}^4 - 15\hat{I}_2(2,2,2)m_c{}^4 + 5\hat{I}_1(2,1,3)m_c{}^4 - 5\hat{I}_1(3,2,1)m_c{}^3m_b \\
& - 5\hat{I}_1(3,1,2)m_c{}^3m_b + 10\hat{I}_0(2,2,2)m_c{}^3m_b - 5\hat{I}_0(3,2,1)m_c{}^3m_b - 10\hat{I}_1(2,2,2)m_c{}^3m_b - 10\hat{I}_2(2,3,1)m_c{}^3m_b \\
& + 10\hat{I}_1^{[0,1]}(3,2,2)m_c{}^3m_b - 5\hat{I}_2(3,2,1)m_c{}^3m_b + 10\hat{I}_0(2,3,1)m_c{}^3m_b - 5/2\hat{I}_1(2,1,3)m_c{}^3m_b + 15\hat{I}_0(4,1,1)m_c{}^3m_b \\
& - 15\hat{I}_1(4,1,1)m_c{}^3m_b + 5\hat{I}_0(3,1,2)m_c{}^3m_b - 10\hat{I}_1^{[0,1]}(3,2,2)m_c{}^3m_b + 10\hat{I}_1(2,3,1)m_c{}^3m_b - 5\hat{I}_2(3,1,2)m_c{}^3m_b \\
& + 15\hat{I}_2(4,1,1)m_c{}^3m_b + 5\hat{I}_2(2,1,3)m_c{}^3m_b - 10\hat{I}_2^{[0,1]}(3,2,2)m_c{}^3m_b + 10\hat{I}_2(2,2,2)m_c{}^3m_b - 5\hat{I}_0(2,2,2)m_c{}^2m_b{}^2
\end{aligned}
$$

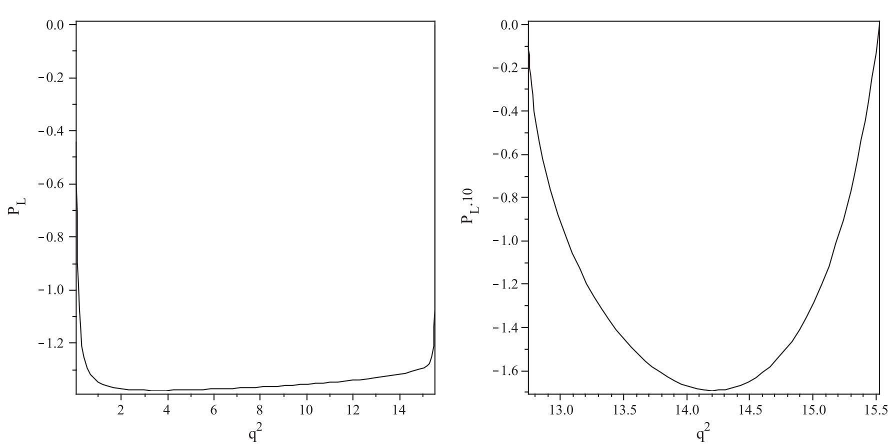

FIG. 9. Longitudinal lepton polarization asymmetry on q^2. The left figure shows this quantity for the $B_c \to D_{s0}^* \mu^+ \mu^-$ decay and the right belongs to the $B_c \to D_{s0}^* \tau^+ \tau^-$.

