# ANALYSIS OF THE $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) P$ AND $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) V$ DECAYS WITHIN THE FACTORIZATION APPROACH IN QCD 

K. AZIZI<br>Physics Division, Faculty of Arts \& Sciences, Dogus University, Acibadem-Kadikoy, 34722 Istanbul, Turkey<br>kazizi@dogus.edu.tr<br>R. KHOSRAVI* ${ }^{*}$ and F. FALAHATI ${ }^{\dagger}$<br>Physics Department, Shiraz University, Shiraz 71454, Iran<br>*khosravi.reza@gmail.com<br>†phy1g832889@shiraz.ac.ir

Received 6 April 2009
Revised 16 July 2009

Using the factorization approach and considering the contributions of the currentcurrent, QCD penguin and electroweak penguin operators at the leading approximation, the decay amplitudes and decay widths of $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) P$ and $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) V$ transitions, where $q=u, d, s$ and $P$ and $V$ are pseudoscalar and vector mesons, are calculated in terms of the transition form factors of the $B_{q} \rightarrow D_{q}$ and $B_{q} \rightarrow D_{q}^{*}$. Having computed those form factors in three-point QCD sum rules, the branching fraction for these decays are also evaluated. A comparison of our results with the predictions of the perturbative QCD as well as the existing experimental data is presented.

Keywords: QCD factorization; QCD sum-rules; current-current; QCD penguin; electroweak penguin operators.

## 1. Introduction

With the chances that a lot of $B_{q}$ mesons will be produced in $B$ factories, ${ }^{1,2}$ it would be possible to check the two-body nonleptonic charmed decay modes $B_{q} \rightarrow$ $D_{q}\left(D_{q}^{*}\right) P$ and $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) V$. Analyzing of such type decays could give valuable information about the origin of the CP violation, hadronic flavor changing neutral currents, test of the Standard Model (SM), constraints on new physic parameters as well as strong interactions among the participating particles which provides valuable tests of the QCD factorization framework.

Theoretically, analyzing of the two-body B-decay amplitudes have been started using the framework of so-called "naive factorization." ${ }^{3-7}$ This method for some decay channels is replaced by QCD factorization ${ }^{8,9}$ since it could not predict direct

CP asymmetries in those decay modes. First, the QCD factorization approach had been applied for the simplest charmless $B \rightarrow \pi \pi$ and $B \rightarrow \pi K$ decays ${ }^{8,10-13}$ then extended to the vector and exotic mesons in final states ${ }^{14-17}$ and $\eta$ or $\eta^{\prime}$ with a pseudoscalar or vector kaon. ${ }^{18}$ In Refs. 19-21, decay modes of $B_{s}$ meson are discussed. A comprehensive study of the exclusive hadronic $B$-meson decays into the final states containing two pseudoscalar mesons $(P P)$ or a pseudoscalar and a vector meson ( $P V$ ) is discussed in Ref. 22. The charmless anti- $B_{s} \rightarrow V V$ decays has also been analyzed in QCD factorization in Ref. 23. The hard-scattering kernels relevant to the negative-helicity decay amplitude in $B$ decays to two vector mesons are calculated in Ref. 24 in the same framework. The two-body hadronic decays of $B$ mesons into pseudoscalar and axial vector mesons have been studied within the framework of QCD factorization in Ref. 25. A detailed study of charmless two-body $B$ decays into final states involving two vector mesons ( $V V$ ) or two axial-vector mesons $(A A)$ or one vector and one axial-vector meson $(V A)$ has also been done within the framework of QCD factorization in Ref. 26. Considering the contributions of both current-current and penguin operators, the amplitudes and branching ratios are recently estimated at the leading approximation for $B_{c} \rightarrow B^{*} P, B V$ in Ref. 27.

In the present work, taking into account the contributions of the currentcurrent, QCD penguin and electroweak penguin operators at the leading approximation, we describe the charmed decays $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) P$ and $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) V$ in the framework of the QCD factorization method. First, using the factorization method, we calculate the decay amplitudes and decay widths of these decays in terms of the transition form factors of the $B_{q} \rightarrow D_{q}$ and $B_{q} \rightarrow D_{q}^{*}$. Having calculated these transition form factors in the framework of the QCD sum rules in our previous works in Refs. 28 and 29, we calculate the branching ratio of these decays. In order to estimate the approximate branching ratios and to have a sense of the order of amplitudes, we make a rough approximation, i.e. at the leading order of $\alpha_{s}$. Within this approximation, the hard-scattering kernel functions become very simple and equal to unity. ${ }^{27}$ In this approximation, the long-distance interactions between the $P(V)$ and $B_{q}-D_{q}\left(D_{q}^{*}\right)$ system could be neglected. The higher order $\alpha_{s}$ corrections might not be small, but calculation of these contributions is not as easy as the light systems in final state and needs much more efforts. Hence, for obtaining the exact results on the considered transitions, those effects should be encountered in the future works. There are several methods in which such type contributions can be studied, QCD factorization, ${ }^{8-10,18,22,24}$ Perturbative QCD ${ }^{30}$ and soft-collinear effective theory. ${ }^{31,32}$ For more detail analysis of NNLO corrections to $B \rightarrow$ light-light systems and higher-order QCD corrections to the charmless $B$ decays see also Refs. 33-38. Note that, some of the considered decays in this paper have been analyzed in the framework of the perturbative QCD (PQCD) ${ }^{30}$ and for some of them, we have some experimental data. ${ }^{39}$

The outline of the paper is as follows. In Sec. 2, we calculate the decay amplitudes and decay widths for $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) P$ and $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) V$ transitions. Finally, Sec. 3 is devoted to the numerical analysis, a comparison of our results
with the predictions of the PQCD as well as the existing experimental data and discussion.

## 2. Decay Amplitudes and Decay Widths

In the present section, we study the decay amplitudes and decay widths for $B_{q} \rightarrow$ $D_{q}\left(D_{q}^{*}\right) P$ and $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) V$ decays, where $q=u, d, s, P=\pi, K, D_{q^{\prime}}$ and $V=K^{*}, D_{q^{\prime}}^{*}\left(q^{\prime}=d, s\right)$. At the quark level, the effective Hamiltonian for $B_{q} \rightarrow$ $D_{q}\left(D_{q}^{*}\right) \pi\left(K, K^{*}\right)$ is given by

$$
\begin{equation*}
H_{\mathrm{eff}}=\frac{G_{F}}{\sqrt{2}}\left\{V_{c b} V_{u q^{\prime}}^{*}\left(C_{1} O_{1}^{u}+C_{2} O_{2}^{u}\right)\right\} . \tag{1}
\end{equation*}
$$

Here $O_{1}^{u}$ and $O_{2}^{u}$ are quark operators and are given by

$$
\begin{equation*}
O_{1}^{u}=\left(\bar{q}_{i}^{\prime} u_{i}\right)_{V-A}\left(\bar{c}_{j} b_{j}\right)_{V-A}, \quad O_{2}^{u}=\left(\bar{q}_{i}^{\prime} u_{j}\right)_{V-A}\left(\bar{c}_{j} b_{i}\right)_{V-A}, \tag{2}
\end{equation*}
$$

where $q^{\prime}=d$, $s$ and $\left(\bar{q}_{1} q_{2}\right)_{V \pm A}=\bar{q}_{1} \gamma^{\mu}\left(1 \pm \gamma_{5}\right) q_{2}$. However, the effective Hamiltonian for $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) D_{q^{\prime}}$ and $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) D_{q^{\prime}}^{*}$ at the quark level can be written as

$$
\begin{equation*}
H_{\mathrm{eff}}=\frac{G_{F}}{\sqrt{2}}\left\{V_{c b} V_{c q^{\prime}}^{*}\left(C_{1} O_{1}^{c}+C_{2} O_{2}^{c}\right)+V_{t b} V_{t q^{\prime}}^{*} \sum_{n=3}^{10} C_{n} O_{n}\right\} . \tag{3}
\end{equation*}
$$

Here $O_{n}$ are quark operators and are given by

$$
\begin{align*}
O_{1}^{c} & =\left(\bar{q}_{i}^{\prime} c_{i}\right)_{V-A}\left(\bar{c}_{j} b_{j}\right)_{V-A}, \\
O_{2}^{c} & =\left(\bar{q}_{i}^{\prime} c_{j}\right)_{V-A}\left(\bar{c}_{j} b_{i}\right)_{V-A}, \\
O_{3(5)} & =\left(\bar{q}_{i}^{\prime} b_{i}\right)_{V-A} \sum_{q}\left(\bar{q}_{j} q_{j}\right)_{V-(+) A}, \\
O_{4(6)} & =\left(\bar{q}_{i}^{\prime} b_{j}\right)_{V-A} \sum_{q}\left(\bar{q}_{j} q_{i}\right)_{V-(+) A},  \tag{4}\\
O_{7(9)} & =\left(\bar{q}_{i}^{\prime} b_{i}\right)_{V-A} \sum_{q} \frac{3}{2} e_{q}\left(\bar{q}_{j} q_{j}\right)_{V+(-) A}, \\
O_{8(10)} & =\left(\bar{q}_{i}^{\prime} b_{j}\right)_{V-A} \sum_{q} \frac{3}{2} e_{q}\left(\bar{q}_{j} q_{i}\right)_{V+(-) A},
\end{align*}
$$

where $\sum_{q}$ sums over $q=u, d, c, s, b$ and $i$ and $j$ are color indices. The operators $O_{1}$ and $O_{2}$ are called the current-current operators, $O_{3} \cdots O_{6}$ are QCD penguin operators and $O_{7} \cdots O_{10}$ are called the electroweak penguin operators.

The Wilson coefficients $C_{n}$ have been calculated in different schemes. ${ }^{40-43}$ In this paper we will use consistently the naive dimensional regularization (NDR) scheme.

