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Exclusive $D_s \to (\eta, \eta')l\nu$ decays in light-cone QCD

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Abstract

Probing the $\bar{s}s$ content of the η and η' mesons and considering mixing between these states as well as gluonic contributions, the form factors responsible for semileptonic $D_s \to (\eta, \eta')l\nu$ transitions are calculated via light-cone QCD sum rules. Corresponding branching fractions and their ratio for different mixing angles are also obtained. Our results are in a good consistency with experimental data as well as predictions of other nonperturbative approaches.

1. Introduction

Based on experimental results, a considerable part of the total decay rate of the D_s meson is related to its decay to η and η' mesons. Therefore, the D_s is a proper meson to study the phenomenology of η and η' mesons and their structures. Due to charm quark, this meson plays an essential role in analyzing of the weak and strong interactions as well as exploring new physics beyond the standard model (SM) which will be probed by the Large Hadron Collider (LHC). The charmed systems are known for very small CP violations in the SM; hence, any detection of CP violations in such systems can be considered as a signal for the presence of new physics (for more information about the D_s meson and its decays see [1]).

In this work, we analyze the semileptonic $D_s \to (\eta, \eta')l\nu$ decays in the framework of light-cone QCD sum rules (LCSR). The η and η' mesons are mixing states [2, 3]:

$$\begin{aligned} |\eta\rangle &= \cos\varphi|\eta_q\rangle - \sin\varphi|\eta_s\rangle, \\ |\eta'\rangle &= \sin\varphi|\eta_q\rangle + \cos\varphi|\eta_s\rangle, \end{aligned} \tag{1}$$

where φ is a single mixing angle. The measured values of φ in the the quark flavor (QF) basis (for more information about this basis see for instance [4–7]) are $\varphi = (39.7 \pm 0.7)^\circ$ and $(41.5 \pm 0.3_{\mathrm{stat}} \pm 0.7_{\mathrm{syst}} \pm 0.6_{\mathrm{th}})^\circ$ with and without the gluonium content for η', respectively [8]. The mixing angle φ has also been obtained as $\varphi = [39.9 \pm 2.6(\mathrm{exp}) \pm 2.3(\mathrm{th})]^\circ$ by recently measured $BR[D(D_s) \to \eta(\eta') + \bar{l} + \nu_l]$ in the light-front quark model [9].

In QF basis,

$$|\eta_q\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle),$$
$$|\eta_s\rangle = |\bar{s}s\rangle. \tag{2}$$

Since the D_s meson decays into η and η' via η_s state, the transition form factors of these decays in the QF basis are written in terms of the transition form factors of $D_s \to \eta_s$ as

$$f_i^{D_s\to\eta} = -\sin\varphi \times f_i^{D_s\to\eta_s}, \qquad f_i^{D_s\to\eta'} = \cos\varphi \times f_i^{D_s\to\eta_s}. \tag{3}$$

For the calculation of $f_i^{D_s\to\eta^{(\prime)}}$ via the LCSR through $f_i^{D_s\to\eta_s}$, information about distribution amplitudes (DA's) of the $|\eta_s\rangle$ state as well as corresponding parameters are needed. These quantities have not been known yet, exactly. However, the same quantities for η meson are available and investigation of $f_i^{D_s\to\eta}$ is possible, directly. On the other hand, according to equation (3), there is a relation between $f_i^{D_s\to\eta}$ and $f_i^{D_s\to\eta'}$:

$$\frac{\left|f_i^{D_s\to\eta}(q^2)\right|}{\left|f_i^{D_s\to\eta'}(q^2)\right|} = \tan\varphi, \tag{4}$$

so, our strategy will be as follows. First, we will calculate the form factors, $f_i^{D_s\to\eta}$ via the LCSR, then using equation (4) and the values of the mixing angle φ, we will evaluate the transition form factors of $D_s \to \eta' l\nu$.

This paper is organized as follows. In the next section, we obtain the LCSR for the transition form factors responsible for $D_s \to \eta l\nu$ decay. Section 3 is devoted to the numerical analysis of the form factors and calculation of branching ratios of the $D_s \to (\eta, \eta')l\nu$ decays. We also compare the obtained results with the existing predictions of the other nonperturbative approaches as well as experimental data.