$$
\begin{aligned}
&- 10\hat{I}_2(3,2,1)m_c{}^2m_b{}^2 + 30\hat{I}_2(1,4,1)m_c{}^2m_b{}^2 + 10\hat{I}_1(3,2,1)m_c{}^2m_b{}^2 + 5\hat{I}_1^{[0,1]}(3,2,2)m_c{}^2m_b{}^2 - 30\hat{I}_1(1,4,1)m_c{}^2m_b{}^2\\
&- 10\hat{I}_1(3,2,1)m_cm_b{}^3 - 10\hat{I}_1(2,3,1)m_cm_b{}^3 + 5\hat{I}_1^{[0,1]}(3,2,2)m_cm_b{}^3 - 30\hat{I}_0(1,4,1)m_cm_b{}^3 - 20\hat{I}_0^{[0,1]}(2,2,2)m_cm_b{}^2\\
&+ 10\hat{I}_0^{[0,2]}(3,2,2)m_cm_b{}^2 - 10\hat{I}_0(1,2,2)m_b{}^3 - 10\hat{I}_0(3,1,1)m_b{}^3 + 10\hat{I}_0(2,1,2)m_b{}^3 - 10\hat{I}_0^{[0,1]}(2,3,1)m_b{}^3 - 50\hat{I}_0(1,3,1)m_b{}^3\\
&- 10\hat{I}_0^{[0,1]}(2,2,2)m_b{}^3 + 20\hat{I}_0^{[0,1]}(2,2,1)m_c - 10\hat{I}_0^{[0,2]}(3,2,1)m_c - 20\hat{I}_0^{[0,1]}(2,1,2)m_c + 10\hat{I}_0^{[0,2]}(3,1,2)m_c + 5\hat{I}_1^{[0,1]}(2,2,2)m_b{}^2\\
&+ 10\hat{I}_2^{[0,1]}(2,2,2)m_b{}^2 + 10\hat{I}_2^{[0,1]}(3,2,1)m_b{}^2 - 5\hat{I}_1(2,2,1)m_b{}^2 - 5\hat{I}_2^{[0,2]}(3,2,2)m_b{}^2 + 10\hat{I}_0(2,2,1)m_b{}^2 - 10\hat{I}_1^{[0,1]}(3,2,1)m_b{}^2\\
&- 5\hat{I}_0(3,1,1)m_b{}^2 - 10\hat{I}_1^{[0,1]}(2,2,2)m_b{}^2 - 30\hat{I}_2^{[0,1]}(1,4,1)m_b{}^2 + 30\hat{I}_1^{[0,1]}(1,4,1)m_b{}^2 - 5\hat{I}_0(2,1,2)m_b{}^2 + 15\hat{I}_2^{[0,1]}(1,3,1)\\
&+ 5\hat{I}_1^{[0,2]}(2,1,3) - 10\hat{I}_2(1,2,1) - 15\hat{I}_2^{[0,1]}(1,3,1) + 5\hat{I}_2^{[0,1]}(2,2,1) - 10\hat{I}_1(1,2,1) - 15\hat{I}_1^{[0,1]}(1,3,1) - 5\hat{I}_1^{[0,1]}(1,1,3)\\
&+ 5\hat{I}_2^{[0,1]}(1,1,3) - 10\hat{I}_1^{[0,1]}(1,2,2) + 5\hat{I}_1^{[0,1]}(2,1,2) + 15\hat{I}_1^{[0,2]}(2,2,2) - 15\hat{I}_2^{[0,2]}(2,2,2) - 5\hat{I}_2^{[0,2]}(2,1,3) + 10\hat{I}_2^{[0,1]}(1,2,2)\\
&+ 10\hat{I}_1^{[0,2]}(3,1,2) - 10\hat{I}_2^{[0,2]}(3,1,2) - 5\hat{I}_1^{[0,1]}(2,2,1) - 5\hat{I}_1^{[0,1]}(2,2,1) + 15\hat{I}_1^{[0,1]}(1,3,1) + 10\hat{I}_0(3,1,1)m_b{}^3 - 10\hat{I}_0(2,1,1)m_c\\