The values of $C_{n}$ at $\mu \approx m_{b}$ with the next-to-leading order (NLO) QCD corrections are given by ${ }^{40-43}$

$$
\begin{array}{ll}
C_{1}=1.117, & C_{2}=-0.257 \\
C_{3}=0.017, & C_{4}=-0.044, \\
C_{5}=0.011, & C_{6}=-0.056  \tag{5}\\
C_{7}=-1 \times 10^{-5}, & C_{8}=5 \times 10^{-4}, \\
C_{9}=-0.010, & C_{10}=0.002
\end{array}
$$

The decay width and the branching ratio of the nonleptonic process $B_{q} \rightarrow$ $D_{q}\left(D^{*}\right) M$, where $M$ stands for the $P$ or $V$ mesons, is given by

$$
\begin{align*}
\Gamma\left(B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) M\right) & =\frac{1}{16 \pi m_{B_{q}}^{3}}|\mathcal{A}|^{2} \sqrt{\lambda\left(m_{B_{q}}^{2}, m_{D_{q}\left(D_{q}^{*}\right)}^{2}, m_{M}^{2}\right)},  \tag{6}\\
\operatorname{Br}\left(B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) M\right) & =\tau_{B_{q}} \Gamma\left(B_{q} \rightarrow D_{q}\left(D^{*}{ }_{q}\right) M\right),
\end{align*}
$$

where, $\lambda(x, y, z)=x^{2}+y^{2}+z^{2}-2 x y-2 x z-2 y z$ is usual triangle function.
To obtain the decay width, we should calculate the amplitude $\mathcal{A}$. This amplitude is obtained using the factorization method and the definition of the related matrix elements in terms of form factors for the $B_{q} \rightarrow D_{q}$ and $B_{q} \rightarrow D_{q}^{*}$ weak transitions as

$$
\begin{align*}
&\left\langle D_{q}\left(p^{\prime}\right)\right| \bar{c} \gamma_{\mu}\left(1-\gamma_{5}\right) b\left|B_{q}(p)\right\rangle \\
&=\left(p+p^{\prime}\right)_{\mu} f_{+}^{B_{q} \rightarrow D_{q}}\left(q^{2}\right)+\left(p-p^{\prime}\right)_{\mu} f_{-}^{B_{q} \rightarrow D_{q}}\left(q^{2}\right),  \tag{7}\\
&\left\langle D_{q}^{*}\left(p^{\prime}, \varepsilon\right)\right| \bar{c} \gamma_{\mu} b\left|B_{q}(p)\right\rangle= \frac{-2 f_{1}^{B_{q} \rightarrow D_{q}^{*}}\left(q^{2}\right)}{m_{D_{q}^{*}}+m_{B_{q}}} \varepsilon_{\mu \nu \alpha \beta} \varepsilon^{* \nu} p^{\alpha} p^{\prime \beta},  \tag{8}\\
&\left\langle D_{q}^{*}\left(p^{\prime}, \varepsilon\right)\right| \bar{c} \gamma_{\mu} \gamma_{5} b\left|B_{q}(p)\right\rangle=-i\left[f_{0}^{B_{q} \rightarrow D_{q}^{*}}\left(q^{2}\right)\left(m_{D_{q}^{*}}+m_{B_{q}}\right) \varepsilon_{\mu}^{*}\right. \\
&+\frac{f_{2}^{B_{q} \rightarrow D_{q}^{*}}}{\left.m_{D_{q}^{*}}+q^{2}\right)}\left(\varepsilon_{B_{q}}^{*} p\right)\left(p+p^{\prime}\right)_{\mu} \\
&\left.+\frac{f_{3}^{B_{q} \rightarrow D_{q}^{*}}\left(q^{2}\right)}{m_{D_{q}^{*}}+m_{B_{q}}}\left(\varepsilon^{*} p\right)\left(p-p^{\prime}\right)_{\mu}\right], \tag{9}
\end{align*}
$$

where $q^{2}$ is transferred momentum square, $q^{2}=\left(p-p^{\prime}\right)^{2}$, and $p$ and $p^{\prime}$ are momentum of the initial and final meson states, respectively.

$$
\text { Analysis of the } B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) P \text { and } B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) V \text { Decays }
$$

We obtain the $\mathcal{A}$ as following:
$\underline{\text { For } B_{q} \rightarrow D_{q} P(P=\pi, K) \text { and } B_{q} \rightarrow D_{q} D_{q^{\prime}}:}$

$$
\begin{align*}
\mathcal{A}^{B_{d(s)} \rightarrow D_{d(s)} P}= & i \frac{G_{F}}{\sqrt{2}} V_{c b} V_{u q^{\prime}}^{*} a_{1} f_{P} F_{1}^{B_{d(s)} \rightarrow D_{d(s)}}\left(m_{P}^{2}\right) \\
\mathcal{A}^{B_{u} \rightarrow D_{u} P}= & i \frac{G_{F}}{\sqrt{2}} V_{c b} V_{u q^{\prime}}^{*}\left[a_{1} f_{P} F_{1}^{B_{u} \rightarrow D_{u}}\left(m_{P}^{2}\right)+a_{2} f_{D_{u}} F_{1}^{B_{u} \rightarrow P}\left(m_{D_{u}}^{2}\right)\right]  \tag{10}\\
\mathcal{A}^{B_{q} \rightarrow D_{q} D_{q^{\prime}}}= & i \frac{G_{F}}{\sqrt{2}} f_{D_{q^{\prime}}} F_{1}^{B_{q} \rightarrow D_{q}}\left(m_{D_{q^{\prime}}}^{2}\right) \\
& \times\left[V_{c b} V_{c q^{\prime}}^{*} a_{1}-V_{t b} V_{t q^{\prime}}^{*}\left(a_{4}+a_{10}+R_{q^{\prime}}\left(a_{6}+a_{8}\right)\right)\right]
\end{align*}
$$

where,

$$
\begin{align*}
F_{1}^{B_{q} \rightarrow P_{1}}\left(m_{P_{2}}^{2}\right) & =\left(m_{B_{q}}^{2}-m_{P_{1}}^{2}\right) f_{+}^{B_{q} \rightarrow P_{1}}\left(m_{P_{2}}^{2}\right)+m_{P_{2}}^{2} f_{-}^{B_{q} \rightarrow P_{1}}\left(m_{P_{2}}^{2}\right) \\
R_{q^{\prime}} & =\frac{2 m_{D_{q^{\prime}}}^{2}}{\left(m_{b}-m_{c}\right)\left(m_{q^{\prime}}+m_{c}\right)} \tag{11}
\end{align*}
$$

$\underline{\text { For } B_{q} \rightarrow D_{q} K^{*} \text { and } B_{q} \rightarrow D_{q} D_{q^{\prime}}^{*}}:$

$$
\begin{align*}
\mathcal{A}^{B_{d(s)} \rightarrow D_{d(s)} K^{*}}= & \frac{2 m_{K^{*}} f_{K^{*}} G_{F}}{\sqrt{2}} V_{c b} V_{u s}^{*} a_{1}\left(p \cdot \varepsilon_{K^{*}}^{*}\right) f_{+}^{B_{d(s)} \rightarrow D_{d(s)}}\left(m_{K^{*}}^{2}\right) \\
\mathcal{A}^{B_{u} \rightarrow D_{u} K^{*}}= & \frac{G_{F}}{\sqrt{2}} V_{c b} V_{u s}^{*}\left(p \cdot \varepsilon_{K^{*}}^{*}\right) \\
& \times\left[2 a_{1} m_{K^{*}} f_{K^{*}} f_{+}^{B_{u} \rightarrow D_{u}}\left(m_{K^{*}}^{2}\right)-a_{2} f_{D_{u}} F_{2}^{B_{u} \rightarrow K^{*}}\left(m_{D_{u}}^{2}\right)\right]  \tag{12}\\
\mathcal{A}^{B_{q} \rightarrow D_{q} D_{q^{\prime}}^{*}}= & \frac{2 m_{D_{q^{\prime}}^{*}} f_{D_{q^{\prime}}^{*}} G_{F}}{\sqrt{2}}\left(p \cdot \varepsilon_{D_{q^{\prime}}^{*}}^{*}\right) f_{+}^{B_{q} \rightarrow D_{q}}\left(m_{D_{q^{\prime}}^{*}}^{2}\right) \\
& \times\left[V_{c b} V_{c q^{\prime}}^{*} a_{1}-V_{t b} V_{t q^{\prime}}^{*}\left(a_{4}+a_{10}\right)\right]
\end{align*}
$$

where,

$$
\begin{align*}
F_{2}^{B_{q} \rightarrow V}\left(m_{P}^{2}\right)= & f_{0}^{B_{q} \rightarrow V}\left(m_{P}^{2}\right)\left(m_{B_{q}}+m_{V}\right)+f_{2}^{B_{q} \rightarrow V}\left(m_{P}^{2}\right)\left(m_{B_{q}}-m_{V}\right) \\
& +\frac{f_{3}^{B_{q} \rightarrow V}\left(m_{P}^{2}\right)}{\left(m_{B_{q}}+m_{V}\right)} m_{P}^{2} \tag{13}
\end{align*}
$$