2. LCSR for $D_s \to \eta$ transition form factors

To calculate the transition form factors of $D_s \to \eta$ in the LCSR method, we consider the following correlation function:

$$\Pi_\mu(p, q) = \mathrm{i}\int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}qx}\langle\eta(p)|T\{\bar{s}(x)\gamma_\mu(1-\gamma_5)c(x)\bar{c}(0)\mathrm{i}(1-\gamma_5)s(0)\}|0\rangle, \tag{5}$$

where we will use the DA's of the η meson. The main reason for choosing the Chiral current, $\bar{c}\mathrm{i}(1-\gamma_5)s$ instead of the usual pseudoscalar (PS), $\bar{c}\mathrm{i}\gamma_5 s$ is to eliminate effectively the contribution of the twist-3 wavefunctions which are poorly known and cause the main uncertainties to the sum rules. This current provides results with less uncertainties (see also [10–14]). Here, we should stress that the Chiral current may enhance the next-to-leading order (NLO) twist-2 contribution and to get more exact results, one should use the DA's of the η mesons up to NLO which are not available yet.

According to the general philosophy of the QCD sum rules and its extension, light-cone sum rules, we should calculate the above correlation function in two different ways. In phenomenological or physical representation, it is calculated in terms of hadronic parameters. In QCD side, it is obtained in terms of DA's and QCD degrees of freedom. The LCSR for the physical quantities like form factors are acquired equating coefficient of the sufficient structures from both representations of the same correlation function through dispersion relation and applying Borel transformation and continuum subtraction to suppress the contributions of the higher states and continuum.

To obtain the phenomenological representation of the correlation function, we insert a complete set of D_s states between the currents. Isolating the pole term of the lowest PS D_s meson, we get

$$\Pi_\mu(p,q) = \frac{\langle \eta(p)|\bar{s}\gamma_\mu(1-\gamma_5)c|D_s(p+q)\rangle\langle D_s(p+q)|\bar{c}\mathrm{i}(1-\gamma_5)s|0\rangle}{m_{D_s}^2-(p+q)^2} + \cdots, \tag{6}$$

where $\cdots$ stands for the contributions of the higher states and continuum. The matrix element, $\langle D_s|\bar{c}\mathrm{i}(1-\gamma_5)s|0\rangle$ is defined as

$$\langle D_s|\bar{c}\mathrm{i}(1-\gamma_5)s|0\rangle = \frac{m_{D_s}^2 f_{D_s}}{m_c+m_s}, \tag{7}$$

where f_{D_s} is leptonic decay constant of D_s meson. The transition matrix element, $\langle \eta(p)|\bar{s}\gamma_\mu(1-\gamma_5)c|D_s(p+q)\rangle$ can be parameterized via Lorentz invariance and parity considerations as [11, 12]

$$\langle \eta(p)|\bar{s}\gamma_\mu(1-\gamma_5)c|D_s(p+q)\rangle = 2f_+^{D_s\to\eta}(q^2)p_\mu + \left(f_+^{D_s\to\eta}(q^2)+f_-^{D_s\to\eta}(q^2)\right)q_\mu, \tag{8}$$

where $f_\pm^{D_s\to\eta}(q^2)$ are transition-form factors responsible for $D_s \to \eta$ decay. Using equations (7) and (8) in equation (6), we obtain

$$\Pi_\mu(p,q) = \Pi_1(q^2,(p+q)^2)p_\mu + \Pi_2(q^2,(p+q)^2)q_\mu, \tag{9}$$

where,

$$\begin{aligned}\Pi_1 &= \frac{2f_+^{D_s\to\eta}(q^2)m_{D_s}^2 f_{D_s}}{(m_c+m_s)\left(m_{D_s}^2-(p+q)^2\right)} + \int_{s_0}^{\infty}\mathrm{d}s\frac{\rho_1^h(s)}{s-(p+q)^2} + \text{subtractions},\\ \Pi_2 &= \frac{\left(f_+^{D_s\to\eta}(q^2)+f_-^{D_s\to\eta}(q^2)\right)m_{D_s}^2 f_{D_s}}{(m_c+m_s)\left(m_{D_s}^2-(p+q)^2\right)} + \int_{s_0}^{\infty}\mathrm{d}s\frac{\rho_2^h(s)}{s-(p+q)^2} + \text{subtractions},\end{aligned} \tag{10}$$

where $\rho_{1,2}^h$ show the spectral densities of the higher resonances and the continuum in hadronic representation. These spectral densities are approximated by evoking the quark–hadron duality assumption,

$$\rho_{1,2}^h(s) = \rho_{1,2}^{\mathrm{QCD}}(s)\theta(s-s_0), \tag{11}$$

where, $\rho_{1,2}^{\mathrm{QCD}}(s) = \frac{1}{\pi}\,\mathrm{Im}\,\Pi^{\mathrm{QCD}}(s)$ are spectral densities in QCD side and s_0 is continuum threshold in the D_s channel.