&+ 10\hat{I}_0^{[0,1]}(3,1,1)m_c - 10\hat{I}_0^{[0,1]}(2,1,2)m_b - 30\hat{I}_0(1,2,1)m_b - 10\hat{I}_0(1,1,3)m_b + 10\hat{I}_0(3,2,1)m_b + 10\hat{I}_0^{[0,1]}(2,2,2)m_b\\
&+ 10\hat{I}_0(1,1,2)m_b,
\end{aligned}
$$

$$
\begin{aligned}
C_2^4 :=\ & 10\hat{I}_0(3,2,2)m_c{}^5m_b{}^2 - 10\hat{I}_0(3,2,2)m_c{}^3m_b{}^4 + 10\hat{I}_0(3,1,2)m_c{}^5 - 10\hat{I}_0(3,2,1)m_c{}^5 + 30\hat{I}_0(4,1,1)m_c{}^3m_b{}^2\\
&- 20\hat{I}_0^{[0,1]}(3,2,2)m_c{}^3m_b{}^2 + 20\hat{I}_0(2,2,2)m_c{}^3m_b{}^2 - 10\hat{I}_0(2,2,2)m_c{}^2m_b{}^3 + 30\hat{I}_0(2,3,1)m_c{}^2m_b{}^3\\
&+ 10\hat{I}_0^{[0,1]}(3,2,2)m_c{}^2m_b{}^3 - 10\hat{I}_0^{[0,1]}(3,2,2)m_cm_b{}^4 + 20\hat{I}_0(3,2,1)m_cm_b{}^4 - 60\hat{I}_0(1,4,1)m_cm_b{}^4 - 10\hat{I}_0(3,2,1)m_b{}^5\\
&- 10\hat{I}_0(2,3,1)m_b{}^5 + 30\hat{I}_0(1,4,1)m_b{}^5 - 20\hat{I}_0^{[0,1]}(3,1,2)m_c{}^3 + 20\hat{I}_0(2,1,2)m_c{}^3 - 20\hat{I}_0(2,2,1)m_c{}^3\\
&+ 20\hat{I}_0^{[0,1]}(3,2,1)m_c{}^3 - 10\hat{I}_0(2,1,2)m_c{}^2m_b - 10\hat{I}_0^{[0,1]}(3,2,1)m_c{}^2m_b + 10\hat{I}_2(3,2,1)m_cm_b{}^3 - 5\hat{I}_1^{[0,1]}(3,2,2)m_cm_b{}^3\\
&+ 30\hat{I}_1(1,4,1)m_cm_b{}^3 - 5\hat{I}_2^{[0,1]}(3,2,2)m_cm_b{}^3 + 10\hat{I}_2(2,3,1)m_cm_b{}^3 + 30\hat{I}_2(1,4,1)m_cm_b{}^3 - 5\hat{I}_0(3,2,1)m_b{}^4\\
&+ 15\hat{I}_0(1,4,1)m_b{}^4 + 5\hat{I}_1(1,1,3)m_c{}^2 + 10\hat{I}_1^{[0,1]}(3,1,2)m_c{}^2 - 10\hat{I}_1^{[0,1]}(3,2,1)m_c{}^2 - 15\hat{I}_2(1,3,1)m_c{}^2\\
&- 5\hat{I}_2(1,1,3)m_c{}^2 + 15\hat{I}_2(1,3,1)m_c{}^2 + 15\hat{I}_2^{[0,1]}(3,1,2)m_c{}^2 - 15\hat{I}_1(1,3,1)m_c{}^2 - 10\hat{I}_2(1,2,2)m_c{}^2\\
&- 15\hat{I}_1^{[0,1]}(4,1,1)m_c{}^2 + 10\hat{I}_1(1,2,2)m_c{}^2 + 5\hat{I}_2^{[0,1]}(2,1,)m_c{}^2 - 30\hat{I}_1^{[0,1]}(2,2,2)m_c{}^2 + 15\hat{I}_2^{[0,1]}(3,2,2)m_c{}^2\\