$\underline{\text { For } B_{q} \rightarrow D_{q}^{*} P(P=\pi, K) \text {, and } B_{q} \rightarrow D_{q}^{*} D_{q^{\prime}}}$ :

$$
\begin{align*}
\mathcal{A}^{B_{q} \rightarrow D_{d(s)}^{*} P}= & -\frac{G_{F}}{\sqrt{2}} V_{c b} V_{u q^{\prime}}^{*} a_{1} f_{P}\left(p \cdot \varepsilon_{D^{*}}^{*}\right) F_{2}^{B_{q} \rightarrow D_{d(s)}^{*}}\left(m_{P}^{2}\right) \\
\mathcal{A}^{B_{u} \rightarrow D_{u}^{*} P}= & -\frac{G_{F}}{\sqrt{2}} V_{c b} V_{u q^{\prime}}^{*}\left(p \cdot \varepsilon_{D^{*}}^{*}\right) \\
& \times\left[a_{1} f_{P} F_{2}^{B_{u} \rightarrow D_{u}^{*}}\left(m_{P}^{2}\right)-2 a_{2} f_{D_{u}^{*}} f_{+}^{B_{u} \rightarrow P}\left(m_{D_{u}^{*}}^{2}\right)\right]  \tag{14}\\
\mathcal{A}^{B_{q} \rightarrow D_{q}^{*} D_{q^{\prime}}}= & -\frac{G_{F}}{\sqrt{2}} f_{D_{q^{\prime}}}\left(p \cdot \varepsilon_{D_{q}^{*}}^{*}\right) F_{2}^{B_{q} \rightarrow D_{q}^{*}}\left(m_{D_{q^{\prime}}}^{2}\right) \\
& \times\left[V_{c b} V_{c q^{\prime}}^{*} a_{1}-V_{t b} V_{t q^{\prime}}^{*}\left(a_{4}+a_{10}+R_{q^{\prime}}^{\prime}\left(a_{6}+a_{8}\right)\right)\right]
\end{align*}
$$

with

$$
\begin{equation*}
R_{q^{\prime}}^{\prime}=-\frac{2 m_{D_{q^{\prime}}}^{2}}{\left(m_{b}+m_{c}\right)\left(m_{q^{\prime}}+m_{c}\right)} \tag{15}
\end{equation*}
$$

$\underline{\text { For } B_{q} \rightarrow D_{q}^{*} K^{*} \text { and } B_{q} \rightarrow D_{q}^{*} D_{q^{\prime}}^{*}}$

$$
\begin{aligned}
\mathcal{A}^{B_{d(s)} \rightarrow D_{d(s)}^{*} K^{*}}= & i \frac{m_{K^{*}} f_{K^{*}} G_{F}}{\sqrt{2}} V_{c b} V_{u s}^{*} a_{1}\left[F_{3}^{B_{d(s)} \rightarrow D_{d(s)}^{*}}\left(m_{K^{*}}^{2}\right)\left(\varepsilon_{D^{*}}^{*} \cdot \varepsilon_{K^{*}}^{*}\right)\right. \\
& +F_{4}^{B_{d(s)} \rightarrow D_{d(s)}^{*}}\left(m_{K^{*}}^{2}\right)\left(p \cdot \varepsilon_{D^{*}}^{*}\right)\left(p \cdot \varepsilon_{K^{*}}^{*}\right) \\
& \left.-i F_{5}^{B_{d(s)} \rightarrow D_{d(s)}^{*}}\left(m_{K^{*}}^{2}\right) \varepsilon_{\mu \nu \rho \sigma} \varepsilon_{K^{*}}^{* \mu} \varepsilon_{D^{*}}^{* \nu} p^{\rho} p_{D^{*}}^{\sigma}\right] \\
\mathcal{A}^{B_{u} \rightarrow D_{u}^{*} K^{*}}= & i \frac{G_{F}}{\sqrt{2}} V_{c b} V_{u s}^{*}\left(a _ { 1 } m _ { K ^ { * } } f _ { K ^ { * } } \left[F_{3}^{B_{u} \rightarrow D_{u}^{*}}\left(m_{K^{*}}^{2}\right)\left(\varepsilon_{D^{*}}^{*} \cdot \varepsilon_{K^{*}}^{*}\right)\right.\right. \\
& +F_{4}^{B_{u} \rightarrow D_{u}^{*}}\left(m_{K^{*}}^{2}\right)\left(p \cdot \varepsilon_{D^{*}}^{*}\right)\left(p \cdot \varepsilon_{K^{*}}^{*}\right) \\
& \left.-i F_{5}^{B_{u} \rightarrow D_{u}^{*}}\left(m_{K^{*}}^{2}\right) \varepsilon_{\mu \nu \rho \sigma} \varepsilon_{K^{*}}^{* \mu} \varepsilon_{D^{*}}^{* \nu} p^{\rho} p_{D^{*}}^{\sigma}\right] \\
& +a_{2} m_{D^{*}} f_{D^{*}}\left[F_{3}^{B_{u} \rightarrow K^{*}}\left(m_{D^{*}}^{2}\right)\left(\varepsilon_{K^{*}}^{*} \cdot \varepsilon_{D^{*}}^{*}\right)\right. \\
& +F_{4}^{B_{u} \rightarrow K^{*}}\left(m_{D^{*}}^{2}\right)\left(p \cdot \varepsilon_{K^{*}}^{*}\right)\left(p \cdot \varepsilon_{D^{*}}^{*}\right) \\
& \left.\left.-i F_{5}^{B_{u} \rightarrow K^{*}}\left(m_{D^{*}}^{2}\right) \varepsilon_{\mu \nu \rho \sigma} \varepsilon_{D^{*}}^{* \mu} \varepsilon_{K^{*}}^{* \nu} p^{\rho} p_{K^{*}} \sigma\right]\right)
\end{aligned}
$$

$$
\begin{align*}
\mathcal{A}^{B_{q} \rightarrow D_{q}^{*} D_{q^{\prime}}^{*}=} & i \frac{G_{F}}{\sqrt{2}} m_{D_{q^{\prime}}^{*}} f_{D_{q^{\prime}}^{*}}\left[F_{3}^{B_{q} \rightarrow D_{q}^{*}}\left(m_{K^{*}}^{2}\right)\left(\varepsilon_{D_{q}^{*}}^{*} \cdot \varepsilon_{D_{q^{\prime}}^{*}}^{*}\right)\right. \\
& +F_{4}^{B_{q} \rightarrow D_{q}^{*}}\left(m_{D_{q^{\prime}}^{*}}^{2}\right)\left(p \cdot \varepsilon_{D_{q}^{*}}^{*}\right)\left(p \cdot \varepsilon_{D_{q^{\prime}}^{*}}^{*}\right) \\
& \left.-i F_{5}^{B_{q} \rightarrow D_{q}^{*}}\left(m_{{D^{\prime}}^{*}}^{2}\right) \varepsilon_{\mu \nu \rho \sigma} \varepsilon_{D_{q^{\prime}}^{*}}^{* \mu} \varepsilon_{D_{q}^{*}}^{* \nu} p^{\rho} p_{D_{q}^{*}}^{\sigma}\right] \\
& \times\left[V_{c b} V_{c q^{\prime}}^{*} a_{1}-V_{t b} V_{t q^{\prime}}^{*}\left(a_{4}+a_{10}\right)\right] \tag{16}
\end{align*}
$$

where

$$
\begin{align*}
& F_{3}^{B_{q} \rightarrow V_{1}}\left(m_{V_{2}}^{2}\right)=\left(m_{B_{q}}+m_{V_{1}}\right) f_{0}^{B_{q} \rightarrow V_{1}}\left(m_{V_{2}}^{2}\right) \\
& F_{4}^{B_{q} \rightarrow V_{1}}\left(m_{V_{2}}^{2}\right)=\frac{2 f_{2}^{B_{q} \rightarrow V_{1}}\left(m_{V_{2}}^{2}\right)}{\left(m_{B_{q}}+m_{V_{1}}\right)}  \tag{17}\\
& F_{5}^{B_{q} \rightarrow V_{1}}\left(m_{V_{2}}^{2}\right)=\frac{-2 f_{1}^{B_{q} \rightarrow V_{1}}\left(m_{V_{2}}^{2}\right)}{\left(m_{B_{q}}+m_{V_{1}}\right)}
\end{align*}
$$

In the above expressions, the $\varepsilon^{*}, \varepsilon^{\prime *}, \varepsilon_{K}$ stand for the polarization of the $D_{q}^{*}, D_{q^{\prime}}^{*}$ and $K^{*}$ mesons, respectively. The quantities $a_{i}$, are given in terms of the coefficient $C_{i}$,

$$
\begin{equation*}
a_{i}=C_{i}+\frac{1}{N_{c}} C_{i+1} \quad(i=\mathrm{odd}) ; \quad a_{i}=C_{i}+\frac{1}{N_{c}} C_{i-1} \quad(i=\text { even }), \tag{18}
\end{equation*}
$$

where $i$ runs from $i=1, \ldots, 10$ and $N_{c}$ is number of color in QCD. In the above equation, the $a_{1}$ and $a_{2}$ are both related to the coefficients $C_{1,2}$, which are very large comparing with the other Wilson coefficients, but we will keep all coefficients to get ride of further approximation.