The correlation function in QCD side, $\Pi^{\mathrm{QCD}}(s)$ is calculated by expanding the T product of the currents in (5) in terms of the DA's of the η meson with increasing twist in the deep Euclidean region, where $(p+q)^2 \ll 0$. After contracting out the c quark pair, we obtain

$$\Pi_\mu(p,q) = \mathrm{i}\int \mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}qx}\langle \eta|\bar{s}\gamma_\mu(1-\gamma_5)S_c(x)(1-\gamma_5)s(0)|0\rangle, \tag{12}$$

where, $S_c(x)$ is the full propagator of c quark.

The light-cone expansion of the quark propagator in the external gluon field is made in [15]. The propagator receives contributions from higher Fock states proportional to the condensates of the operators $\bar{q}Gq$, $\bar{q}GGq$, and $\bar{q}q\bar{q}q$. In this paper, we neglect contributions with two gluons as well as four quark operators due to the fact that their contributions are small [16]. In this approximation, the $S_c(x)$ is given as

$$\begin{aligned}S_c(x) = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}kx}\frac{\not{k}+m_c}{k^2-m_c^2} - \mathrm{i}g_s\int\frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}kx}\\ \times\int_0^1 \mathrm{d}u\left[\frac{1}{2}\frac{\not{k}+m_c}{\left(m_c^2-k^2\right)^2}G_{\mu\nu}(ux)\sigma^{\mu\nu} + \frac{1}{m_c^2-k^2}ux_\mu G^{\mu\nu}(ux)\gamma_\nu\right],\end{aligned} \tag{13}$$

where $G_{\mu\nu}$ is the gluonic field strength tensor and g_s is the strong coupling constant. We can rewrite equation (12) as

$$\Pi_\mu(p,q) = \frac{i}{4}\int d^4x\, e^{iqx}[\mathrm{Tr}\,\gamma_\mu(1-\gamma_5)S_c(x)(1-\gamma_5)\Gamma_i]\langle\eta|\bar{s}\Gamma^i s|0\rangle, \qquad (14)$$

where Γ^i is the full set of the Dirac matrices, $\Gamma^i = (I, \gamma_5, \gamma_\alpha, \gamma_\alpha\gamma_5, \sigma_{\alpha\beta})$. As is clear from equation (14), to proceed to calculate the theoretical side of the correlation function, we need to know the matrix elements of the nonlocal operators between vacuum and η meson states. Up to twist-4, the η meson DA's are defined as [17]

$$\begin{aligned}\langle\eta(p)|\bar{q}\gamma_\mu\gamma_5 q|0\rangle &= -i f_\eta p_\mu \int_0^1 du\, e^{-iupx}\left[\varphi_\eta(u) + \frac{1}{16}m_\eta^2 x^2 A(u)\right]\\ &\quad - \frac{i}{2} f_\eta m_\eta^2 \frac{x_\mu}{px}\int_0^1 du\, e^{-iupx} B(u),\end{aligned} \qquad (15)$$

$$\begin{aligned}\langle\eta(p)|\bar{q}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)q(0)|0\rangle &= f_\eta m_\eta^2\left[p_\beta\left(g_{\alpha\mu} - \frac{x_\alpha p_\mu}{px}\right) - p_\alpha\left(g_{\beta\mu} - \frac{x_\beta p_\mu}{px}\right)\right]\\ &\quad\times\int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i)\, e^{-ipx(\alpha_1+u\alpha_3)} + f_\eta m_\eta^2 \frac{p_\mu}{px}(p_\alpha x_\beta - p_\beta x_\alpha)\\ &\quad\times\int \mathcal{D}\alpha_i \varphi_\parallel(\alpha_i)\, e^{-ipx(\alpha_1+u\alpha_3)},\end{aligned} \qquad (16)$$

$$\begin{aligned}\langle\eta(p)|\bar{q}(x)g_s\tilde{G}_{\alpha\beta}(vx)\gamma_\mu q(0)|0\rangle &= i f_\eta m_\eta^2\left[p_\beta\left(g_{\alpha\mu} - \frac{x_\alpha p_\mu}{px}\right) - p_\alpha\left(g_{\beta\mu} - \frac{x_\beta p_\mu}{px}\right)\right]\\ &\quad\times\int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i)\, e^{-ipx(\alpha_1+u\alpha_3)} + i f_\eta m_\eta^2 \frac{p_\mu}{px}(p_\alpha x_\beta - p_\beta x_\alpha)\\ &\quad\times\int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i)\, e^{-ipx(\alpha_1+u\alpha_3)},\end{aligned} \qquad (17)$$