&- 15\hat{I}_2^{[0,2]}(3,2,2)m_c{}^2 - 15\hat{I}_1^{[0,1]}(3,2,1)m_c{}^2 - 10\hat{I}_1^{[0,1]}(2,3,1)m_cm_b - 10\hat{I}_1^{[0,1]}(2,2,2)m_cm_b - 5\hat{I}_2^{[0,1]}(3,2,1)m_cm_b\\
&- 10\hat{I}_2(1,2,2)m_cm_b - 5\hat{I}_2(2,1,2)m_cm_b - 15\hat{I}_1^{[0,1]}(3,2,1)m_cm_b + 5\hat{I}_2(2,2,1)m_b{}^2 + 5\hat{I}_1^{[0,2]}(3,2,2)m_b{}^2\\
&+ 5\hat{I}_1^{[0,1]}(2,2,2)m_b{}^2 + 10\hat{I}_2^{[0,1]}(2,2,2)m_b{}^2 + 10\hat{I}_2^{[0,1]}(3,2,1)m_b{}^2 - 5\hat{I}_1(2,2,1)m_b{}^2 - 5\hat{I}_2^{[0,2]}(3,2,2)m_b{}^2\\
&+ 10\hat{I}_0(2,2,1)m_b{}^2 - 10\hat{I}_1^{[0,1]}(3,2,1)m_b{}^2 - 5\hat{I}_0(3,1,1)m_b{}^2 - 10\hat{I}_1^{[0,1]}(2,2,2)m_b{}^2 - 30\hat{I}_2^{[0,1]}(1,4,1)m_b{}^2\\
&+ 30\hat{I}_1^{[0,1]}(1,4,1)m_b{}^2 - 5\hat{I}_0(2,1,2)m_b{}^2 + 15\hat{I}_2^{[0,1]}(1,3,1) + 5\hat{I}_1^{[0,2]}(2,1,3) - 10\hat{I}_2(1,2,1) - 15\hat{I}_2^{[0,1]}(1,3,1)\\
&+ 5\hat{I}_2^{[0,1]}(2,2,1) - 10\hat{I}_1(1,2,1) - 15\hat{I}_1^{[0,1]}(1,3,1) - 5\hat{I}_1^{[0,1]}(1,1,3) + 5\hat{I}_2^{[0,1]}(1,1,3) - 10\hat{I}_1^{[0,1]}(1,2,2)\\
&- 15\hat{I}_1^{[0,1]}(3,1,2)m_c{}^2 + 10\hat{I}_0(2,2,1)m_c{}^2 - 5\hat{I}_1^{[0,1]}(2,1,3)m_c{}^2 + 15\hat{I}_1(1,3,1)m_c{}^2 - 10\hat{I}_0(2,1,2)m_c{}^2\\
&+ 30\hat{I}_2^{[0,1]}(2,2,2)m_c{}^2 + 15\hat{I}_2^{[0,1]}(3,2,1)m_c{}^2 + 15\hat{I}_2^{[0,1]}(4,1,1)m_c{}^2 - 20\hat{I}_0(2,2,1)m_cm_b + 5\hat{I}_1(2,1,2)m_cm_b\\
&- 5\hat{I}_0(2,1,2)m_cm_b - 5\hat{I}_1^{[0,2]}(3,2,2)m_cm_b + 10\hat{I}_0(1,3,1)m_cm_b + 10\hat{I}_2^{[0,1]}(2,3,1)m_cm_b - 10\hat{I}_2(1,3,1)m_cm_b\\
&+ 15\hat{I}_1(1,3,1)m_cm_b - 15\hat{I}_1^{[0,1]}(3,1,2)m_cm_b - 5\hat{I}_1(1,1,3)m_cm_b - 5\hat{I}_2^{[0,1]}(2,1,3)m_cm_b - 20\hat{I}_2(2,2,1)m_cm_b\\
&+ 40\hat{I}_0(2,2,1)m_c{}^2m_b + 10\hat{I}_0^{[0,1]}(3,1,2)m_c{}^2m_b - 60\hat{I}_0(1,3,1)m_cm_b{}^2 + 20\hat{I}_0(3,1,1)m_cm_b{}^2\\