Now we can calculate the decay widths for $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) P$ and $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) V$ decays. The explicit expressions for decay widths are given as follow:

$$
\begin{align*}
\Gamma\left(B_{d(s)}\right. & \left.\rightarrow D_{d(s)} P(P=\pi, K)\right) \\
= & \frac{G_{F}^{2}}{32 \pi m_{B_{d(s)}}^{3}}\left|V_{c b} V_{u q^{\prime}}^{*}\right|^{2} a_{1}^{2} f_{P}^{2} \sqrt{\lambda\left(m_{B_{d(s)}}^{2}, m_{D_{d(s)}}^{2}, m_{P}^{2}\right)} \\
& \times\left[F_{1}^{\left.B_{d(s)} \rightarrow D_{d(s)}\left(m_{P}^{2}\right)\right]^{2},}\right.  \tag{19}\\
\Gamma\left(B_{u} \rightarrow\right. & \left.D_{u} P(P=\pi, K)\right) \\
= & \frac{G_{F}^{2}}{32 \pi m_{B_{u}}^{3}}\left|V_{c b} V_{u q^{\prime}}^{*}\right|^{2} \sqrt{\lambda\left(m_{B_{u}}^{2}, m_{D_{u}}^{2}, m_{P}^{2}\right)} \\
& \times\left[a_{1}^{2} f_{P}^{2}\left[F_{1}^{B_{u} \rightarrow D_{u}}\left(m_{P}^{2}\right)\right]^{2}+a_{2}^{2} f_{D_{u}}^{2}\left[F_{1}^{B_{u} \rightarrow P}\left(m_{D_{u}}^{2}\right)\right]^{2}\right. \\
& \left.+2 a_{1} a_{2} f_{D_{u}} f_{P}\left[F_{1}^{B_{u} \rightarrow D_{u}}\left(m_{P}^{2}\right) F_{1}^{B_{u} \rightarrow P}\left(m_{D_{u}}^{2}\right)\right]\right], \tag{20}
\end{align*}
$$

$$
\begin{align*}
& \Gamma\left(B_{q} \rightarrow D_{q} D_{q^{\prime}}\right) \\
& =\frac{G_{F}^{2}}{32 \pi m_{B_{q}}^{3}} f_{D_{q^{\prime}}}^{2} \sqrt{\lambda\left(m_{B_{q}}^{2}, m_{D_{q}}^{2}, m_{D_{q^{\prime}}}^{2}\right)}\left[F_{1}^{B_{q} \rightarrow D_{q}}\left(m_{D_{q^{\prime}}}^{2}\right)\right]^{2} \\
& \times\left|V_{c b} V_{c q^{\prime}}^{*} a_{1}-V_{t b} V_{t q^{\prime}}^{*}\left[a_{4}+a_{10}+R_{q^{\prime}}\left(a_{6}+a_{8}\right)\right]\right|^{2},  \tag{21}\\
& \Gamma\left(B_{d(s)} \rightarrow D_{d(s)} K^{*}\right) \\
& =\frac{G_{F}^{2}}{32 \pi m_{B_{d(s)}}^{3}}\left|V_{c b} V_{u s}^{*}\right|^{2} a_{1}^{2} f_{K^{*}}^{2} \lambda\left(m_{B_{d(s)}}^{2}, m_{D_{d(s)}}^{2}, m_{K^{*}}^{2}\right)^{\frac{3}{2}} \\
& \times\left[f_{+}^{B_{d(s)} \rightarrow D_{d(s)}}\left(m_{K^{*}}^{2}\right)\right]^{2},  \tag{22}\\
& \Gamma\left(B_{u} \rightarrow D_{u} K^{*}\right) \\
& =\frac{G_{F}^{2}}{32 \pi m_{B_{u}}^{3}}\left|V_{c b} V_{u s}^{*}\right|^{2} \frac{\lambda\left(m_{B_{u}}^{2}, m_{D_{u}}^{2}, m_{K^{*}}^{2}\right)^{\frac{3}{2}}}{4 m_{K^{*}}^{2}} \\
& \times\left[4 a_{1}^{2} m_{K^{*}}^{2} f_{K^{*}}^{2}\left[f_{+}^{B_{u} \rightarrow D_{u}}\left(m_{K^{*}}^{2}\right)\right]^{2}+a_{2}^{2} f_{D_{u}}^{2}\left[F_{2}^{B_{u} \rightarrow K^{*}}\left(m_{D_{u}}^{2}\right)\right]^{2}\right. \\
& \left.-4 a_{1} a_{2} m_{K^{*}} f_{K^{*}} f_{D_{u}} f_{+}^{B_{u} \rightarrow D_{u}}\left(m_{K^{*}}^{2}\right) F_{2}^{B_{u} \rightarrow K^{*}}\left(m_{D_{u}}^{2}\right)\right],  \tag{23}\\
& \Gamma\left(B_{q} \rightarrow D_{q} D_{q^{\prime}}^{*}\right) \\
& =\frac{G_{F}}{32 \pi m_{B_{q}}^{3}} f_{D_{q^{\prime}}^{*}}^{2} \lambda\left(m_{B_{q}}^{2}, m_{D_{q}}^{2}, m_{D_{q^{\prime}}^{*}}^{2}\right)^{\frac{3}{2}}\left[f_{+}^{B_{q} \rightarrow D_{q}}\left(m_{D_{q^{\prime}}^{*}}^{2}\right)\right]^{2} \\
& \times\left|V_{c b} V_{c q^{\prime}}^{*} a_{1}-V_{t b} V_{t q^{\prime}}^{*}\left[a_{4}+a_{10}\right]\right|^{2},  \tag{24}\\
& \Gamma\left(B_{d(s)} \rightarrow D_{d(s)}^{*} K^{*}\right) \\
& =\frac{G_{F}^{2}}{32 \pi m_{B_{d(s)}}^{3}}\left|V_{c b} V_{u s}^{*}\right|^{2} a_{1}^{2} m_{K^{*}}^{2} f_{K^{*}}^{2} \lambda\left(m_{B_{d(s)}}^{2}, m_{D_{d(s)}^{*}}^{2}, m_{K^{*}}^{2}\right)^{\frac{1}{2}} \\
& \times\left(\left[F_{3}^{B_{d(s)} \rightarrow D_{d(s)}^{*}}\left(m_{K^{*}}^{2}\right)\right]^{2}\left[\frac{\lambda\left(m_{B_{d(s)}}^{2}, m_{D_{d(s)}^{*}}^{2}, m_{K^{*}}^{2}\right)}{4 m_{D_{d(s)}^{*}}^{2} m_{K^{*}}^{2}}+3\right]\right. \\
& +\left[F_{4}^{B_{d(s)} \rightarrow D_{d(s)}^{*}}\left(m_{K^{*}}^{2}\right)\right]^{2}\left[\frac{\lambda\left(m_{B_{d(s)}}^{2}, m_{D_{d(s)}^{*}}^{2}, m_{K^{*}}^{2}\right)^{2}}{16 m_{D_{d(s)}^{*}}^{2} m_{K^{*}}^{2}}\right] \\
& +\left[F_{5}^{B_{d(s)} \rightarrow D_{d(s)}^{*}}\left(m_{K^{*}}^{2}\right)\right]^{2}\left[\frac{\lambda\left(m_{B_{d(s)}}^{2}, m_{D_{d(s)}^{*}}^{2}, m_{K^{*}}^{2}\right)}{2}\right]
\end{align*}
$$

$$
\begin{align*}
& +\left[F_{3}^{B_{d(s)} \rightarrow D_{d(s)}^{*}}\left(m_{K^{*}}^{2}\right)\right]\left[F_{4}^{B_{d(s)} \rightarrow D_{d(s)}^{*}}\left(m_{K^{*}}^{2}\right)\right] \\
& \left.\times\left(m_{B_{d(s)}}^{2}-m_{D_{d(s)}^{*}}^{2}-m_{K^{*}}^{2}\right)\left[\frac{\lambda\left(m_{B_{d(s)}}^{2}, m_{D_{d(s)}^{*}}^{2}, m_{K^{*}}^{2}\right)}{4 m_{D_{d(s)}^{*}}^{2} m_{K^{*}}^{2}}\right]\right),  \tag{25}\\
& \Gamma\left(B_{u} \rightarrow D_{u}^{*} K^{*}\right) \\
& =\frac{G_{F}^{2}}{32 \pi m_{B_{u}}^{3}}\left|V_{c b} V_{u s}^{*}\right|^{2} \lambda\left(m_{B_{u}}^{2}, m_{D_{u}^{*}}^{2}, m_{K^{*}}^{2}\right)^{\frac{1}{2}}\left(\left[a_{1} m_{K^{*}} f_{K^{*}} F_{3}^{B_{u} \rightarrow D_{u}^{*}}\left(m_{K^{*}}^{2}\right)\right.\right. \\
& \left.+a_{2} m_{D_{u}^{*}} f_{D_{u}^{*}} F_{3}^{B_{u} \rightarrow K^{*}}\left(m_{D_{u}^{*}}^{2}\right)\right]^{2}\left[\frac{\lambda\left(m_{B_{u}}^{2}, m_{D_{u}^{*}}^{2}, m_{K^{*}}^{2}\right)}{4 m_{D_{u}^{*}}^{2} m_{K^{*}}^{2}}+3\right] \\
& +\left[a_{1} m_{K^{*}} f_{K^{*}} F_{4}^{B_{u} \rightarrow D_{u}^{*}}\left(m_{K^{*}}^{2}\right)+a_{2} m_{D_{u}^{*}} f_{D_{u}^{*}} F_{4}^{B_{u} \rightarrow K^{*}}\left(m_{D_{u}^{*}}^{2}\right)\right]^{2} \\
& \times\left[\frac{\lambda\left(m_{B_{u}}^{2}, m_{D_{*}^{*}}^{2}, m_{K^{*}}^{2}\right)^{2}}{16 m_{D_{u}^{*}}^{2} m_{K^{*}}^{2}}\right]+\left[a_{1} m_{K^{*}} f_{K^{*}} F_{5}^{B_{u} \rightarrow D_{u}^{*}}\left(m_{K^{*}}^{2}\right)\right. \\
& +a_{2} m_{D_{u}^{*}} f_{D_{u}^{*}} F_{5}^{B_{u} \rightarrow K^{*}}\left(m_{D_{u}^{*}}^{2}\right]^{2}\left[\frac{\lambda\left(m_{B_{u}}^{2}, m_{D_{u}^{*}}^{2}, m_{K^{*}}^{2}\right)}{2}\right] \\
& +\left[a_{1} m_{K^{*}} f_{K^{*}} F_{3}^{B_{u} \rightarrow D_{u}^{*}}\left(m_{K^{*}}^{2}\right)+a_{2} m_{D_{u}^{*}} f_{D_{u}^{*}} F_{3}^{B_{u} \rightarrow K^{*}}\left(m_{D_{u}^{*}}^{2}\right)\right] \\
& \times\left[a_{1} m_{K^{*}} f_{K^{*}} F_{4}^{B_{u} \rightarrow D_{u}^{*}}\left(m_{K^{*}}^{2}\right)+a_{2} m_{D_{u}^{*}} f_{D_{u}^{*}} F_{4}^{B_{u} \rightarrow K^{*}}\left(m_{D_{u}^{*}}^{2}\right)\right] \\
& \left.\times\left(m_{B_{u}}^{2}-m_{D_{u}^{*}}^{2}-m_{K^{*}}^{2}\right)\left[\frac{\lambda\left(m_{B_{u}}^{2}, m_{D_{D^{*}}}^{2}, m_{K^{*}}^{2}\right)}{4 m_{D_{u}^{*}}^{2} m_{K^{*}}^{2}}\right]\right),  \tag{26}\\
& \Gamma\left(B_{q} \rightarrow D_{q}^{*} D_{q^{\prime}}^{*}\right) \\
& =\frac{G_{F}^{2}}{32 \pi m_{B_{q}}^{3}} m_{D_{q^{\prime}}^{*}}^{2}, f_{D_{q^{\prime}}^{*}}^{2} \lambda\left(m_{B_{q}}^{2}, m_{D_{q}^{*}}^{2}, m_{D_{q^{\prime}}^{*}}^{2}\right)^{\frac{1}{2}} \\
& \times\left(\left[F_{3}^{B Q_{q} \rightarrow D_{q}^{*}}\left(m_{D_{q^{\prime}}^{\prime}}^{2}\right)\right]^{2}\left[\frac{\lambda\left(m_{B_{q}}^{2}, m_{D_{q}^{*}}^{2}, m_{D_{q^{\prime}}^{*}}^{2}\right)}{4 m_{D_{q}^{*}}^{2} m_{D_{q^{\prime}}^{*}}^{2}}+3\right]\right. \\
& +\left[F_{4}^{B{ }_{q} \rightarrow D_{q}^{*}}\left(m_{D_{q^{\prime}}^{*}}^{2}\right)\right]^{2}\left[\frac{\lambda\left(m_{B_{q}}^{2}, m_{D_{q}^{*}}^{2}, m_{D_{q^{\prime}}^{*}}^{2}\right)^{2}}{16 m_{D_{q}^{*}}^{2} m_{D_{q^{\prime}}}^{2}}\right] \\
& +\left[F_{5}^{B_{q} \rightarrow D_{q}^{*}}\left(m_{D_{q^{*}}^{*}}^{2}\right)\right]^{2}\left[\frac{\lambda\left(m_{B_{q}}^{2}, m_{D_{q}^{*}}^{2}, m_{D_{q^{\prime}}^{*}}^{2}\right)}{2}\right]
\end{align*}
$$