where, $\tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\sigma\lambda}G^{\sigma\lambda}$ and $\mathcal{D}\alpha_i = d\alpha_1\, d\alpha_2\, d\alpha_3 \delta(1-\alpha_1-\alpha_2-\alpha_3)$. Since we use the chiral current, the twist-3 wavefunctions do not give any contribution. In equations (15)–(17), $\varphi_\eta(u)$ is the leading twist-2, $A(u)$ and part of $B(u)$ are two particle twist-4, $\varphi_\parallel(\alpha_i)$, $\varphi_\perp(\alpha_i)$, $\tilde{\varphi}_\parallel(\alpha_i)$, and $\tilde{\varphi}_\perp(\alpha_i)$ are three particle twist-4 DA's. Here we should stress that using the identity

$$\gamma_\mu\sigma_{\alpha\beta} = i(g_{\mu\alpha}\gamma_\beta - g_{\mu\beta}\gamma_\alpha) + \epsilon_{\mu\alpha\beta\rho}\gamma^\rho\gamma_5, \qquad (18)$$

and due to the parity invariance of strong interactions, the matrix element,

$$\langle\eta(p)|\bar{s}\gamma_\mu G^{\alpha\beta}(ux)\sigma_{\alpha\beta}s|0\rangle = 0, \qquad (19)$$

and has no contribution. For extracting the QCD or theoretical side of the correlation function, we insert the expression of the charm quark full propagator as well as the DA's of the η meson into equation (14) and carry out the Fourier transformation.

Now, we proceed to get the LCSR for our form factors equating the coefficients of the corresponding p_μ and q_μ structures from both phenomenological and QCD sides of the correlation function and applying Borel transform with respect to the variable $(p+q)^2$ in order to suppress the contributions of the higher states and continuum as well as eliminate the subtraction terms. As a result, the following sum rules for the form factors $f_+^{D_s\to\eta}$ and $f_+^{D_s\to\eta} + f_-^{D_s\to\eta}$ are obtained:

$$f_+^{D_s\to\eta}(q^2) = \frac{m_c^2 m_\eta^2 f_\eta}{2m_{D_s}^2 f_{D_s}} \mathrm{e}^{\frac{m_{D_s}^2}{M^2}} \left\{ \int_\delta^1 \frac{\mathrm{d}u}{u} \left(\frac{2\varphi_\eta(u)}{m_\eta^2} + \frac{3A(u)}{4uM^2} - \frac{m_c^2 A(u)}{2u^2 M^4} \right) \mathrm{e}^{\frac{-s(u)}{M^2}} \right.$$
$$+ 2\int_\delta^1 \mathrm{d}u \int_0^u \mathrm{d}t \frac{B(u)}{tM^2} \mathrm{e}^{\frac{-s(u)}{M^2}} - \int_\delta^1 \mathrm{d}u \int \mathcal{D}\alpha_i$$
$$\times \frac{8\varphi_\perp(\alpha_i) + 2\varphi_\parallel(\alpha_i) - 8\tilde{\varphi}_\perp(\alpha_i) - 2\tilde{\varphi}_\parallel(\alpha_i)}{k^2 M^2} \mathrm{e}^{\frac{-s(k)}{M^2}} + 4m_\eta^2 \int_\delta^1 \mathrm{d}u \int \mathcal{D}\alpha_i \int_0^k \mathrm{d}t$$
$$\left. \times \frac{\varphi_\perp(\alpha_i) + \varphi_\parallel(\alpha_i) - 2\tilde{\varphi}_\perp(\alpha_i) - 2\tilde{\varphi}_\parallel(\alpha_i)}{t^2 M^4} \mathrm{e}^{\frac{-s(t)}{M^2}} \right\}, \quad (20)$$

$$f_+^{D_s\to\eta}(q^2) + f_-^{D_s\to\eta}(q^2) = \frac{m_c^2 m_\eta^2 f_\eta}{m_{D_s}^2 f_{D_s}} \mathrm{e}^{\frac{m_{D_s}^2}{M^2}} \left\{ 2\int_\delta^1 \mathrm{d}u \int_0^u \mathrm{d}t \frac{B(u)}{t^2 M^2} \mathrm{e}^{\frac{-s(t)}{M^2}} - 4m_\eta^2 \int_\delta^1 \mathrm{d}u \int \mathcal{D}\alpha_i \right.$$
$$\left. \times \int_0^k \mathrm{d}t \frac{2\varphi_\perp(\alpha_i) + 2\varphi_\parallel(\alpha_i) - \tilde{\varphi}_\perp(\alpha_i) - \tilde{\varphi}_\parallel(\alpha_i)}{t^3 M^4} \mathrm{e}^{\frac{-s(t)}{M^2}} \right\}, \quad (21)$$