&- 20\hat{I}_0^{[0,1]}(2,2,2)m_cm_b{}^2 + 10\hat{I}_0^{[0,2]}(3,2,2)m_cm_b{}^2 - 10\hat{I}_0(1,2,2)m_b{}^3 - 10\hat{I}_0(3,1,1)m_b{}^3 + 30\hat{I}_0(1,3,1)m_b{}^3\\
&+ 10\hat{I}_0^{[0,1]}(2,2,2)m_b{}^3 + 10\hat{I}_0^{[0,1]}(2,3,1)m_b{}^3 - 10\hat{I}_0(2,1,2)m_b{}^3 + 20\hat{I}_0^{[0,1]}(2,2,1)m_c - 10\hat{I}_0^{[0,2]}(3,2,1)m_c\\
&- 20\hat{I}_0^{[0,1]}(2,1,2)m_c + 10\hat{I}_0^{[0,2]}(3,1,2)m_c - 10\hat{I}_0(1,1,2)m_b + 10\hat{I}_0(1,2,1)m_b + 10\hat{I}_0^{[0,1]}(2,1,2)m_b,
\end{aligned}
$$

$$
\begin{aligned}
C_T^4 :=& -5\hat{I}_2(3,2,2)m_c{}^6 + 5\hat{I}_1(3,2,2)m_c{}^6 + 5\hat{I}_0(3,2,2)m_c{}^5 m_b + 5\hat{I}_2(3,2,2)m_c{}^5 m_b + 15\hat{I}_1(4,1,1)m_c{}^4 - 5\hat{I}_2(3,2,1)m_c{}^4 \\
& - 5\hat{I}_2(3,1,2)m_c{}^4 + 5\hat{I}_1(3,1,2)m_c{}^4 + 15\hat{I}_1(2,2,2)m_c{}^4 - 15\hat{I}_2(4,1,1)m_c{}^4 + 15\hat{I}_2^{[0,1]}(3,2,2)m_c{}^4 \\
& - 15\hat{I}_1^{[0,1]}(3,2,2)m_c{}^4 + 10\hat{I}_1^{[0,1]}(3,2,2)m_c{}^3 m_b - 5\hat{I}_2(3,2,1)m_c{}^3 m_b + 10\hat{I}_0(2,3,1)m_c{}^3 m_b - 5\hat{I}_1(2,1,3)m_c{}^3 m_b \\
& + 15\hat{I}_0(4,1,1)m_c{}^3 m_b - 15\hat{I}_1(4,1,1)m_c{}^3 m_b + 5\hat{I}_0(3,1,2)m_c{}^3 m_b - 10\hat{I}_0^{[0,1]}(3,2,2)m_c{}^3 m_b + 10\hat{I}_1(2,3,1)m_c{}^3 m_b \\
& - 5\hat{I}_2(3,1,2)m_c{}^3 m_b + 15\hat{I}_2(4,1,1)m_c{}^3 m_b + 5\hat{I}_2(2,1,3)m_c{}^3 m_b - 10\hat{I}_2^{[0,1]}(3,2,2)m_c{}^3 m_b + 10\hat{I}_2(2,2,2)m_c{}^3 m_b \\
& - 5\hat{I}_0(2,2,2)m_c{}^2 m_b{}^2 - 10\hat{I}_2(3,2,1)m_c{}^2 m_b{}^2 + 10\hat{I}_2(3,2,1)m_c m_b{}^3 - 5\hat{I}_0^{[0,1]}(3,2,2)m_c m_b{}^3 + 30\hat{I}_1(1,4,1)m_c m_b{}^3 \\
& - 5\hat{I}_2^{[0,1]}(3,2,2)m_c m_b{}^3 + 5\hat{I}_1(1,1,3)m_c{}^2 + 10\hat{I}_0^{[0,1]}(3,1,2)m_c{}^2 - 10\hat{I}_0^{[0,1]}(3,2,1)m_c{}^2 - 15\hat{I}_2(1,3,1)m_c{}^2 \\