$$
\begin{align*}
&+\left[F_{3}^{B_{q} \rightarrow D_{q}^{*}}\left(m_{D_{q^{\prime}}^{*}}^{2}\right)\right]\left[F_{4}^{B_{q} \rightarrow D_{q}^{*}}\left(m_{D_{q^{\prime}}^{*}}^{2}\right)\right] \\
&\left.\times\left(m_{B_{q}}^{2}-m_{D_{q}^{*}}^{2}-m_{D_{q^{\prime}}^{*}}^{2}\right)\left[\frac{\lambda\left(m_{B_{q}}^{2}, m_{D_{q}^{*}}^{2}, m_{D_{q^{\prime}}^{*}}^{2}\right.}{4 m_{D_{q}^{*}}^{2} m_{D_{q^{\prime}}^{*}}^{2}}\right]\right) \\
& \times\left|V_{c b} V_{c q^{\prime}}^{*} a_{1}-V_{t b} V_{q^{\prime}}^{*}\left[a_{4}+a_{10}\right]\right|^{2},  \tag{27}\\
& \Gamma\left(B_{d(s)} \rightarrow D_{d(s)}^{*} P(P=\pi, K)\right) \\
&= \frac{G_{F}^{2}}{128 \pi m_{B_{d(s)}}^{3} m_{D_{d(s)}^{*}}^{2}}\left|V_{c b} V_{u q^{\prime}}^{*}\right|^{2} a_{1}^{2} f_{P}^{2} \lambda\left(m_{B_{d(s)}}^{2}, m_{D_{d(s)}^{*}}^{2}, m_{P}^{2}\right)^{\frac{3}{2}} \\
& \times\left[F_{2}^{\left.B_{d(s)} \rightarrow D_{d(s)}^{*}\left(m_{P}^{2}\right)\right]^{2}},\right.  \tag{28}\\
& \Gamma\left(B_{u} \rightarrow\right.\left.D_{u}^{*} P(P=\pi, K)\right) \\
&= \frac{G_{F}^{2}}{32 \pi m_{B_{u}}^{3}}\left|V_{c b} V_{u q^{\prime}}^{*}\right|^{2} \lambda\left(m_{B_{u}}^{2}, m_{D_{u}^{*}}^{2}, m_{P}^{2}\right)^{\frac{3}{2}} \\
& \times\left(4 m_{D_{u}^{*}}^{2}\right. \\
&-4 a_{2}^{2} m_{D_{u}^{*}}^{2} f_{D_{u}^{*}}^{2}\left[f_{+}^{B_{u} \rightarrow P}\left(m_{D_{u}^{*}}^{2} a_{2} m_{D_{u}^{*}}^{*} f_{P} f_{D_{u}^{*}}^{2}+f_{+}^{B_{u} \rightarrow P}\left(m_{1}^{2} f_{P}^{2}\left[F_{2}^{*}\right) F_{2}^{B_{u} \rightarrow D_{u} \rightarrow D_{u}^{*}}\left(m_{P}^{2}\right)\right]^{2}\right)\right)  \tag{29}\\
& \Gamma\left(B_{q} \rightarrow\right.\left.D_{q}^{*} D_{q^{\prime}}\right) \\
&= \frac{G_{F}^{2}}{128 \pi m_{B_{q}}^{3} m_{D_{q}^{*}}^{2}} f_{D_{q^{\prime}}}^{2} \lambda\left(m_{B_{q}}^{2}, m_{D_{q}^{*}}^{2}, m_{D_{q^{\prime}}}^{2}\right)^{\frac{3}{2}}\left[F_{2}^{B_{q} \rightarrow D_{q}^{*}}\left(m_{D_{q^{\prime}}}^{2}\right)\right]^{2} \\
& \times\left|V_{c b} V_{c q^{\prime}}^{*} a_{1}-V_{t b} V_{t q^{\prime}}^{*}\left[a_{4}+a_{10}+R_{q^{\prime}}^{\prime}\left(a_{6}+a_{8}\right)\right]\right|^{2} \tag{30}
\end{align*}
$$

## 3. Numerical Analysis

This section encompasses our numerical analysis, comparison of our results with the predictions of the PQCD as well as the existing experimental data and discussion. The expressions of the amplitudes and decay widths depict that the main input parameters entering the expressions are Wilson coefficients presented in Sec. 2, elements of the CKM matrix, leptonic decay constants, Borel parameters $M_{1}^{2}$ and $M_{2}^{2}$ as well as the continuum thresholds $s_{0}$ and $s_{0}^{\prime}{ }^{28,29}$ In further numerical analysis, we choose the numerical values as presented in Tables 1-3. The Borel mass squares $M_{1}^{2}$ and $M_{2}^{2}$ and continuum thresholds $s_{0}$ and $s_{0}^{\prime}$ are auxiliary parameters, hence the physical quantities should be independent of them. The parameters $s_{0}$ and $s_{0}^{\prime}$, are determined from the conditions that guarantees the sum rules for form factors to have the best stability in the allowed $M_{1}^{2}$ and $M_{2}^{2}$ region. The working regions

Analysis of the $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) P$ and $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) V$ Decays 5855
Table 1. Values of the masses in MeV .39

| $m_{\pi}$ | $m_{K}$ | $m_{D^{ \pm}}$ | $m_{\bar{D}^{0}}$ | $m_{D_{s}}$ | $m_{K^{*}(892)}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 139.570 | $493.677 \pm 0.016$ | $1869.62 \pm 0.20$ | $1864.84 \pm 0.17$ | $1968.49 \pm 0.34$ | $891.66 \pm 0.26$ |
| $m_{D^{*} \pm}$ | $m_{\bar{D}^{* 0}}$ | $m_{D_{s}^{*}}$ | $m_{B^{ \pm}}$ | $m_{B^{0}}$ | $m_{B_{s}}$ |
| $2010.27 \pm 0.17$ | $2006.97 \pm 0.19$ | $2112.3 \pm 0.5$ | $5279.2 \pm 0.3$ | $5279.5 \pm 0.3$ | $5366.3 \pm 0.6$ |

Table 2. Values of the decay constants in MeV .

| $f_{\pi}$ Ref. 39 | $f_{K}$ Ref. 39 | $f_{D^{ \pm}}$Ref. 39 | $f_{\bar{D}^{0}}$ Ref. 39 | $f_{D_{s}}$ Ref. 39 | $f_{K^{*}}$ Ref. 44 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $130.7 \pm 0.46$ | $159.8 \pm 1.84$ | $222.6 \pm 19.5$ | $222.6 \pm 19.5$ | $294 \pm 27$ | $217 \pm 5$ |
| $f_{D^{*} \pm}$ Ref. 39 | $f_{\bar{D}^{* 0}}$ Ref. 39 | $f_{D_{s}^{*}}$ Ref. 45 | $f_{B^{ \pm}}$Ref. 39 | $f_{B^{0}}$ Ref. 39 | $f_{B_{s}}$ Ref. 46 |
| $230 \pm 20$ | $230 \pm 20$ | $266 \pm 32$ | $176 \pm 42$ | $176 \pm 42$ | $206 \pm 10$ |