where, M^2 is the Borel parameter and

$$\begin{aligned} s(x) &= \frac{m_c^2 - q^2\bar{x} + m_\eta^2 x\bar{x}}{x}, \\ \bar{x} &= 1 - x, \\ k &= \alpha_1 + u\alpha_3, \\ \delta &= \frac{1}{2m_\eta^2}\left[\left(m_\eta^2 + q^2 - s_0\right) + \sqrt{\left(s_0 - m_\eta^2 - q^2\right)^2 - 4m_\eta^2\left(q^2 - m_c^2\right)}\right]. \end{aligned} \quad (22)$$

3. Numerical analysis

In this section, we numerical analyze the form factors, $f_\pm^{D_s\to(\eta,\eta')}(q^2)$, and calculate branching fractions of $D_s \to (\eta, \eta')l\nu$ decays and their ratio. We also compare the results of the considered observables with predictions of the other nonperturbative approaches as well as existing experimental data. As we mentioned before, using equation (4), the transition form factors of $D_s \to \eta' l\nu$ decay are calculated by the help of the transition form factors of $D_s \to \eta l\nu$ decay easily. Hence, we will discuss only the $f_\pm^{D_s\to\eta}(q^2)$ form factors. From the LCSR for these form factors, it follows that the main input parameters are the DA's of the η meson. The explicit expressions of the wavefunctions, $\varphi_\eta(u)$, $A(u)$, $B(u)$ and $\varphi_\parallel(\alpha_i)$, $\varphi_\perp(\alpha_i)$, $\tilde{\varphi}_\parallel(\alpha_i)$, and $\tilde{\varphi}_\perp(\alpha_i)$ as well as related parameters are given as [17]

$$\begin{aligned} &\varphi_\eta(u) = 6u\bar{u}\left(1 + a_2^\eta C_2^{\frac{3}{2}}(2u-1)\right), \\ &\tilde{\varphi}_\parallel(\alpha_i) = 120\alpha_1\alpha_2\alpha_3(v_{00} + v_{10}(3\alpha_3 - 1)), \\ &\varphi_\parallel(\alpha_i) = 120\alpha_1\alpha_2\alpha_3(a_{10}(\alpha_2 - \alpha_1)), \\ &\tilde{\varphi}_\perp(\alpha_i) = -30\alpha_3^2\left[h_{00}(1-\alpha_3) + h_{01}(\alpha_3(1-\alpha_3) - 6\alpha_2\alpha_1) + h_{10}\left(\alpha_3(1-\alpha_3) - \tfrac{3}{2}\left(\alpha_1^2 + \alpha_2^2\right)\right)\right], \\ &\varphi_\perp(\alpha_i) = 30\alpha_3^2(\alpha_1 - \alpha_2)\left[h_{00} + h_{01}\alpha_3 + \tfrac{1}{2}h_{10}(5\alpha_3 - 3)\right], \\ &B(u) = g_\eta(u) - \varphi_\eta(u), \\ &g_\eta(u) = g_0 C_0^{\frac{1}{2}}(2u-1) + g_2 C_2^{\frac{1}{2}}(2u-1) + g_4 C_4^{\frac{1}{2}}(2u-1), \end{aligned}$$

$$
\begin{aligned}
A(u) = 6u\bar{u}\Big[&\tfrac{16}{15} + \tfrac{24}{35}a_2^{\eta} + 20\eta_3 + \tfrac{20}{9}\eta_4 + \left(-\tfrac{1}{15} + \tfrac{1}{16} - \tfrac{7}{27}\eta_3 w_3 - \tfrac{10}{27}\eta_4\right)C_2^{\frac{3}{2}}(2u-1) \\
&+ \left(-\tfrac{11}{210}a_2^{\eta} - \tfrac{4}{135}\eta_3 w_3\right)C_4^{\frac{3}{2}}(2u-1)\Big] \\
&+ \left(-\tfrac{18}{5}a_2^{\eta} + 21\eta_4 w_4\right)\big[2u^3(10 - 15u + 6u^2)\ln u \\
&+ 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln\bar{u} + u\bar{u}(2 + 13u\bar{u})\big],
\end{aligned} \tag{23}
$$

where $C_n^k(x)$ are the Gegenbauer polynomials:

$$
\begin{aligned}
h_{00} &= v_{00} = -\tfrac{1}{3}\eta_4, \\
a_{10} &= \tfrac{21}{8}\eta_4 w_4 - \tfrac{9}{20}a_2^{\eta}, \\
v_{10} &= \tfrac{21}{8}\eta_4 w_4, \\
h_{01} &= \tfrac{7}{4}\eta_4 w_4 - \tfrac{3}{20}a_2^{\eta}, \\
h_{10} &= \tfrac{7}{4}\eta_4 w_4 + \tfrac{3}{20}a_2^{\eta}, \\
g_0 &= 1, \\
g_2 &= 1 + \tfrac{18}{7}a_2^{\eta} + 60\eta_3 + \tfrac{20}{3}\eta_4, \\
g_4 &= -\tfrac{9}{28}a_2^{\eta} - 6\eta_3 w_3.
\end{aligned} \tag{24}
$$

The constants in equations (23) and (24) were calculated at the renormalization scale $\mu = 1$ GeV^2 using QCD sum rules and are given as $a_2^{\eta} = 0.2$, $\eta_3 = 0.013$, $\eta_4 = 0.5$, $w_3 = -3$, and $w_4 = 0.2$.

The values of the other input parameters appearing in sum rules for form factors are: quark masses at the scale of about 1 GeV $m_s = 0.14$ GeV, $m_c = 1.3$ GeV [18], meson masses $m_\eta = 0.5478$ GeV, $m_{\eta'} = 0.9578$ GeV, $m_{D_s} = 1.9685$ GeV, $V_{cs} = 1.023 \pm 0.036$ [19] and $f_{D_s} = (0.274 \pm 0.013 \pm 0.007)$ GeV [20].

The sum rules for form factors also contain two auxiliary parameters, s_0 and M^2. The continuum threshold is not totally arbitrary but it depends on the energy of the first excited state. We choose, $s_0 = (6.5 \pm 0.5)$ GeV^2 (see also [21]). Now, we are looking for a working region for M^2, where according to sum rules philosophy, our numerical results be stable for a given continuum threshold s_0. The working region for the Borel mass parameter is determined requiring that not only contributions of the higher states and continuum effectively suppress, but also contributions of the DA's with higher twists are small. Our numerical analysis shows that the suitable region is 2.5 GeV$^2 \leqslant M^2 \leqslant 3.5$ GeV^2. The dependence of the form factors $f_+^{D_s\to\eta}$ and $f_-^{D_s\to\eta}$ on M^2 are shown in figure 1. This figure shows that the form factors weakly depend on the Borel mass parameter in its working region.

Now, we proceed to find the q^2 dependence of the form factors. It should be stressed that in the region, $q^2 \geqslant 1.4$ GeV^2, the applicability of the LCSR is problematic. In order to extend our results to the whole physical region, we look for a parametrization of the form factors such that in the region, $0 \leqslant q^2 \leqslant 1.4$ GeV^2, the results obtained from the above-mentioned parametrization coincide well with the light-cone QCD sum rules predictions. The most simple parametrization of the q^2 dependence of the form factors is expressed in terms of three parameters in the following form:

$$
f_\pm(q^2) = \frac{f_\pm(0)}{1 - \alpha\hat{q} + \beta\hat{q}^2}, \tag{25}
$$

where, $\hat{q} = q^2/m_{D_s}^2$. The values of the parameters, $f_\pm^{D_s\to\eta}(0)$, α and β are given in table 1. This table also contains predictions of the light-front quark model (LFQM) for $f_+^{D_s\to\eta}(0)$ for two sets (for details see [23]). The errors presented in this table are due to variation of the

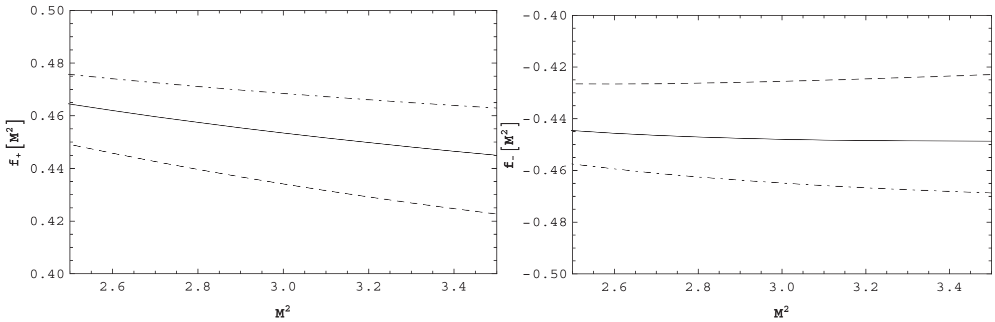

Figure 1. The dependence of the form factors on M^2 at $q^2 = 0$. The dashed, solid, and dashed–dotted lines correspond to the $s_0 = 5.5$ GeV^2, $s_0 = 6$ GeV^2, and $s_0 = 6.5$ GeV^2, respectively.