& - 5\hat{I}_2(1,1,3)m_c{}^2 + 15\hat{I}_2(1,3,1)m_c{}^2 + 15\hat{I}_2^{[0,1]}(3,1,2)m_c{}^2 - 15\hat{I}_1(1,3,1)m_c{}^2 - 30\hat{I}_1^{[0,1]}(2,2,2)m_c{}^2 \\
& + 15\hat{I}_1^{[0,2]}(3,2,2)m_c{}^2 - 15\hat{I}_2^{[0,2]}(3,2,2)m_c{}^2 - 15\hat{I}_1^{[0,1]}(3,2,1)m_c{}^2 - 10\hat{I}_0(2,1,2)m_c{}^2 + 30\hat{I}_2^{[0,1]}(2,2,2)m_c{}^2 \\
& + 15\hat{I}_2^{[0,1]}(3,2,1)m_c{}^2 + 15\hat{I}_2^{[0,1]}(4,1,1)m_c{}^2 - 20\hat{I}_0(2,2,1)m_c m_b + 5\hat{I}_1(2,1,2)m_c m_b - 5\hat{I}_0(2,1,2)m_c m_b \\
& - 5\hat{I}_1^{[0,2]}(3,2,2)m_c m_b + 5\hat{I}_1^{[0,1]}(3,2,1)m_c m_b - 10\hat{I}_2^{[0,1]}(2,2,2)m_c m_b + 20\hat{I}_1(2,2,1)m_c m_b - 15\hat{I}_2(1,3,1)m_c m_b \\
& + 5\hat{I}_2^{[0,2]}(3,2,2)m_c m_b + 10\hat{I}_1(1,2,2)m_c m_b + 15\hat{I}_1^{[0,1]}(3,1,2)m_c m_b + 10\hat{I}_1(1,3,1)m_c m_b - 10\hat{I}_0^{[0,1]}(2,3,1)m_c m_b \\
& - 10\hat{I}_0^{[0,1]}(2,2,2)m_c m_b - 5\hat{I}_2^{[0,1]}(3,2,1)m_c m_b - 10\hat{I}_2(1,2,2)m_c m_b - 5\hat{I}_2(2,1,2)m_c m_b - 15\hat{I}_0^{[0,1]}(3,2,1)m_c m_b \\
& + 5\hat{I}_2(2,2,1)m_b{}^2 + 5\hat{I}_1^{[0,2]}(3,2,2)m_b{}^2 + 5\hat{I}_0^{[0,1]}(2,2,2)m_b{}^2 + 10\hat{I}_2^{[0,1]}(2,2,2)m_b{}^2 + 10\hat{I}_2^{[0,1]}(3,2,1)m_b{}^2 \\
& - 5\hat{I}_1(2,2,1)m_b{}^2 + 15\hat{I}_2^{[0,1]}(1,3,1) + 5\hat{I}_1^{[0,2]}(2,1,3) - 10\hat{I}_2(1,2,1) - 15\hat{I}_2^{[0,1]}(1,3,1) + 5\hat{I}_2^{[0,1]}(2,2,1) \\
& - 10\hat{I}_1(1,2,1) - 15\hat{I}_1^{[0,1]}(1,3,1) - 5\hat{I}_1^{[0,1]}(1,1,3) + 5\hat{I}_2^{[0,1]}(1,1,3) - 10\hat{I}_1^{[0,1]}(1,2,2) + 10\hat{I}_1^{[0,2]}(3,2,1) \\
& - 10\hat{I}_2^{[0,2]}(3,2,1),
\end{aligned}
$$

where

$$
\hat{I}_n^{[i,j]}(a,b,c) = (M_1^2)^i (M_2^2)^j \frac{d^i}{d(M_1^2)^i} \frac{d^j}{d(M_2^2)^j} [(M_1^2)^i (M_2^2)^j \hat{I}_n(a,b,c)].
$$

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