Table 3. Values of the elements of the CKM matrix. ${ }^{39}$

| $\left\|V_{u d}\right\|$ | $\left\|V_{u s}\right\|$ | $\left\|V_{c d}\right\|$ | $\left\|V_{c s}\right\|$ | $\left\|V_{c b}\right\|$ |
| :---: | :---: | :---: | :---: | :---: |
| $0.97377 \pm 0.00027$ | $0.2257 \pm 0.0021$ | $0.230 \pm 0.011$ | $0.957 \pm 0.110$ | $0.0416 \pm 0.0006$ |
| $\left\|V_{u b}\right\|$ | $\left\|V_{t d}\right\|$ | $\left\|V_{t b}\right\|$ | $\left\|V_{t s}\right\|$ |  |
| $0.00431 \pm 0.00030$ | $0.0074 \pm 0.0008$ | $0.77 \pm 0.18$ | $0.0406 \pm 0.0027$ |  |

Table 4. The values of form factors $f_{ \pm}$at different values of $q^{2}$.

| $q^{2}$ | $m_{\pi \pm}^{2}$ | $m_{K}^{2} \pm$ | $m_{K^{*}(892)}^{2} \pm$ | $m_{D \pm}^{2}$ | $m_{D_{s}}^{2}$ | $m_{D^{*} \pm}^{2}$ | $m_{D_{S}^{*}}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{+}^{B+} \rightarrow \bar{D}^{0}\left(q^{2}\right)$ | $0.59 \pm 0.14$ | $0.60 \pm 0.15$ | $0.63 \pm 0.16$ | $0.86 \pm 0.22$ | $0.92 \pm 0.23$ | $0.96 \pm 0.24$ | $1.08 \pm 0.27$ |
| $f_{-}^{B+} \rightarrow \bar{D}^{0}\left(q^{2}\right)$ | $-0.20 \pm 0.05$ | $-0.21 \pm 0.05$ | $-0.22 \pm 0.06$ | $-0.38 \pm 0.10$ | $-0.44 \pm 0.11$ | $-0.49 \pm 0.12$ | $-0.69 \pm 0.17$ |
| $f_{+}^{B^{0}} \rightarrow D^{+}\left(q^{2}\right)$ | $0.58 \pm 0.15$ | $0.59 \pm 0.15$ | $0.63 \pm 0.17$ | $0.86 \pm 0.22$ | $0.92 \pm 0.22$ | $0.95 \pm 0.23$ | $1.08 \pm 0.27$ |
| $f_{-}^{B^{0}} \rightarrow D^{+}\left(q^{2}\right)$ | $-0.20 \pm 0.05$ | $-0.21 \pm 0.05$ | $-0.22 \pm 0.06$ | $-0.37 \pm 0.10$ | $-0.44 \pm 0.11$ | $-0.49 \pm 0.12$ | $-0.69 \pm 0.17$ |
| $f_{+}^{B_{s}^{0} \rightarrow D_{s}^{+}}\left(q^{2}\right)$ | $0.26 \pm 0.06$ | $0.27 \pm 0.06$ | $0.28 \pm 0.07$ | $0.35 \pm 0.09$ | $0.38 \pm 0.10$ | $0.39 \pm 0.10$ | $0.42 \pm 0.11$ |
| $f_{-}^{B}{ }_{s}^{0} \rightarrow D_{s}^{+}\left(q^{2}\right)$ | $-0.11 \pm 0.03$ | $-0.12 \pm 0.03$ | $-0.13 \pm 0.03$ | $-0.15 \pm 0.04$ | $-0.16 \pm 0.04$ | $-0.17 \pm 0.05$ | $-0.18 \pm 0.05$ |

for $M_{1}^{2}$ and $M_{2}^{2}$ as well as the values for continuum thresholds are determined in Refs. 28 and 29. Here, we choose the values $s_{0}=35 \pm 5 \mathrm{GeV}^{2}, s_{0}^{\prime}=7 \pm 1 \mathrm{GeV}^{2}$, $M_{1}^{2}=17.0 \pm 2.5 \mathrm{GeV}^{2}, M_{2}^{2}=7 \pm 1 \mathrm{GeV}^{2}$ from those working values for auxiliary parameters. The values of the form factors $f_{ \pm}$and $f_{0,1,2,3}$ at different values of $q^{2}$ which we need in the expressions for decay widths are presented in Tables 4 and 5 , respectively. Using the expressions for total decay widths, the values of branching fractions for $B_{q} \rightarrow D_{q} P, B_{q} \rightarrow D_{q}^{*} P, B_{q} \rightarrow D_{q} V$ and $B_{q} \rightarrow D_{q}^{*} V$ are found. We depict the values of the branching ratios in Tables 6-9. Here, we should

Table 5. The values of form factors $f_{0,1,2,3}$ at different values of $q^{2}$.

| $q^{2}$ | $m_{\pi \pm}^{2}$ | $m_{K}^{2} \pm$ | $m_{K^{*}(892)}^{2} \pm$ | $m_{D \pm}^{2}$ | $m_{D_{s}}^{2}$ | $m_{D^{*} \pm}^{2}$ | $m_{D_{S}^{*}}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{0}^{B^{+}} \rightarrow{\overline{D^{*}}}^{\text {( }}\left(q^{2}\right)$ | $0.76 \pm 0.19$ | $0.78 \pm 0.19$ | $0.80 \pm 0.20$ | $0.97 \pm 0.24$ | $1.01 \pm 0.25$ | $1.09 \pm 0.26$ | $1.13 \pm 0.27$ |
| $f_{1}^{B^{+}} \rightarrow{\overline{D^{*}}}^{\left(q^{2}\right)}$ | $0.62 \pm 0.15$ | $0.63 \pm 0.15$ | $0.67 \pm 0.16$ | $0.98 \pm 0.24$ | $1.08 \pm 0.26$ | $1.14 \pm 0.27$ | $1.27 \pm 0.29$ |
| $f_{2}^{B^{+}} \rightarrow{\overline{D^{*}}}^{\left(q^{2}\right)}$ | $0.90 \pm 0.22$ | $0.96 \pm 0.23$ | $0.99 \pm 0.24$ | $1.50 \pm 0.38$ | $1.60 \pm 0.40$ | $1.82 \pm 0.46$ | $2.00 \pm 0.50$ |
| $f_{3}^{B^{+}} \rightarrow{\overline{D^{*}}}^{\left(q^{2}\right)}$ | $-1.51 \pm 0.38$ | $-1.62 \pm 0.40$ | $-1.65 \pm 0.40$ | $-2.01 \pm 0.50$ | $-2.30 \pm 0.55$ | $-2.50 \pm 0.60$ | $-2.65 \pm 0.61$ |
| $f_{0}^{B^{0}} \rightarrow D^{*+}\left(q^{2}\right)$ | $0.76 \pm 0.19$ | $0.78 \pm 0.19$ | $0.81 \pm 0.20$ | $0.97 \pm 0.24$ | $1.02 \pm 0.25$ | $1.10 \pm 0.26$ | $1.13 \pm 0.27$ |
| $f_{1}^{B^{0}} \rightarrow D^{*+}\left(q^{2}\right)$ | $0.61 \pm 0.15$ | $0.63 \pm 0.15$ | $0.66 \pm 0.16$ | $0.98 \pm 0.24$ | $1.07 \pm 0.26$ | $1.14 \pm 0.27$ | $1.27 \pm 0.29$ |
| $f_{2}^{B^{0}} \rightarrow D^{*+}\left(q^{2}\right)$ | $0.90 \pm 0.22$ | $0.95 \pm 0.23$ | $0.99 \pm 0.24$ | $1.51 \pm 0.38$ | $1.61 \pm 0.40$ | $1.82 \pm 0.46$ | $2.01 \pm 0.50$ |
| $f_{3}^{B^{0}} \rightarrow D^{*+}\left(q^{2}\right)$ | $-1.52 \pm 0.38$ | $-1.62 \pm 0.40$ | $-1.66 \pm 0.40$ | $-2.01 \pm 0.50$ | $-2.31 \pm 0.55$ | $-2.51 \pm 0.60$ | $-2.65 \pm 0.61$ |
| $f_{0}^{B_{s}^{0}} \rightarrow D_{s}^{*+}\left(q^{2}\right)$ | $0.38 \pm 0.10$ | $0.40 \pm 0.11$ | $0.41 \pm 0.11$ | $0.58 \pm 0.15$ | $0.62 \pm 0.16$ | $0.67 \pm 0.17$ | $0.70 \pm 0.18$ |
| $f_{1}^{B}{ }_{s}^{0} \rightarrow D_{s}^{*+}\left(q^{2}\right)$ | $0.33 \pm 0.08$ | $0.36 \pm 0.08$ | $0.40 \pm 0.11$ | $0.67 \pm 0.17$ | $0.71 \pm 0.17$ | $0.74 \pm 0.18$ | $0.76 \pm 0.18$ |
| $f_{2}^{B}{ }_{s}^{0} \rightarrow D_{s}^{*+}\left(q^{2}\right)$ | $0.43 \pm 0.11$ | $0.47 \pm 0.12$ | $0.50 \pm 0.13$ | $0.81 \pm 0.21$ | $0.85 \pm 0.22$ | $0.88 \pm 0.23$ | $0.91 \pm 0.23$ |
| $f_{3}^{B}{ }_{s}^{0} \rightarrow D_{s}^{*+}\left(q^{2}\right)$ | $-0.67 \pm 0.17$ | $-0.69 \pm 0.17$ | $-0.72 \pm 0.18$ | $-1.29 \pm 0.32$ | $-1.42 \pm 0.34$ | $-1.49 \pm 0.35$ | $-1.55 \pm 0.36$ |

Table 6. Values for the branching ratio of $B_{q} \rightarrow D_{q} P$.