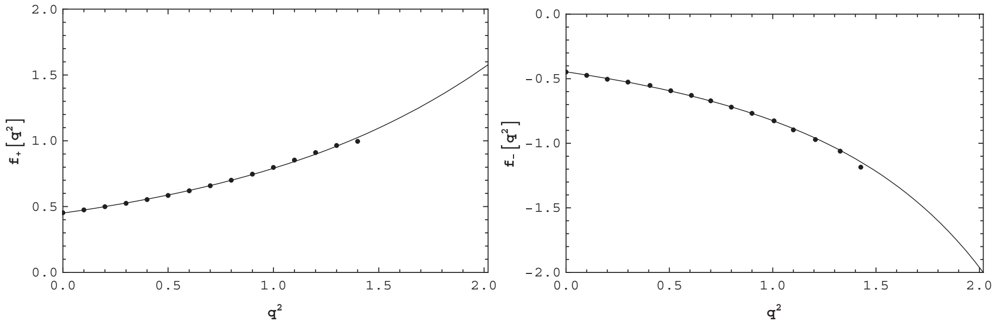

Figure 2. The dependence of the form factors of $D_s \to \eta$ on q^2. The circle points correspond to the values obtained directly from sum rules and the solid lines belong to the fit parametrization of the form factors.

Table 1. Parameters appearing in the fit function for form factors of $D_s \to \eta$ in two approaches.

Model	$f_-^{D_s\to\eta}(0)$	α	β
This work (LCSR)	-0.44 ± 0.13	2.05 ± 0.65	1.08 ± 0.35
	$f_+^{D_s\to\eta}(0)$	α	β
This work (LCSR)	0.45 ± 0.14	1.96 ± 0.63	1.12 ± 0.36
LFQM(I) [23]	0.50	1.17	0.34
LFQM(II) [23]	0.48	1.11	0.25

continuum threshold s_0, variation of the Borel parameter M^2, and uncertainties coming from the DA's and other input parameters.

The dependence of the form factors, $f_+(q^2)$ and $f_-(q^2)$ for $D_s \to \eta$ on q^2 extracted from the fit parametrization are shown in figure 2. This figure also contains the form factors obtained directly from our sum rules in the reliable region. We see that, the aforementioned fit parametrization describe well our form factors. The values of $f_+^{D_s\to(\eta,\eta')}(q^2)$ form factors at $q^2 = 0$ extracted from fit parametrization and using equation (4) are shown in table 2. Note that for massless leptons, the form factors, $f_-^{D_s\to(\eta,\eta')}(q^2)$, do not contribute to the decay rate formula, so we present only the $f_+^{D_s\to(\eta,\eta')}(q^2)$ in this table. For comparison, the predictions

Table 2. The $f_+^{D_s\to(\eta,\eta')}(q^2)$ form factors at $q^2 = 0$ in different approaches: this work (LCSR), three-point QCD sum rules (3PSR) and LFQM. Our results for $f_+^{D_s\to\eta'}$ correspond to $\varphi = 39.7^\circ(41.5^\circ)$.

Form factor	This work (LCSR)	3PSR [22]	LFQM(I) [23]	LFQM(II) [23]
$f_+^{D_s\to\eta}(0)$	0.45 ± 0.14	0.50 ± 0.04	0.50	0.48
$f_+^{D_s\to\eta'}(0)$	$0.55 \pm 0.18(0.51 \pm 0.16)$	–	0.62	0.60

Table 3. The branching ratios in different models and experiment. Our values correspond to $39.7^\circ(41.5^\circ)$.

Mode	This work	3PSP [22]	LFQM(I) [23]	LFQM(II) [23]	EXP [19]
$\mathrm{Br}(D_s \to \eta l \nu) \times 10^2$	3.15 ± 0.97	2.3 ± 0.4	2.42	2.25	2.9 ± 0.6
$\mathrm{Br}(D_s \to \eta' l \nu) \times 10^2$	$0.97 \pm 0.38(0.84 \pm 0.34)$	1.0 ± 0.2	0.95	0.91	1.02 ± 0.33

Table 4. The R_{D_s} with respect to mixing angle, φ for different models and experimental value.