| $B_{q} \rightarrow D_{q} P$ | Present work | PQCD $^{30}$ | Expt. ${ }^{39}$ |
| :--- | :---: | :---: | :---: |
| $B^{ \pm} \rightarrow \bar{D}^{0} \pi^{ \pm}$ | $(5.95 \pm 1.95) \times 10^{-3}$ | $5.11_{-2.07-0.75-0.15}^{+2.95+0+0.43+0.15} \times 10^{-3}$ | $(4.92 \pm 0.20) \times 10^{-3}$ |
| $B^{ \pm} \rightarrow \overline{D^{0}} K^{ \pm}$ | $(4.31 \pm 1.52) \times 10^{-4}$ | $4.00_{-1.64-0.93-0.12}^{+2.35+0.63+0.12} \times 10^{-4}$ | $(4.08 \pm 0.24) \times 10^{-4}$ |
| $B^{ \pm} \rightarrow \overline{D^{0}} D^{ \pm}$ | $(3.44 \pm 1.22) \times 10^{-4}$ | - | $(4.80 \pm 1.00) \times 10^{-4}$ |
| $B^{ \pm} \rightarrow \bar{D}^{0} D_{s}{ }^{ \pm}$ | $(2.03 \pm 0.85) \times 10^{-2}$ | - | $(1.09 \pm 0.27) \%$ |
| $B^{0} \rightarrow \bar{D}^{ \pm} \pi^{\mp}$ | $(5.69 \pm 1.70) \times 10^{-3}$ | $2.69_{-1.17-0.73-0.08}^{+1.78+0.55+0.08} \times 10^{-3}$ | $(3.40 \pm 0.90) \times 10^{-3}$ |
| $B^{0} \rightarrow \bar{D}^{ \pm} K^{\mp}$ | $(3.53 \pm 1.23) \times 10^{-4}$ | $2.43_{-1.01-0.71-0.07}^{1.56+0.63+0.07} \times 10^{-4}$ | $(2.00 \pm 0.60) \times 10^{-4}$ |
| $B^{0} \rightarrow \bar{D}^{ \pm} D^{\mp}$ | $(2.87 \pm 0.89) \times 10^{-4}$ | - | $(1.90 \pm 0.60) \times 10^{-4}$ |
| $B^{0} \rightarrow \bar{D}^{ \pm} D_{s} \mp$ | $(8.88 \pm 2.82) \times 10^{-3}$ | - | $(6.50 \pm 2.10) \times 10^{-3}$ |
| $B_{s}^{0} \rightarrow D_{s}^{ \pm} \pi^{\mp}$ | $(1.42 \pm 0.57) \times 10^{-3}$ | $2.13_{-0.81-0.68-0.06}^{+1.14+0.69+0.06} \times 10^{-3}$ | $(3.80 \pm 0.30) \times 10^{-3}$ |
| $B_{s}^{0} \rightarrow D_{s}{ }^{ \pm} K^{\mp}$ | $(1.03 \pm 0.51) \times 10^{-4}$ | $1.71_{-0.65-0.55-0.05}^{+0.92+0.58+0.05} \times 10^{-4}$ | - |
| $B_{s}^{0} \rightarrow D_{s}^{ \pm} D^{\mp}$ | $(1.20 \pm 0.73) \times 10^{-4}$ | - | - |
| $B_{s}^{0} \rightarrow D_{s}^{ \pm} D_{s} \mp$ | $(2.17 \pm 0.82) \times 10^{-3}$ | - | - |

stress that, as we mentioned before, our results depicted in the tables are approximate results since we considered the observable only at the leading order of $\alpha_{s}$. To obtain more exact results the higher order $\alpha_{s}$ corrections should be taken into account. However, the presented uncertainties in the results are belong to the uncertainties in the values of the input parameters as well as variations in form factors

Table 7. Values for the branching ratio of $B_{q} \rightarrow D_{q}^{*} P$.

| $B \rightarrow D_{q}^{*} P$ | Present work | $\mathrm{PQCD}^{30}$ | Expt. $^{39}$ |
| :--- | :---: | :---: | :---: |
| $B^{ \pm} \rightarrow \bar{D}^{* 0} \pi^{ \pm}$ | $(4.89 \pm 1.52) \times 10^{-3}$ | $5.04_{-2.04-0.73-0.15}^{+2.92+0.44+0.15} \times 10^{-3}$ | $(4.60 \pm 0.40) \times 10^{-3}$ |
| $B^{ \pm} \rightarrow \bar{D}^{* 0} K^{ \pm}$ | $(3.38 \pm 1.04) \times 10^{-4}$ | $3.60_{-1.62-0.92-0.12}^{+2.33+0.62+0.12} \times 10^{-4}$ | $(3.70 \pm 0.40) \times 10^{-4}$ |
| $B^{ \pm} \rightarrow \bar{D}^{* 0} D^{ \pm}$ | $(2.57 \pm 0.88) \times 10^{-4}$ | - | - |
| $B^{ \pm} \rightarrow \bar{D}^{* 0} D_{s}{ }^{ \pm}$ | $(11.03 \pm 2.91) \times 10^{-3}$ | - | $(10.00 \pm 4.00) \times 10^{-3}$ |
| $B^{0} \rightarrow \bar{D}^{* \pm} \pi^{\mp}$ | $(3.45 \pm 1.75) \times 10^{-3}$ | $2.60_{-1.14-0.70-0.07}^{+1.73+0.53+07} \times 10^{-3}$ | $(2.76 \pm 0.21) \times 10^{-3}$ |
| $B^{0} \rightarrow \bar{D}^{* \pm} K^{\mp}$ | $(2.08 \pm 0.68) \times 10^{-4}$ | $2.37_{-0.99-0.69-0.07}^{+1.52+0.62+0.07} \times 10^{-4}$ | $(2.14 \pm 0.20) \times 10^{-4}$ |
| $B^{0} \rightarrow \bar{D}^{* \pm} D^{\mp}$ | $(3.14 \pm 1.46) \times 10^{-4}$ | - | $(9.30 \pm 1.50) \times 10^{-4}$ |
| $B^{0} \rightarrow \bar{D}^{* \pm} D_{s} \mp$ | $(8.69 \pm 2.88) \times 10^{-3}$ | - | $(8.80 \pm 1.60) \times 10^{-3}$ |
| $B_{s}^{0} \rightarrow D_{s}^{* \pm} \pi^{\mp}$ | $(2.11 \pm 0.73) \times 10^{-3}$ | $2.42_{-0.72+0.77-0.07}^{+1.12+0.78+0.07} \times 10^{-3}$ | - |
| $B_{s}^{0} \rightarrow D_{s}^{* \pm} K^{\mp}$ | $(1.59 \pm 0.67) \times 10^{-4}$ | $1.65_{-0.63-0.53-0.05}^{+0.90+0.56+0.05} \times 10^{-4}$ | - |
| $B_{s}^{0} \rightarrow D_{s}^{* \pm} D^{\mp}$ | $(0.30 \pm 0.11) \times 10^{-4}$ | - | - |
| $B_{s}^{0} \rightarrow D_{s}^{* \pm} D_{s}^{\mp}$ | $(2.54 \pm 0.57) \times 10^{-3}$ | - | - |

Table 8. Values for the branching ratio of $B_{q} \rightarrow D_{q} V$.

| $B_{q} \rightarrow D_{q} V$ | Present work | $\mathrm{PQCD}^{30}$ | Expt. ${ }^{39}$ |
| :--- | :---: | :---: | :---: |
| $B^{ \pm} \rightarrow \bar{D}^{0} K^{*}(892)^{ \pm}$ | $(2.90 \pm 0.88) \times 10^{-4}$ | $6.49_{-2.68-1.58-0.20}^{+3.86+0.12+0.20} \times 10^{-4}$ | $(6.30 \pm 0.80) \times 10^{-4}$ |
| $B^{ \pm} \rightarrow \bar{D}^{0} D^{* \pm}$ | $(5.61 \pm 1.88) \times 10^{-4}$ | - | $(4.60 \pm 0.90) \times 10^{-4}$ |
| $B^{ \pm} \rightarrow \bar{D}^{0} D_{s}^{* \pm}$ | $(7.01 \pm 2.09) \times 10^{-3}$ | - | $(7.20 \pm 2.60) \times 10^{-3}$ |
| $B^{0} \rightarrow \bar{D}^{ \pm} K^{*}(892)^{\mp}$ | $(3.20 \pm 1.15) \times 10^{-4}$ | $4.07_{-1.69-1.11-0.12}^{+2.61+0.94+0.12} \times 10^{-4}$ | $(4.50 \pm 0.70) \times 10^{-4}$ |
| $B^{0} \rightarrow \bar{D}^{ \pm} D^{* \mp}$ | $(8.18 \pm 2.84) \times 10^{-4}$ | - | - |
| $B^{0} \rightarrow \bar{D}^{ \pm} D_{s}^{* \mp}$ | $(9.23 \pm 2.67) \times 10^{-3}$ | - | $(8.60 \pm 3.40) \times 10^{-3}$ |
| $B_{s}^{0} \rightarrow D_{s}^{ \pm} K^{*}(892)^{\mp}$ | $(0.50 \pm 0.22) \times 10^{-4}$ | $3.02_{-1.16-0.91-0.10}^{+1.62+0.88+0.10} \times 10^{-4}$ | - |
| $B_{s}^{0} \rightarrow D_{s}^{ \pm} D^{* \mp}$ | $(1.07 \pm 0.59) \times 10^{-4}$ | - | - |
| $B_{s}^{0} \rightarrow D_{s}^{ \pm} D_{s}^{* \mp}$ | $(2.62 \pm 0.93) \times 10^{-3}$ | - | - |

which are related to the errors in determination of the auxiliary parameters namely, Borell mass parameters $M_{1}^{2}$ and $M_{2}^{2}$ and continuum thresholds $s_{0}$ and $s_{0}^{\prime}$. These tables also include a comparison of our results with the existing predictions of the PQCD as well as the experimental data. From these tables, we see a good consistency among two nonperturbative approaches and the experiment in order of magnitude. In many cases, the presented results out of order of magnitude from three approaches coincide especially, when we consider the uncertainties in the results. The best consistency between our results and predictions of the PQCD

Table 9. Values for the branching ratio of $B_{q} \rightarrow D_{q}^{*} V$.

| $B_{q} \rightarrow D_{q}^{*} V$ | Present work | $\mathrm{PQCD}^{30}$ | Expt. $^{39}$ |
| :--- | :---: | :---: | :---: |
| $B^{ \pm} \rightarrow \bar{D}^{* 0} K^{*}(892)^{ \pm}$ | $(5.07 \pm 2.61) \times 10^{-4}$ | $6.82_{-2.80-1.65-0.21}^{+4.14+1.22+0.21} \times 10^{-4}$ | $(8.30 \pm 1.50) \times 10^{-4}$ |
| $B^{ \pm} \rightarrow \bar{D}^{* 0} D^{* \pm}$ | $(0.11 \pm 0.07) \times 10^{-2}$ | - | $<1.1 \%$ |
| $B^{ \pm} \rightarrow \bar{D}^{* 0} D_{s}^{* \pm}$ | $(6.85 \pm 2.98) \times 10^{-2}$ | - | $2.20 \pm 0.70 \%$ |
| $B^{0} \rightarrow \bar{D}^{* \pm} K^{*}(890)^{\mp}$ | $(3.55 \pm 1.25) \times 10^{-4}$ | $4.88_{-2.08-1.41-0.15}^{+3.18+1.16+0.15} \times 10^{-4}$ | $(3.30 \pm 0.60) \times 10^{-4}$ |
| $B^{0} \rightarrow \bar{D}^{* \pm} D^{* \mp}$ | $(8.78 \pm 2.50) \times 10^{-4}$ | - | $(8.30 \pm 1.01) \times 10^{-4}$ |
| $B^{0} \rightarrow \bar{D}^{* \pm} D_{s}^{* \mp}$ | $(8.17 \pm 2.93) \times 10^{-2}$ | - | $(1.79 \pm 0.16) \%$ |
| $B_{s}^{0} \rightarrow D_{s}^{* \pm} K^{*}(890)^{\mp}$ | $(1.63 \pm 0.86) \times 10^{-4}$ | $3.47_{-1.35-1.66-0.11}^{+1.96+1.07+0.11} \times 10^{-4}$ | - |
| $B_{s}^{0} \rightarrow D_{s}^{* \pm} D^{* \mp}$ | $(6.76 \pm 2.69) \times 10^{-4}$ | - | - |
| $B_{s}^{0} \rightarrow D_{s}^{* \pm} D_{s}^{* \mp}$ | $(2.77 \pm 0.76) \times 10^{-2}$ | - | $\left(23_{-13}^{+21}\right) \%$ |

is related to the $B_{s}^{0} \rightarrow D_{s}^{* \pm} K^{\mp}$ transition and $B_{s}^{0} \rightarrow D_{s}{ }^{ \pm} K^{*}(892)^{\mp}$ transition shows the biggest discrepancy between two methods. Our central value prediction on $B^{0} \rightarrow \bar{D}^{* \pm} K^{\mp}$ is approximately the same as the experimental result, however, the central experimental result on the branching ratio of $B_{s}^{0} \rightarrow D_{s}^{* \pm} D_{s}^{* \mp}$ depicts a big discrepancy comparing that of our prediction. The presented predictions from PQCD are related to the charm-charmless cases in the final states and we have no predictions on the charm-charm cases from this approach. In this approach, the wave functions of the participating mesons, which are available with higher order corrections, have been used to calculate the amplitudes. ${ }^{30}$ Therefore, over all agreement between our results and predictions of PQCD for charm-light cases in the final state and the experimental data for both charm-light and charm-charm cases, could be considered as a good test of the QCD factorization at leading order of $\alpha_{s}$ for related transitions. However, for exact comparison, much more efforts are needed in the future works, which may include the higher order corrections. Our results of some decay modes which have not been measured in the experiment can be tested in the future experiments at LHCb and other $B$ factories.

In conclusion, using the QCD factorization approach and taking into account the contributions of the current-current, QCD penguin and the electroweak penguin operators at the leading approximation, the decay amplitudes and decay widths of $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) P$ and $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right) V$ transitions were calculated in terms of the transition form factors of the $B_{q} \rightarrow D_{q}\left(D_{q}^{*}\right)$. Having computed those form factors in the framework of the three-point QCD sum rules in our previous works, the branching fraction for these decays were also evaluated. A comparison of our results with the predictions of the perturbative QCD as well as the existing experimental data was presented. Our results are over all in a good agreement with the predictions of the PQCD and the existing experimental data. Our predictions on some transitions, which have no experimental data can be checked by future experiments at LHCb or
other $B$ factories. To get more exact results from the QCD factorization method, higher order $\alpha_{s}$ corrections should be considered in the future works.

## Acknowledgments

The authors would like to thank T. M. Aliev and A. Ozpineci for useful discussions. One of the authors (K. Azizi) thanks Turkish Scientific and Research Council (TUBITAK) for their partial financial support provided under the project 108 T 502.

## References

1. BaBar Collab. (B. Aubert et al.), Nucl. Instrum. Methods A 479, 1 (2002).
2. Belle Collab. (S. Mori et al.), Nucl. Instrum. Methods A 479, 117 (2002).
3. M. Bauer, B. Stech and M. Wirbel, Z. Phys. C 34, 103 (1987).
4. M. Neubert and B. Stech, Adv. Ser. Direct. High Energy Phys. 15, 294 (1998).
5. A. Ali and C. Greub, Phys. Rev. D 57, 2996 (1998).
6. A. Ali, G. Kramer and C. D. Lu, Phys. Rev. D 58, 094009 (1998).
7. Y. H. Chen, H. Y. Cheng, B. Tseng and K. C. Yang, Phys. Rev. D 60, 094014 (1999).
8. M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999).
9. M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 591, 313 (2000).
10. M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 606, 245 (2001).
11. T. Muta, A. Sugamoto, M. Z. Yang and Y. D. Yang, Phys. Rev. D 62, 094020 (2000).
12. D. S. Du, D. S. Yang and G. H. Zhu, Phys. Lett. B 488, 46 (2000).
13. D. S. Du, H. U. Gong, J. F. Sun, D. S. Yang and G. H. Zhu, Phys. Rev. D 65, 074001 (2002).
14. M. Z. Yang and Y. D. Yang, Phys. Rev. D 62, 114019 (2000).
15. H. Y. Cheng and K. C. Yang, Phys. Rev. D 64, 074004 (2001).
16. H. Y. Cheng and K. C. Yang, Phys. Lett. $B$ 511, 40 (2001).
17. M. Diehl and G. Hiller, JHEP 0106, 067 (2001).
18. M. Beneke and M. Neubert, Nucl. Phys. B 651, 225 (2003).
19. D. S. Du, H. J. Gong, J.-F. Sun, D. S. Yang and G. H. Zhu, Phys. Rev. D 65, 094025 (2002).
20. D. S. Du, J. F. Sun, D. S. Yang and G. H. Zhu, Phys. Rev. D 67, 014023 (2003).
21. J. F. Sun, G. H. Zhu and D. S. Du, Phys. Rev. D 68, 054003 (2003).
22. M. Beneke and M. Neubert, Nucl. Phys. B 675, 333 (2003).
23. X. Q. Li, G. Lu and Y. D. Yang, Phys. Rev. D 68, 114015 (2003).
24. M. Beneke, J. Rohrer and D. Yang, Nucl. Phys. B 774, 64 (2007).
25. H. Y. Cheng and K. C. Yang, Phys. Rev. D 76, 114020 (2007).
26. H. Y. Cheng and K. C. Yang, arXiv:hep-ph/0805.0329.
27. J. Sun, Y. Yang, W. Du and H. Ma, Phys. Rev. D 77, 114004 (2008).
28. K. Azizi, Nucl. Phys. B 801, 70 (2008).
29. K. Azizi and M. Bayar, Phys. Rev. D 78, 054011 (2008).
30. R.-H. Li, C.-D. Lu and H. Zou, Phys. Rev. D 78, 014018 (2008).
31. C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D 70, 054015 (2004).
32. M. Beneke and Th. Feldmann, Nucl. Phys. B 685, 249 (2004).
33. G. Bell, Nucl. Phys. B 812, 264 (2009).
34. G. Bell, Nucl. Phys. B 795, 1 (2008).
35. G. Bell, arXiv:0907.1016.
36. G. Bell, arXiv:0902.1915.
37. G. Bell, arXiv:0705.3133.
38. V. Pilipp, Nucl. Phys. B 794, 154 (2008).
39. Particle Data Group (C. Amsler et al.), Phys. Lett. B 667, 1 (2008).
40. G. Buchalla, A. Buras and M. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).
41. A. Buras, M. Jamin and M. Lautenbacher, Nucl. Phys. B 400, 75 (1993).
42. M. Ciuchini et al., Nucl. Phys. B 415, 403 (1994).
43. N. Deshpande and X.-G. He, Phys. Lett. B 336, 471 (1994).
44. M. Neubert and B. Stech, to appear in Heavy Flavors, 2nd. edn., eds. A. J. Buras and M. Lindner (World Scientific, Singapore), preprint CERN-TH/97-99, arXiv:hepph/9705292.
45. P. Colangelo, F. De Fazio and A. Ozpineci, Phys. Rev. D 72, 074004 (2005).
46. J. Rolf, M. Della Morte, S. Durr, J. Heitger, A. Juttner, H. Molke, A. Shindler and R. Sommer, Nucl. Phys. B (Proc. Suppl.) 129, 322 (2004).