Model	Angle (φ°)	R_{D_s}
This work (LCSR)	$39.7^\circ(41.5^\circ)$	$0.32 \pm 0.02(0.27 \pm 0.01)$
3PSR [22]	40°	0.44 ± 0.01
LFQM(I) [23]	39°	0.39
LFQM(II) [23]	39°	0.41
EXP [19]	–	0.35 ± 0.12

of the other approaches are also presented in this table. From this table, we see a good consistency among the results predicted by different approaches.

Now, we would like to evaluate the branching ratios for the considered decays. Using the parametrization of the transition matrix elements in terms of form factors, in massless lepton case, we get

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}(D_s \to (\eta, \eta')l\nu_l) = \frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s}^3}\left[\left(m_{D_s}^2 + m_{\eta^{(\prime)}}^2 - q^2\right)^2 - 4m_{D_s}^2 m_{(\eta,\eta')}^2\right]^{3/2}\left|f_+^{D_s\to\eta^{(\prime)}}(q^2)\right|^2, \tag{26}$$

where G_F is the Fermi constant. Integrating equation (26) over q^2 in the whole physical region and using the total mean lifetime, $\tau_{D_s} = (0.5 \pm 0.007)\,ps$ [19], the branching ratios of the $D_s \to (\eta, \eta')l\nu$ decays are obtained as presented in table 3.

This table also includes a comparison of our results and predictions of the other nonperturbative approaches including the LFQM and 3PSR and experimental values [19]. From this table, we see a good consistency between our results and predictions of the different approaches especially experimental data.

At the end of this section, we would like to compare also the ratio: $\mathrm{R}_{D_s} = \frac{\mathrm{Br}(D_s\to\eta' l\nu)}{\mathrm{Br}(D_s\to\eta l\nu)}$ in table 4 for different approaches as well as experimental value. This table also depicts a good consistency among the values, specially between our prediction with $\varphi = 39.7^\circ$ and experimental value. This can be considered as a good test for correctness of the considered internal structure for the D_s meson as well as the mixing angle between η and η' states.

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References

[1] Ryd A and Petrov A A 2009 arXiv:0910.1265 [hep-ph]
[2] Feldmann T, Kroll P and Stech B 1998 *Phys. Rev.* D **58** 114006
Feldmann T, Kroll P and Stech B 1999 *Phys. Lett.* B **449** 339
Feldmann T 2000 *Int. J. Mod. Phys.* A **15** 159
[3] De Fazio F and Pennington M R 2000 *J. High Energy Phys.* JHEP07(2000)051
[4] Choi H M 2010 *J. Phys. G: Nucl. Part. Phys.* **37** 085005
[5] Carlucci M V, Colangelo P and De Fazio F 2009 *Phys. Rev.* D **80** 055023
[6] Mathieu V and Vento V 2010 *Phys. Rev.* D **81** 034004
[7] Schechter J, Subbaraman A and Weigel H 1993 *Phys. Rev.* D **48** 339
[8] Ambrosino F *et al (KLOE Collaboration)* 2007 *Phys. Lett.* B **648** 267
[9] Ke H-W, Li X-Q and Wei Z-T 2010 *Eur. Phys. J.* C **69** 133
[10] Huang T, Li Z H and Wu X Y 2001 *Phys. Rev.* D **63** 094001
[11] Huang T, Li Z H and Zuo F 2009 *Eur. Phys. J.* C **60** 63
[12] Aliev T M, Kanik I and Ozpineci A 2003 *Phys. Rev.* D **67** 094009
[13] Aliev T M, Ozpineci A and Savci M 2004 arXiv:0401181 [hep-ph]
[14] Chernyak V L and Zhitnitsky I R 1990 *Nucl. Phys.* B **345** 137
[15] Balitsky I I and Braun V M 1989 *Nucl. Phys.* B **311** 541
[16] Braun V M and Filyanov I E 1990 *Z. Phys.* C **48** 239
[17] Ball P 1999 *J. High Energy Phys.* JHEP01(1999)010 (arXiv:hep-ph/9812375)
[18] Ioffe B L 2006 *Prog. Part. Nucl. Phys.* **56** 232
[19] Nakamura K *et al (Particle Data Group)* 2010 *J. Phys. G: Nucl. Part. Phys.* **37** 075021
[20] Artuso M *et al (CLEO Collaboration)* 2007 *Phys. Rev. Lett.* **99** 071802
[21] Colangelo P, De Fazio F and Ozpineci A 2005 *Phys. Rev.* D **72** 074004
[22] Colangelo P and De Fazio F 2001 *Phys. Lett.* B **520** 78
[23] Wei Z T, Ke H W and Yang X F 2009 *Phys. Rev.* D **80** 015022